

M1 : LMI, Optimization and Polynomial Methods
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This is a course for graduate students or researchers with a background in linear control systems, linear algebra and convex optimization. The focus is on semidefinite programming (SDP), or optimization over linear matrix inequalities (LMIs), an extension of linear programming to the cone of positive semidefinite matrices. Since the 1990s, LMI methods have found numerous applications mostly in combinatorial optimization, systems control and signal processing.

Outline

The course starts with fundamental mathematical features of linear matrix inequalities:

- Part I.0: general introduction, course outline and material (0h30)
- Part I.1: historical developments of LMIs, convexity, cones, duality, semidefinite programming (2h30)

Then we cover latest achievements in semidefinite programming and real algebraic geometry:

- Part I.2: classification of convex semialgebraic sets that can be represented with LMIs, lift and project techniques (1h)
- Part I.2b: non-convex BMIs (0h30)
- Part I.3: Shor's relaxation, polynomial moments and sum-of-squares, Lasserre's hierarchy of LMI relaxations to solve non-convex polynomial optimization problems, including BMIs (1h30)

For the labs, we use the YALMIP interface, SeDuMi and PENBMI to define and solve LMI and BMI problems under the Matlab environment (1h30). We also briefly survey recent developments in semidefinite solvers and software packages:

- Part I.4: basics of interior-point algorithms, latest achievements in software and solvers for LMIs and BMIs (1h30)

The end of the course focuses on the application of LMI techniques to solve several control problems traditionally deemed as difficult, such as robustness analysis of linear and nonlinear systems, or design of fixed-order robust controllers with H-infinity specifications. The originality of the approach is in the simultaneous use of algebraic or polynomial techniques (as opposed to classical state-space methods) and modern convex optimization techniques:

- Part II.1 and Part II.2: State-space methods - stability analysis: Lyapunov stability, pole placement in LMI regions, uncertain systems and robustness analysis - controller design: H2, Hinf, robust state-feedback and output-feedback design (3h)
- Part II.3 and Part II.4: Polynomial methods - stability analysis: polynomials in control, robust stability of polynomials - controller design: robust fixed-order controller design (2h)



Didier Henrion received the "Diplome de Doctorat" (Ph.D. degree) from INSA in October 1999. Since 2000 Didier Henrion has been a CNRS researcher at LAAS in the research group MAC under the scientific responsibility of Sophie Tarbouriech. He is also holding a secondary appointment as a researcher at the Katedra Ridici Techniky (Department of Automatic Control) of the Fakulta Elektrotechnicka (Faculty of Electrical Engineering) of the Ceske Vysoke Ucení Technike v Praze (CVUT, Czech Technical University in Prague) under the scientific responsibility of Michael Sebek. In 2001 he was awarded a two-year Young Researcher Grant from CNRS. In 2004 he was awarded the Bronze Medal from CNRS.

From 1999 to 2004 Didier Henrion was an Associate Editor at the Conference Editorial Board of the IEEE Control Systems Society. Since 2003 he has been an Associate Editor of the IFAC journal Automatica. Since 2005 he has been an Associate Editor of the EUCA European Journal of Control and IEEE Transactions on Automatic Control. Didier Henrion is chairing the IEEE Technical Committee on Computer Aided Control System Design, vice-chairing the IFAC Technical Committee on Control Design, and he is a member of the IFAC Technical Committees on Robust Control and Aerospace. He is also a regular member of the panel of reviewers of the AMS Mathematical Reviews and Zentralblatt MATH.