Conference on Geometry and Algebra of Linear Matrix Inequalities
proposed by Didier Henrion and Monique Laurent
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Technical programme as of November 11, 2013

Tuesday November 12

9:30 - 9:45 Introduction

10:00 - 10:45 **Jean-Bernard Lasserre** (LAAS-CNRS and Inst. Math. Univ. Toulouse, France)
The moment-LP and moment-SOS hierarchies
We review basic properties of the moment-LP and moment-SOS hierarchies for polynomial optimization and compare them. We also illustrate how to use such a methodology in two applications outside optimization. Namely:

- for approximating (as closely as desired in a strong sense) sets defined with quantifiers of the form

$$\begin{aligned} R_f &= \{x \in B : f(x,y) \leq 0 \text{ for all } y \text{ such that } (x,y) \in K\} \\ D_f &= \{x \in B : f(x,y) \leq 0 \text{ for some } y \text{ such that } (x,y) \in K\} \end{aligned}$$

  by a hierarchy of *inner sublevel set approximations*

$$\Theta_k = \{x \in B : J_k(x) \leq 0\} \subset R_f,$$

  or *outer sublevel set approximations*

$$\Theta_k = \{x \in B : J_k(x) \leq 0\} \supset D_f,$$

  for some polynomials (J_k) of increasing degree;

- for computing *convex polynomial underestimators* of a given polynomial f on a box $B \subset \mathbb{R}^n$.

11:15 - 12:00 **Jean-Baptiste Hiriart-Urruty** (Univ. Paul Sabatier Toulouse, France)
How (even professional) mathematicians can make mistakes or fall into traps...
Mistakes are part of the progressing process in mathematics, as well as in science in general. It is generally not accepted that students can make mistakes or fall into traps, even less professional mathematicans. Mathematicians make mistakes, unconsciously of course. There are essentially two types of mistakes: those which are technical and are corrected in errata, and those, more profound, which give rise to new things. After having recalled a couple of historiacl examples or anecdotes, I shall give an example which happened to me and a co-author, where a mistake remained unnoticed (by referees, editors, other filters, etc.) until the end of the process of publication, but which happened to be useful for making a result more precise and pertinent.

12:30 - 14:00 Lunch

16:00 - 16:45 **Konstantin Avrachenkov** (INRIA Méditerannée Nice, France)
Singular perturbations in optimization
We consider linear and nonlinear mathematical programs where the data depends

on a perturbation parameter. We are interested in the behaviour of "solutions" as the perturbation parameter tends to zero. Our particular focus is on the case of singular perturbations when there is a loss of continuity or analyticity with respect to the perturbation parameter. Often at the limit the original ill-posed problem can be substituted by a reduced well-posed problem. For linear programming we propose an asymptotic simplex method, which finds not only a limiting optimal solution but also an asymptotic (uniformly) optimal solution.

17:15 - 18:00 **Rida Laraki** (LAMSADE-CNRS and Ecole Polytechnique Palaiseau, France)
Inertial game dynamics and applications to constrained optimization
We derive a class of inertial dynamics for games and constrained optimization problems over simplices by building on the well-known "heavy ball with friction" method. In the single-agent case, the dynamics are generated by endowing the problem's configuration space with a Hessian–Riemannian structure and then deriving the equations of motion for a particle moving under the influence of the problem's objective (viewed as a potential field); for normal form games, the procedure is similar, but players are instead driven by the unilateral gradient of their payoff functions. By specifying an explicit Nash–Kuiper embedding of the simplex, we show that global solutions exist if and only if the interior of the simplex is mapped isometrically to a closed hypersurface of some ambient Euclidean space, and we characterize those Hessian–Riemannian structures which have this property. For such structures, low-energy solutions are attracted to isolated minimizers of the potential, showing in particular that pure Nash equilibria of potential games are attracting; more generally, when the game is not a potential one, we establish an inertial variant of the folk theorem of evolutionary game theory, showing that strict Nash equilibria attract all nearby strategy profiles. Joint work with Panayotis Merticopoulos (CNRS, LIG).

18:30 - 19:15 **Frank Vallentin** (Univ. Köln, Germany)
Efficient distributions of points
How efficiently can one distribute points in a geometric space? This is a fundamental question in discrete geometry with applications in mathematics, information theory, physics and material science. Measures for efficiency are: packing density, covering density, energy, regularity. The geometric space can be the surface of a sphere, it can be the Euclidean space or a matrix group. In this talk I will introduce universal methods to solve these problems computationally but rigorously, with the help of convex optimization (SOS) and harmonic analysis. I will report on progress on the Kepler problem and the packing of general convex bodies.

19:30 - 21:00 Dinner

Wednesday November 13

9:00 - 9:45 **Bernd Sturmfels** (Univ. California at Berkeley, USA)
Quartic spectrahedra
We discuss ongoing work with John-Kristian Ottem, Kristian Ranestad and Cynthia Vinzant on the geometry of three-dimensional spectrahedra given by symmetric 4x4-matrices. Their boundaries are quartic surfaces with up to 16 nodes, of which any number between 0 and 10 can lie on the spectrahedron. Their planar shadows are defined by degree 12 curves with 46 singular points.

10:15 - 11:00 **Thorsten Theobald** (Goethe Univ. Frankfurt, Germany)
A semidefinite hierarchy for containment of spectrahedra
Polyhedra (or in the bounded cases polytopes) are the feasible regions of linear programs. Generalizing this notion, spectrahedra are defined as the feasible regions of semidefinite programs. In this talk, we report on joint work with Kai Kellner and Christian Trabandt on the computational question whether a given polytope or spectrahedron S_A (as given by the positive semidefiniteness region of a linear matrix pencil $A(x)$) is contained in another one S_B. In particular, we present and study a hierarchy of sufficient semidefinite conditions to certify the containment of a spectrahedron in another one. The hierarchical criterion is stronger than a solitary semidefinite criterion discussed earlier by Helton, Klep, and McCullough as well as by the authors. Moreover, several exactness results for the solitary criterion can be brought forward to the hierarchical approach.

11:30 - 12:15 **Stéphane Dauzère-Pérès** (Ecole des Mines de Saint-Etienne, France)
Recent advances on the integration of lot-sizing and scheduling
Jean-Bernard Lasserre proposed in the beginning of the 90s an original model and approach to tackle a rather general integrated lot-sizing and scheduling problem. The approach was first developed and validated in my PhD thesis, and later on extended in the PhD thesis of William Roux. The approach was however limited on some aspects, in particular in the size of the lot-sizing problems that could be solved. A novel model, solved by a Lagrangian relaxation heuristic, has been proposed in the PhD thesis of Cathy Wolosewicz. This approach has been again improved and extended in the PhD thesis of Edwin Gómez Urrutia.

12:30 - 14:00 Lunch

16:00 - 16:45 **Mathieu Claeys** (Univ. Cambridge, UK)
Occupation measures and semi-definite relaxations for optimal control
This talk shows how Lasserre's hierarchy of moment relaxations can be used to solve optimal control problems. This is done by relaxing the control problem as a generalized moment problem, to which the semi-definite relaxations can be applied. The approach is first applied to the impulsive control of linear time varying systems, by modeling the controls by a measure. The resulting semi-definite conditions circumvent time discretiziation and related difficulties. By the use of occupation measures, the method is then extended to a class of impulsive non-linear problems. This results in a monotone sequence of lower bounds to the original control problem. Finally, those results are transposed to switched system, by modeling each mode by a corresponding occupation measure. This allows for large computational gains with respect to the classical approach, where the control space is measured.

17:15 - 18:00 **Christophe Prieur** (Gipsalab-CNRS Grenoble, France)
Design of switching rules for linear systems of conservation laws
In this talk, the exponential stability in L^2-norm is considered for a class of switched linear systems of conservation laws. The state equations and the boundary conditions are both subject to switching. We consider the problem of designing stabilizing switching controllers. By means of Lyapunov techniques, three control strategies are developed based on steepest descent selection, possibly combined with a hysteresis and a low-pass filter. For these design methods, the assumption is given by a convex problem, written in terms of a linear matrix inequality. Some numerical results

are given to illustrate our approach, firstly by means an academic example with unstable modes without switch, and secondly for the stabilization of the Shallow Water fluid in a network of two reaches. This is a joint work with Pierre-Olivier Lamare and Antoine Girard (both from Laboratoire Jean Kuntzmann, Université de Grenoble).

18:30 - 19:15 **Emmanuel Trélat** (Univ. Pierre et Marie Curie, Paris, France)
Finite-dimensional predictor feedback stabilization of heat equations with boundary input delay
We consider a controlled heat equation, where the control acts on the boundary and is subject to a constant delay. Such a model is a paradigm for more general parabolic systems coupled with a transport equation. We prove that this is possible to stabilize (in H^1 norm) this process by means of an explicit feedback control that is designed from a finite-dimensional subsystem. The implementation is then very efficient and is based on standard tools of pole placement. The feedback acts on the system as a finite-dimensional predictor. We compare our approach with other methods based on LMI, which require however some specific assumptions, either on the system or on the delay. In contrast our approach does not require any specific assumption. This is an ongoing work with Christophe Prieur (Gipsalb-CNRS Grenoble).

19:30 - 21:00 Dinner

Thursday November 14

9:00 - 9:45 **Greg Blekherman** (Georgia Inst. Tech. Atlanta, USA)
Homogeneous truncated moment problem and positive rank
Let L be a linear functional on the vector space of homogeneous polynomials of degree $2d$ in n variables. The functional L comes from integration with respect to a measure if and only if L can be written as a positive linear combination of point evaluations. I will call the minimal number of point evaluations needed to express L the positive rank of L. I will discuss finding bounds on the positive rank in terms of the number of variables n and degree $2d$ and how these bounds can lead to algorithms for the truncated moment problem.

10:15 - 11:00 **Tim Netzer** (Univ. Leipzig, Germany)
The moment problem is still not completely solved
Within the last 20 years, great progress has been made concerning the moment problem for semialgebraic sets. Results of Schmüdgen apply to "small" sets, giving positive solutions to the moment problem. Results of Scheiderer et al. apply to "large" sets, showing that there is no finite solution to the moment problem. Whether sets are "small" or "large" is often not easy to decide. We first provide a method to decide the "size"-question for a large class of sets. But maybe more interesting is the fact that there exist sets which are neither "small" nor "large". To these kinds of sets, none of the existing results apply, and we thus cannot decide the moment problem so far.

11:30 - 12:15 **Pablo Parrilo** (MIT Boston, USA)
Approximation quality of SOS relaxations
Sums of squares (SOS) relaxations provide efficiently computable lower bounds for minimization of multivariate polynomials. Practical experience has shown that these

bounds usually outperform most other available techniques, but a fully satisfactory theoretical justification is still lacking. In this talk, we discuss several results (new and old) about the approximation quality of these SOS bounds, focusing on the case of polynomial optimization on the sphere

12:30 - 14:00 Lunch

16:00 - 16:45 **Martin Mevissen** (IBM Research Lab Dublin, Ireland)
Data-driven distributionally robust polynomial optimization
We consider robust optimization for polynomial optimization problems where the uncertainty set is a set of candidate probability density functions. This set is data-driven, i.e. it is a norm-ball around a density function estimated from data samples. We show that by employing polynomial and histogram density estimates, we can introduce robustness of polynomial programs with respect to distributional uncertainty sets without making the problem harder. We show that the distributionally robust counterpart is equivalent to a generalized problem of moments. We also give finite-sample consistency guarantees for the data-driven uncertainty sets. Finally, we apply our approach to a network optimization problem. Joint work with Emanuele Ragnoli, Jia Yuan Yu.

17:15 - 18:00 **Cordian Riener** (Aalto Univ. Helsinki, Finland)
Efficiently optimizing with symmetric polynomials
This talk will present some new results related to exploiting structure in the context of polynomial optimization with symmetric polynomials. We will show an efficient way of building a block diagonal moment matrix approach using harmonic polynomials. This will also shade some light on a theorem by Procesi and Schwarz. Further a generalization of the degree principle will be presented.

18:30 - 19:15 **Dima Pasechnik** (Univ. Oxford, UK)
A moment problem and secondary polytopes of nonconvex polyhedra
A generating function (known as a Fantappiè transformation) for moments of a constant density measure supported on a non-convex compact polyhedron P allow one to show, subject to mild extra assumptions, that P admits real-weighted triangulations on a well-defined invariant set of vertices (Steiner points are not needed). We attept to investigate such real-weighted triangulations by introducing an analog of secondary polytope for P.

19:30 - 21:00 Bouillabaisse

Friday November 15

9:00 - 9:45 **Rekha Thomas** (Univ. Washington Seattle, USA)
The Euclidean distance degree of an algebraic variety
It is a common problem in optimization to minimize Euclidean distance to a given set. In joint work with Jan Draisma, Emil Horobet, Giorgio Ottaviani and Bernd Sturmfels, we study the number of critical points on an algebraic variety to the squared Euclidean distance function, called the Euclidean distance degree of the variety. This is an intrinsic measure of the complexity of this polynomial optimization problem. Algebraic geometry offers powerful tools that give rise to formulas for this degree in many situations and applications.

10:15 - 11:00 **Tomás Prieto-Rumeau** (UNED Madrid, Spain)
Numerical approximations for average cost Markov decision processes
Given a discrete-time Markov control process with general state and action spaces, we propose numerical techniques to estimate an optimal policy and the optimal value. By sampling an underlying probability measure, we approximate the original control model with a controlled Markov process with finite state and action spaces. This control problem can be solved by means of two nested linear programming problems. Using suitable concentration inequalities, it is shown that the approximation errors decrease exponentially in probability.

11:30 - 12:15 **Sinai Robins** (Nanyang Tech. Univ., Singapore)
A Fourier approach to the Brion identities
We prove the Brion identities in the discrete geometry of polytopes by using Gaussians and Fourier techniques. In this way, the localization problems that usually arise in the non-convergent rational function approach are alleviated, and a more global analysis ensues. We show first how to prove the continuous Brion identity and then how to use it to prove the discrete Brion identity, using the Poisson summation formula.

12:30 - 14:00 Lunch

16:00 - 16:45 **Markus Schweighofer** (Univ. Konstanz, Germany)
Pure states and preorder membership
In 1926, Grete Hermann showed that ideal membership (in polynomial rings) is decidable. Today most students learn this from the theory of Gröbner bases. Ideals relate to equations. In the same way preorders (or more general quadratic modules) relate to inequalities. Except in very special cases, it is not known whether the membership problem for finitely generated preorders is decidable. In many concrete situations, one can however decide membership of a polynomial in a preorder by using the tool of pure states on ideals. Given such a membership problem, one can try to fit a geometric space to it in such a way that membership becomes equivalent to positivity on this space. We will try to outline the very basic ideas of this approach to the preorder membership problem. (Joint work with Sabine Burgdorf and Claus Scheiderer)

17:15 - 18:00 **Bernard Mourrain** (INRIA Méditerannée Nice, France)
Certified relaxation for polynomial optimization on semi-algebraic sets
About a decade ago, J.-B. Lasserre proposed a relaxation approach to compute the minimum of a polynomial function over a semi-algebraic set. He showed that the solutions of a hierarchy of SemiDefinite Programming (SDP) problems converge to this minimum and that in some cases the minimum is attained in a finite number of steps. In this talk, we consider a polynomial optimization problem over a general semi-algebraic set. Here are questions we may ask if we want to apply Lasserre's relaxation approach:

- Is the hierarchy of optimization problems that we consider exact, i.e. does it converge in a finite number of steps ?
- Is there a criterion to detect when the optimum is reached, that will eventually be satisfied ?

- When the convergence criterion is satisfied, how can we recover all the points where the optimum is achieved ?

We show that any relaxation hierarchy which "contains" the projection of the Karush-Kuhn-Tucker relaxation stops in a finite number of steps when the minimum is reached and the ideal defining the minimizers is generated by the kernel of the associated moment matrix in that degree. Assuming the minimizer ideal is zero-dimensional, we give a new criterion to detect when the minimum is reached and we prove that this criterion is satisfied for a sufficiently high degree. This exploits new representation of positive polynomials as elements of the preordering modulo the KKT ideal, which only involves polynomials in the initial set of variables. Some examples illustrate the method. (Joint work with M. Abril Bucero).

18:30 - 19:15 **Murray Marshall** (Univ. Saskatchewan, Saskatoon, Canada)
Application of localization to the multivariate moment problem
It is explained how the localization technique introduced by M. in 2003 leads to a useful reformulation of the multivariate moment problem in terms of extension of positive semidefinite linear functionals to positive semidefinite linear functionals on the localization of the real polynomial ring $R[x_1, \ldots, x_n]$ at $p = (1+x_1^2)\ldots(1+x_n^2)$ or $p' = (1+x_1^2)\ldots(1+x_{n-1}^2)$. It is explained how this reformulation can be exploited to prove new results concerning existence and uniqueness of the measure μ and density of the complex polynomial ring $C[x_1, \ldots, x_n]$ in the space of measurable functions $L^s(\mu)$, $1 \leq s < \infty$, and, at the same time, to give new proofs of old results of Fuglede, Nussbaum, Petersen and Schmüdgen, results which were proved previously using the theory of strongly commuting self-adjoint operators on Hilbert space.

19:30 - 21:00 Dinner

Saturday November 16

9:00 - 9:30 **Amir Ali Ahmadi** (IBM Watson Research Center, USA)
DSOS and SDSOS: more tractable alternatives to SOS
We present linear and second order inner approximations to the sum of squares cone and new hierarchies for polynomial optimization problems that are amenable to LP and SOCP and considerably more scalable than the SDP-based sum of squares approaches.

9:30 - 10:00 **Milan Korda** (EPFL Lausanne, Switzerland)
Region of attraction approximation for polynomial dynamical systems
We address the long-standing problem of computing the region of attraction (ROA) of a target set (typically a neighborhood of an equilibrium point) of a controlled nonlinear system with polynomial dynamics and semialgebraic state and input constraints. We show that the ROA can be computed by solving an infinite-dimensional convex linear programming (LP) problem over the space of measures. In turn, this problem can be solved approximately via a classical converging hierarchy of convex finite-dimensional linear matrix inequalities (LMIs). Our approach is genuinely primal in the sense that convexity of the problem of computing the ROA is an outcome of optimizing directly over system trajectories. The dual infinite-dimensional LP on

nonnegative continuous functions (approximated by polynomial sum-of-squares) allows us to generate a hierarchy of semialgebraic outer approximations of the ROA at the price of solving a sequence of LMI problems with asymptotically vanishing conservatism. This sharply contrasts with the existing literature which follows an exclusively dual Lyapunov approach yielding either nonconvex bilinear matrix inequalities or conservative LMI conditions. The approach is simple and readily applicable as the outer approximations are the outcome of a single semidefinite program with no additional data required besides the problem description. Joint work with Didier Henrion and Colin N. Jones.

10:30 - 11:00 **Simone Naldi** (LAAS-CNRS Toulouse and Univ. Pierre et Marie Curie Paris)
Real root finding for determinants of linear matrices
In our work we are interested in the study of algebraic varieties given as the zero locus of the determinant of a matrix whose entries are linear forms with rational coefficients. Our main contribution concerns the construction of efficient algo- rithms for finding real points in every connected component of this determinantal variety, starting from a geometric characterization of the problem. The resolution of this problem in a good complexity class, taking advantage of the geometric structure of the problem, is required in many scientific areas, like convex optimization and real convex algebraic geometry. Joint work with Didier Henrion and Mohab Safey El Din.

11:00 - 11:30 **Daniel Plaumann** (Univ. Konstanz, Germany)
Determinantal representations of hyperbolic curves via polynomial homotopy continuation
We study symmetric determinantal representations of hyperbolic forms in three variables and thus descriptions of certain three-dimensional cones by linear matrix inequalities. Such representations always exist by the Helton-Vinnikov theorem but are hard to compute in practice. In this talk, we will show how to use polynomial homotopy continuation to find numerical solutions. (Work in progress with Anton Leykin).

11:30 - 12:00 **João Gouveia** (Univ. Coimbra, Portugal)
Approximate cone factorizations and lifts of polytopes
Recently there has been a renewed interest on the relationship between efficient conic representations of a polytope and (exact) cone factorizations of its slack matrix. Since in practice, one only obtains numerical approximations of cone factorizations, the question of what do these approximations imply in terms of representing the polytope gains importance. In this talk we will present inner and outer convex approximations of a polytope obtained from approximate cone factorizations of a slack matrix of the polytope, and show that if the quality of the approximated factorization is good, so is the approximated lift. Joint work with Pablo Parrilo and Rekha Thomas.

12:15 - 14:00 Lunch