Polynomial LPV synthesis applied to turbofan engines

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Abstract

Results on polynomial fixed order controller design are extended to SISO gain-scheduling with guaranteed stability and H_∞ performance over the whole scheduling parameter range. Salient features of the approach are (a) the use of polynomials as modeling objects; (b) the use of flexible LMI conditions allowing polynomial dependence of the open-loop system and controller transfer functions in the scheduling parameters; and (c) the decoupling in the LMI conditions between the Lyapunov variables and the controller variables, allowing both parameter-dependent Lyapunov functions and fixed-order controller design. The synthesis procedure is integrated into the ATOL framework developed by the manufacturer of aircraft and space engines Snecma to systematically design reduced complexity gain-scheduled control laws for aircraft turbofan engines.

Key words: Turbofan engine control, linear parameter varying (LPV) systems, fixed-order controller design, linear matrix inequalities (LMI), polynomials

1 Introduction

Gain-scheduled control laws are widely applied to industrial plants whose dynamical properties strongly vary with the operating point or the environment. There exist various techniques for gain-scheduling, from ad hoc tuning procedures to more sophisticated techniques ensuring stability and performances over the whole operating range, see Leith and Leithead (2000); Rugh and Shamma (2000) for comprehensive surveys. In the latter category, linear parameter-varying (LPV) controller design via convex optimization over linear matrix inequalities (LMI) can be found. This methodology has found various applications especially in the aerospace industry. In particular, as shown recently in Balas (2002) or Bruzelius (2004) these techniques can succesfully be applied to control aircraft turbofan engines, which are characterized by a strong sensitivity of the system dynamics with respect to the flight envelope (power lever angle, aircraft speed, altitude) or the environment (inlet pressure, temperature).

The main drawbacks of currently available design procedures for LPV/LMI gain-scheduling are as follows:

- a linearizing change of variables is used to ensure convexity of the LMI design conditions, and the controller must be reconstructed using tedious linear algebra, see Scherer and Weiland (2000) for a good overview;
- the order of the controller must be equal to the order of the plant plus the order of the weighting functions, which is contradictory with the sometimes stringent reduced complexity specifications ubiquitous in embedded aerospace control systems;
- an appropriate state-space canonical form must be obtained over the whole operating range.

All these issues are explicitly and fairly discussed in the recent paper Wassink et al. (2005) where an LPV/LMI gain-scheduled control law is designed for a mechatronics system.

In this paper, an original gain-scheduling design procedure is proposed which is aimed at removing the above drawbacks:

- the controller variables appear explicitly as decision variables in the design LMI;
- the order of the controller as well as its structure are fixed from the outset, independently of the open-loop system order and weighting functions;
- the design conditions are directly formulated in the polynomial setting.

The main objective is to provide a relatively simple design methodology integrated into the ATOL control design framework developed by Snecma[1], see Vary and Reberga (2005).

Earlier works resulted in a collection of linearized models at various operating points, see Henrion et al. (2004), as well as LPV models for turbofan engine transfer functions, Reberga et al. (2005). In this paper, these models are used to validate the new gain-scheduling design procedure.

2 Problem statement

A continuous SISO open-loop plant is considered, modeled by a transfer function $G(s, \theta) = B(s, \theta)/\ A(s, \theta)$ where A and B are polynomials in the Laplace indeterminate $s \in \mathbb{C}$, both parametrized in a vector $\theta \in \mathbb{R}^p$, capturing the evolution of the model with respect to its environment. θ is assumed to be a time-varying vector of exogeneous variables that can be measured in real-time. It belongs to a semialgebraic set

$$\Theta = \{\theta \in \mathbb{R}^q : g_i(\theta) \geq 0,\ i = 1, ..r\} \tag{1}$$

where g_i are given multivariate polynomials.

[1] Snecma is, within the SAFRAN Group, dedicated to design, development and production of engines for civil aircrafts, military aircrafts, launch vehicles and satellites.

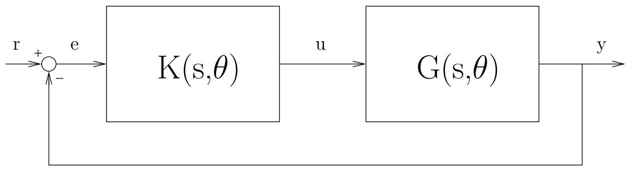

Fig. 1. Negative feedback configuration

Let $K(s, \theta) = Y(s, \theta)/X(s, \theta)$ be a controller depending also on θ. This controller is placed in a negative feedback configuration as shown in Figure 1. It is important to outline that neither the order of the controller nor its structure depend on the open-loop plant order or structure.

$M(s, \theta) = N(s, \theta)/D(s, \theta)$ denotes any closed-loop transfer function with this configuration. M can be the closed-loop transfer function $BY/(AX + BY)$ between reference signal r and output signal y, or the sensitivity transfer function $AX/(AX + BY)$ between r and error signal e. Note that in any case, M can be represented by two polynomials where the numerator and the denominator are both linear in controller polynomials X and Y.

With these notations, the problem to solve can be expressed as follows.

LPV design problem: Given open-loop plant $G(s, \theta)$, find controller $K(s, \theta)$ such that the closed-loop system $M(s, \theta)$ is stable and its H_∞ norm is less than a given bound γ, for all θ in Θ.

To be adressed, this LPV design problem is decomposed into three steps:

(1) Sufficient LMI conditions are derived to guarantee stability of a polynomial.

(2) These conditions are extended to robust stability of a polynomial affected by additive unstructured norm-bounded uncertainty.

(3) The conditions are further extended to the case of polynomials with coefficients depending polynomially on θ.

The approach presented in this paper relies on Lyapunov stability theory. It is described in this paper in a transfer function framework, whereas most of the references of the technical literature describe it in a state-space framework. The time-derivative of the Lyapunov function can be taken into account explicitly in the stability conditions. For simplicity reasons, a Lyapunov function which does not depend on the parameters (the so-called quadratic stability framework) is chosen in this paper. Hence, arbitrarily fast time-variation of the parameters are allowed. This is how the notion of LPV transfer function and LPV poles used in this paper should be interpreted: it is a formalism allowing time-varying parameters, even if it is more convenient to fix values of the parameters to use LTI analysis tools.

3 LPV problem

In the sequel, any polynomial $R(s) = \sum_{i=0}^{d} r_i s^i$ of degree d is associated to the vector of its coefficients $R = [r_0 \ r_1 \ \ldots \ r_d]$.

Consider a transfer function $M(s) = N(s)/D(s)$, with no dependence on parameter θ, where $D(s) = \sum_{i=0}^{d} d_i s^i$ and $N(s) = \sum_{i=0}^{d} n_i s^i$ are polynomials of degree d. $y_M(t)$ and $u_M(t)$ are defined as the output and input signals of this transfer function so that: $Y_M(s)/U_M(s) = N(s)/D(s)$, where $U_M(s)$ and $Y_M(s)$ are respectively the Laplace transforms of signals $u_M(t)$ and $y_M(t)$.

Sufficient conditions for stability of this transfer function are now given, based on Lyapunov's stability theory, see e.g. de Oliveira and Skelton (2001). An alternative proof based on polynomials positivity can be found in Henrion et al. (2003a,b).

Define a state-vector:

$x(t) = [y_M; dy_M/dt; \dots \; d^{d-1}y_M/(dt)^{d-1}]$ and

$\xi(t) = [x; d^d y_M/(dt)^d]$.

Let

$$\Pi_1 = \begin{bmatrix} 0 \; 1 & & \\ \vdots & \ddots & \\ 0 & & 1 \end{bmatrix}, \Pi_2 = \begin{bmatrix} 1 & & 0 \\ & \ddots & \vdots \\ & & 1 \; 0 \end{bmatrix}$$

be d-by-$(d+1)$ matrices so that: $x(t) = \Pi_2 \xi(t)$ and $\dot{x}(t) = \Pi_1 \xi(t)$. Consider a quadratic Lyapunov function V such that : $V(t) = x^*(t)Px(t) = \xi^*(t)\Pi_2^* P \Pi_2 \xi(t)$ and where the star denotes transposition. Finally, define the linear mapping $F(P) = -\Pi_1^* P \Pi_2 - \Pi_2^* P \Pi_1$.

Theorem 1 *Given a stable polynomial $C(s)$, polynomial $D(s)$ is stable if there exists a symmetric matrix P such that:*

$$C^*D + D^*C - F(P) \succeq 0$$

where $\succeq 0$ stands for positive semidefinite, i.e. all eigenvalues real nonnegative.

Polynomial $C(s)$ is called the central polynomial. It plays a key role, see section 4.1.

Proof of Theorem 1: It closely follows de Oliveira and Skelton (2001). First, for zero initial conditions, recall that asymptotic stability of the dynamical system described in the Laplace domain by the algebraic relation $D(s)Y_M(s) = N(s)U_M(s)$ depends only on the roots of polynomial $D(s)$, which must belong to the open left half-plane.

Following Lyapunov's second approach, asymptotic stability of this system is also equivalent to the sign conditions $V(t) > 0$ and $\dot{V}(t) \leq 0$ along system trajectories, i.e. for all vectors $\xi(t)$ such that $D\xi(t) = 0$. By construction of the linear mapping F, the time-derivative of the Lyapunov function is given by $\dot{V}(t) = -\xi^*(t)F(P)\xi(t)$. Hence the stability conditions can be summarized as:

$$\begin{aligned} V > 0 \quad &\Longleftrightarrow \quad \xi^* P \xi > 0, \\ \dot{V} \leq 0 \quad &\Longleftrightarrow \quad \xi^* F(P) \xi \leq 0, \\ \text{Along system trajectories} \quad &\Longleftrightarrow \quad D\xi = 0. \end{aligned}$$

Applying Finsler's Lemma to the last two quadratic inequalities, it follows that there exists a vector C (same dimensions as D) such that:

$$\begin{aligned} C^* D + D^* C - F(P) \succeq 0 \\ P \succ 0. \end{aligned} \quad (2)$$

The condition $P \succ 0$ can be removed since $C(s)$ is stable, see Henrion et al. (2003a). □

3.2 Discrete-time stability

Consider now a discrete-time SISO LTI transfer function $M_d(z) = N_d(z)/D_d(z)$, where $D_d(z) = \sum_{i=0}^{d} d_i z^i$ and $N_d(z) = \sum_{i=0}^{d} n_i z^i$ are polynomials of degree d of the shift operator z.

$\{y_M[k]\}$ and $\{u_M[k]\}$ are defined as sequences of output and input signals of this transfer function so that: $Y_M(z)/U_M(z) = N_d(z)/D_d(z)$, where $U_M(z)$ and $Y_M(z)$ are respectively the Z-transforms of $\{u_M[k]\}$ and $\{y_M[k]\}$.

Based on the same idea of the previous subsection, define a state-vector $x[k] = [\ y_M[k]\ y_M[k-1]\ ...\ y_M[k-d+1]]'$ and $\xi[k] = [\ x[k]\ y_M[k-d]]'$.

With these notations, $x[k] = \Pi_2\xi[k]$ and $x[k+1] = \Pi_1\xi[k]$. Consider a quadratic Lyapunov function V such that: $V[k] = x^*[k]Px[k] = \xi^*[k]\Pi_2^*P\Pi_2\xi[k]$.

The conditions ensuring asymptotic stability of the system are:

- $V[k] > 0$
- $V[k+1] - V[k] \leq 0$, along system trajectories.

The latter inequality is equivalent to:

$$\xi^*[k]\Pi_1^*P\Pi_1\xi[k] - \xi^*[k]\Pi_2^*P\Pi_2\xi[k] \leq 0$$

$$\Longleftrightarrow \xi^*[k]F_d(P)\xi[k] \leq 0$$

where $F_d(P) = \Pi_1^*P\Pi_1 - \Pi_2^*P\Pi_2$.

Finally, following the same approach as in the continuous-time case, Corollary 1 is obtained.

Corollary 1 *Given a stable polynomial $C_d(z)$, polynomial $D_d(z)$ is stable if* *there exists a symmetric matrix P such that:*

$$C_d^* D_d + D_d^* C_d - F_d(P) \succeq 0 \tag{3}$$

Discrete-time LMI (3) has the same form as the continuous-time LMI of Theorem 1. The only difference is the linear mapping $F_d(P)$. The rest of the approach remains the same. The following extensions to uncertainties and to the LPV case are also valid for discrete-time systems.

3.3 H_∞ performance

Suppose now that polynomial $D(s)$ is affected by an additive norm-bounded uncertainty: $D_\Delta(s) = D(s) + \Delta N(s)$, $\|\Delta\|_\infty \leq \gamma^{-1}$, where Δ is a matrix of unstructured uncertainty and γ is a given positive scalar.

According to the small-gain theorem, see e.g. Skogestad and Postlethwaite (2005), robust stability of polynomial $D_\Delta(s)$ is equivalent to the H_∞ performance constraint:

$$\|N(s)D^{-1}(s)\|_\infty < \gamma. \tag{4}$$

Theorem 2 *Given a stable polynomial $C(s)$, the transfer function $N(s)/D(s)$ is stable and satisfies the H_∞ performance constraint (4) if there exists a* *symmetric matrix P and a scalar λ such that:*

$$\begin{bmatrix} C^*D + D^*C - F(P) - \lambda C^*C & N^* \\ N & \lambda\gamma^2 I_n \end{bmatrix} \succeq 0. \tag{5}$$

Proof of Theorem 2: The proof is derived in two steps. Applying first Theorem 1 to uncertain polynomial $D_\Delta(s)$ yields the uncertain LMI

$$C^*(D + \Delta N) + (D^* + N^*\Delta^*)C - F(P) \succeq 0$$

or equivalently, the quadratic inequality

$$\xi^* \left(C^*(D + \Delta N) + (D^* + N^*\Delta^*)C - F(P)\right)\xi \geq 0$$

that must hold for all vectors ξ.

Denoting $z = \Delta^* C\xi$, this can be expressed as:

$$\begin{bmatrix} \xi \\ z \end{bmatrix}^* \begin{bmatrix} C^*D + D^*C - F(P) & N^* \\ N & 0 \end{bmatrix} \begin{bmatrix} \xi \\ z \end{bmatrix} \geq 0. \tag{6}$$

Moreover, the assumption on the complex uncertainty block $\|\Delta\|_\infty \leq \gamma$ can be rewritten as

$$\gamma^{-2} I - \Delta\Delta^* \succeq 0$$

or equivalently, as the quadratic inequality

$$\xi^* C^* C \xi - \gamma^2 z^* z \geq 0. \tag{7}$$

Then, applying, as in de Oliveira and Skelton (2001), the S-procedure on quadratic inequalities (6) and (7) yields robust LMI (5).□

LMI (5) is solved for a given scalar γ. A typical good value of γ is around 1.2, see Åström (2000).

3.4 LPV systems

Suppose now that polynomials $N(s)$ and $D(s)$ depend also polynomially on a vector of parameter θ defined as in Section 2. This dependence is denoted as follows:

$$D(s,\theta) = \sum_{\alpha \in \mathbb{N}^q} D_\alpha(s)\theta^\alpha, \quad N(s,\theta) = \sum_{\alpha \in \mathbb{N}^q} N_\alpha(s)\theta^\alpha$$

where $D_\alpha(s)$ and $N_\alpha(s)$ are polynomials of degree d in s.

Recall that polynomials $N(s)$ and $D(s)$ depend linearly on the coefficients of controller polynomials $X(s)$ and $Y(s)$, which are the decision variables. Let us gather these variables into a vector k, together with the entries of matrix P and scalar λ. Inequality (5) becomes an LMI in k polynomially parametrized in θ:

$$L(k,\theta) = \sum_\alpha L_\alpha(k)\theta^\alpha = \begin{bmatrix} C^*D(\theta) + D(\theta)^*C - F(P) - \lambda C^*C & N(\theta)^* \\ N(\theta) & \lambda\gamma^2 I_n \end{bmatrix} \succeq 0. \tag{8}$$

The problem here is to find parameter k ensuring positive semidefiniteness of this matrix inequality for all values of parameter θ in Θ. This is a parametrized LMI problem, a semi-infinite convex optimization problem which is difficult to solve in general. Several numerical methods have been proposed recently, see for instance the survey by Scherer (2006). The technique presented here is based on a matrix extension of a representation result proposed by Putinar for positive polynomials on semialgebraic sets.

The non-restrictive assumption that the semialgebraic set Θ is compact and that it is included in a ball of radius ρ centered around the origin is made, so that in the description (1) the redundant constraint $g_{r+1}(\theta) = \rho^2 - \|\theta\|_2^2 \geq 0$ can be added.

Lemma 1 *There exists k such that $L(k,\theta) \succ 0$ for all $\theta \in \Theta$ if and only if there exist sum-of-squares matrix polynomials $M_i(\theta)$ such that $L(k,\theta) = M_0(\theta) + \sum_{i=1}^{r+1} g_i(\theta)M_i(\theta)$.*

Proof of Lemma 1: See Theorem 24 in Scherer (2006), an extension to the matrix case of a scalar result by Putinar used for polynomial optimization in Lasserre (2001).□

The discrepancy between the strict inequality in Lemma 1 and the non-strict inequality (8) is not relevant numerically speaking since $L(k, \theta) \succ 0$ can be written as $L(k, \theta) \succeq \epsilon I$ for an arbitrarily small positive scalar ϵ.

Checking if a multivariate matrix polynomial of given degree is a sum-of-squares amounts to solving an LMI problem Scherer (2006), but the degree of the polynomials $M_i(\theta)$ is not known in advance. So, in practice, solving parametrized LMI (8) amounts to solving a hierarchy of LMI of increasing size, in the same spirit as in Lasserre (2001).

3.5 Remarks

Since the Lyapunov matrix P is chosen to be constant, LMI (8) ensure quadratic stability, see Scherer and Weiland (2000). It could also depend on θ. In that case, the polynomial matrix $L(k, \theta)$ in (8) is slightly modified but the rest of the approach is valid.

Although the approach is presented for SISO transfer functions, it is worth mentioning that it is still relevant for MIMO (Multiple Input Multiple Output) systems. The only additional assumption is to consider MIMO transfer functions with scalar denominators. Indeed, in the general MIMO case, as the product of matrices is not commutative, in a closed-loop transfer $M(s,\theta)$ the polynomial matrices $X(s)$ and $Y(s)$ of the controller do not typically enter affinely the numerator and the denominator. Choosing scalar denominators is non-restrictive: one can always take the least common multiple of denominators of the scalar transfer functions.

4 Application to a turbofan engine

This technique is now applied to the control of a turbofan engine. In Henrion et al. (2004); Reberga (2005) continuous-time LPV models of different loops of a turbofan were obtained. The models depend on the scalar variable $\theta = \mathrm{P}S32$, representing the pressure in the combustion chamber. In this paper, two of these models are used to study two particular transfers. The first one involves the fuel flow input WF32 and the compressor speed XN25. The second is between WF32 and the low pressure fan speed XN2. In both cases, the polynomial approach is applied to the sensitivity function of the closed-loop system. A low-pass filter is added in the design phase to get better performances.

The LMI problems were parsed by the YALMIP interface 3.0 of Löfberg (2001) which handles polynomial matrix sum-of-squares problems. Resulting semidefinite programs were solved by SeDuMi 1.1 of Sturm (1999) on Matlab 6.5. LPV controllers are validated on a nonlinear Simulink turbofan engine model provided by Snecma. Different points of the flight envelope (altitude and Mach number combinations) can be chosen to check robustness of the system. Due to saturation limits for WF32, a basic anti-windup scheme described in Gomes da Silva Jr. and Tarbouriech (2005) is added for the validations. For confidentiality reasons, the units and scales do not appear on the figures. Models and controllers equations are given for θ values normalised between $[-1, 1]$.

4.1 Central polynomial tuning

As mentioned in Henrion et al. (2003b), it is hard to give a general formula for the choice of the central polynomial. Somehow, it can be compared to the choice of the weighting filters of the mixed sensitivity problem, see Skogestad and Postlethwaite (2005) for example. The only necessary condition is to choose a stable polynomial.

It can be proven theoretically that $C(s)$ can be considered as a possible characteristic polynomial, see Yang et al. (2007). Figure 2 shows that there is a genuine link between the roots of the central polynomial and the poles of the closed-loop system: the poles of the LPV closed-loop system tend to be close to the roots chosen for $C(s)$. The 3 root locii correspond to the study of 3 different 4^{th} order closed-loop, obtained with a same model but with 3 different LPV controllers. The only difference in the design of the LPV controllers is the choice of the central polynomial. It is given on the top of each plot. Each of the root locii represents the migration of 4 closed-loop poles for various operating points, that is, various values of the scheduling parameter θ.

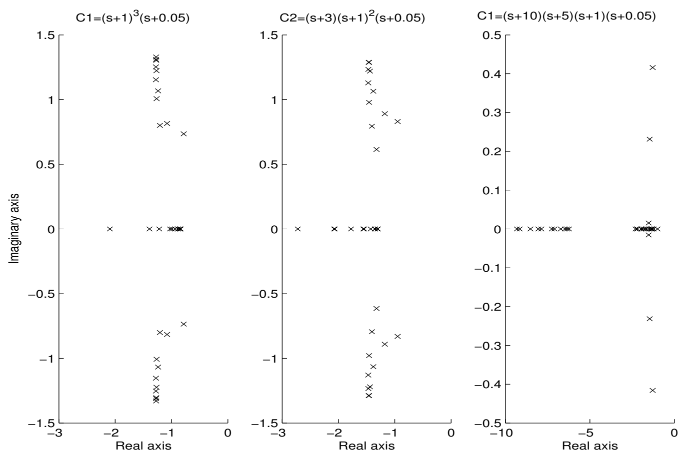

Fig. 2. Compared closed-loop system poles for various operating points and three different central polynomials.

In the case of an application, a good knowledge of the process is also useful. Keeping in mind the natural good dynamics of the model gives indications on the desired closed-loop system and so on the roots of the central polynomial.

Undoubtedly, these indications remain vague. As explained before, it is the designer's task to choose the adequate central polynomial. Since there is no criterion over $C(s)$ to optimize in the design LMI, several iterations are necessary to find out a consistent central polynomial. This is the key to a successful synthesis.

4.2 WF32-XN25 loop

The polynomial approach is applied to control the following second order model with an affine dependence on θ:

$$G_1(s,\theta) = \frac{(-0.06 + 0.06\theta)s^2 + (3.41 + 1.53\theta)s - 53.74\theta + 86.6}{s^2 + (47.98 + 3.72\theta)s + 27.88\theta + 47.54}$$

An LPV PI controller is sought, whose transfer function can be written:

$$K(s,\theta) = (k_{i1} + k_{i2}\theta)\frac{1}{s} + (k_{p1} + k_{p2}\theta). \tag{9}$$

With this controller, the order of the closed-loop system is equal to 3 plus the order of the filter, that is 4. Hence, choosing a 4^{th} order central polynomial equal to $C(s) = (s + 0.005)(s + 3.7)(s + 4)(s + 5)$, the LPV controller (10) is obtained.

$$K(s,\theta) = (0.76 + 0.43\theta)\frac{1}{s} + (0.56 + 0.02\theta) \tag{10}$$

Closed-loop simulations of the LPV system are shown on Figures 3 and 5. The corresponding variations of θ are respectively presented on Figure 4 and 6. Note that the values of θ are sampled on the Simulink simulator for implementation reasons.

Tracking is ensured by the integrator in the controller, and the time-response behavior is monitored by a suitable choice of $C(s)$.

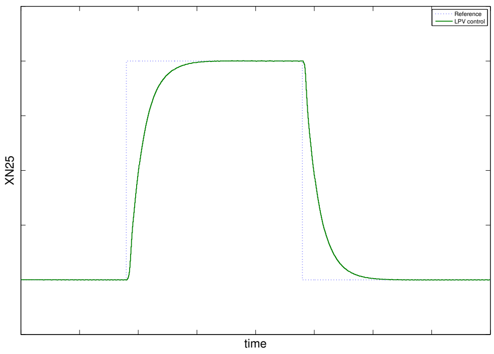

Fig. 3. Control of the WF32-XN25 loop. Step response. Low Altitude/ High Mach number.

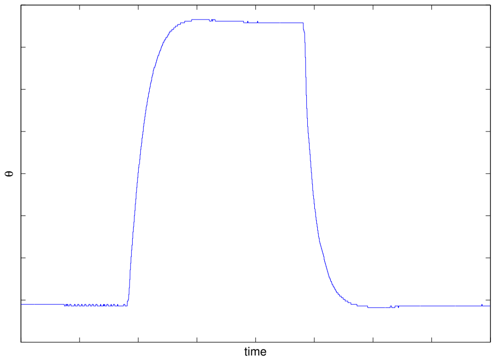

Fig. 4. Control of the WF32-XN25 loop. Evolution of the scheduling parameter θ - Low Altitude/ High Mach number.

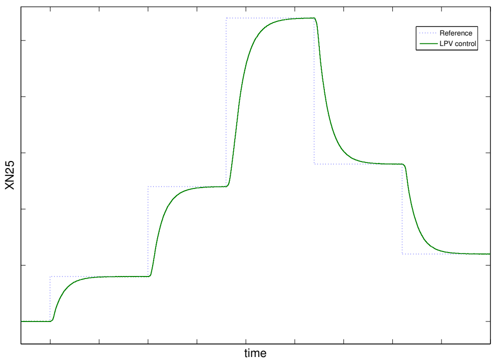

Fig. 5. Control of the WF32-XN25 loop. Step response. High Altitude/ Low Mach number.

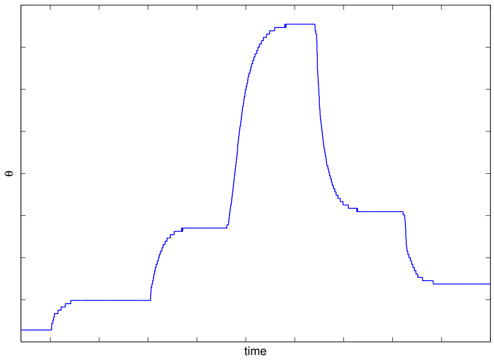

Fig. 6. Control of the WF32-XN25 loop. Evolution of the scheduling parameter θ - High Altitude/ Low Mach number.

4.3 WF32-XN2 loop

A first order open-loop LPV model of this loop with a second order dependence on θ is considered. It is defined by:

$$G_2(s,\theta) = \frac{(0.06 - 0.13\theta + 0.26\theta^2)s + 2.17 - 0.26\theta - 0.23\theta^2}{s + 1.2 + 0.9\theta - 0.11\theta^2}$$

It is controlled with a second order LPV controller, whose dependance on θ is affine. With the central polynomial $C(s) = (s + 0.005)(s + 2)(s + 3)(s + 28)$, the following controller is obtained:

$$K(s,\theta) = \frac{(16.13 + 1.89\theta) + (11.12 - 3.43\theta)s}{s(s + 11.8 + 1.055\theta)} \quad (11)$$

Figures 7 and 9 represents the closed-loop system for variations of parameter θ respectively given by Figures 8 and 10. To best evaluate the performances obtained with the LPV control, it is compared with the nominal control, that is the control currently applied by Snecma.

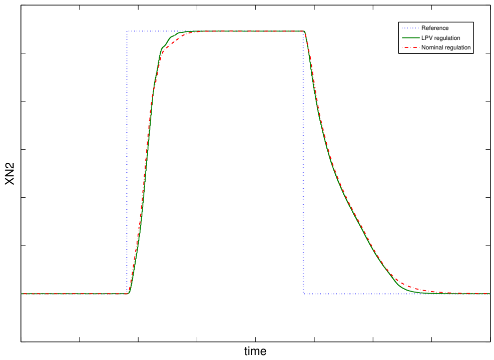

Fig. 7. Control of the WF32-XN2 loop. Step response. Ground conditions.

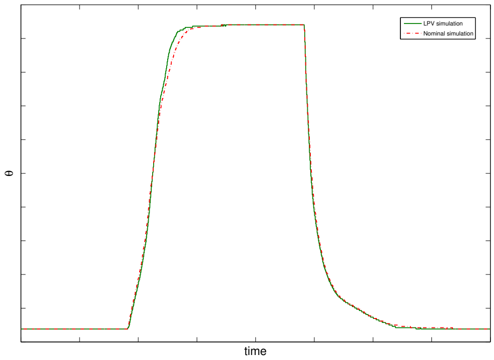

Fig. 8. Control of the WF32-XN2 loop. Evolution of the scheduling parameter θ - Ground conditions.

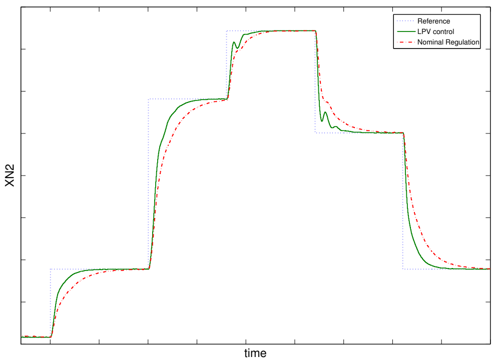

Fig. 9. Control of the WF32-XN2 loop. Step response. High Altitude/ High Mach number.

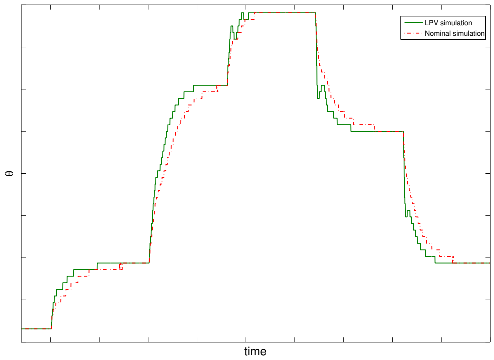

Fig. 10. Control of the WF32-XN2 loop. Evolution of the scheduling parameter θ - High Altitude/ High Mach number.

5 Conclusion

An extension of the fixed-order controller design procedure of Henrion et al. (2003b) to SISO gain-scheduling design with guaranteed H_∞ performance over the whole parameter range has been described. The idea of central polynomial, developed in Henrion et al. (2003b) in order to derive an LMI formulation, is used, with the following characteristics:

- the controller variables appear explicitly as decision variables in the design LMI, thus avoiding the use of tedious linear algebra to reconstruct controller parameters;
- the order and the structure of the controller can be fixed from the outset, independently of the open-loop system order and weighting functions;
- polynomial dependence of the open-loop system and controller transfer functions in the scheduling parameters are allowed;
- a decoupling between the Lyapunov variables and the controller variables

allows the use of parameter-dependent Lyapunov functions.

The gain-scheduling design procedure was validated in the scope of an on-going industrial research project on aircraft turbofan engine control. It is integrated within the ATOL software framework developed by Snecma, see Vary and Reberga (2005).

The main difficulty of the approach is of course the appropriate choice of the central polynomial. Roughly speaking, as explained in Henrion et al. (2003b), the central polynomial enforces the pole dynamics desired in closed-loop. For a given choice of a central polynomial, if the design LMI are infeasible, then it may mean that the desired closed-loop dynamics are not achievable. Unfortunately, it may also happen that the desired dynamics are achievable, but that the design LMI are too conservative to retrieve the appropriate controller. This is the main limitation of the whole approach.

As in the mixed sensitivity problem, several transfers can be considered simultaneously (sensitivity and mixed sensitivity functions for example). In that case, several LMIs like LMI (5) must be solved to find the LPV controller. Yet, this is completely different from the extension to the MIMO case. Applying a unique H_∞ constraint to a MIMO system is different from applying several H_∞ constraints to each of its transfers, see Skogestad and Postlethwaite (2005) for more details.

Besides, difficulties are not expected when extending the approach to MIMO systems, as soon as the linearity of the numerator and denominator polynomials in the controller coefficients is preserved.

Open issues and future research directions related with this work can be sketched as follows:

- better characterize the impact of the choice of the central polynomial on the achievable closed-loop performance;
- integrate time-domain constraints (overshoot, saturation) and/or combine with anti-windup design in the scope of aircraft turbofan engine control;
- study numerical aspects related with conditioning of the design LMI, study alternative choices of polynomial bases.

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