A review of the book "From vector spaces to function spaces - Introduction to functional analysis with applications" by Yutaka Yamamoto, SIAM, Philadelphia, 2012.

This is a very accessible account of modern functional analytic techniques aimed at graduate students, scientists and engineers. The author's motivation is to provide a conceptual understanding of key mathematical notions, and to motivate the introduction of technical concepts. The focus is more on the intuition behind the abstract mathematical notions rather than on the technical details and recipes used in the proofs. The most technical proofs are only sketched in the book, but the main ideas are described so that the interested and motivated reader can fill in the details.

The author is a renowned researcher in systems and control theory, and indeed it was optimal control (in the 1960s) and then robust control (in the 1990s) that largely contributed to the use and development of modern techniques of functional analysis in engineering. Consistently, the book culminates with a final chapter on fundamentals of linear system control which motivates much of the mathematical material described in the previous chapters.

The book can be structured into three parts. The first part (Chapters 1-5) deals with vector spaces and linear operators, following the classical pattern of topological spaces, Banach spaces, and then Hilbert spaces. Chapter 1 focuses on linearity while chapter 2 focuses on topology. Sections 2.4 and 2.5 offer a particularly transparent, elegant and short treatment of Banach's open mapping and closed graph theorems, as well as Baire's category theorem. Chapter 3 focuses on projections and best approximations in Hilbert spaces. Chapter 4 revisits duality and weak topologies, culminating with Alaoglu's theorem on the weak-star compactness of the unit ball, a central result of 20th century mathematics whose potential is not yet fully exploited in engineering circles, in this reviewer's opinion. Chapter 5 offers a basic self-contained account of linear operator theory.

The second part of the book (Chapters 6-8) describes the modern mathematical toolkit of systems engineers, namely distributions (Chapter 6, closely following the elementary account originally proposed by Laurent Schwartz), Fourier series and Fourier transform (Chapter 7) and Laplace transform (Chapter 8). This part gives a good overview of what is taught (or should be taught) as applied analysis in technical universities and engineering schools.

The third part (Chapters 9-10) contains more advanced material on Hardy spaces, the Nehari approximation problem, Sarason's theorem, Nehari's theorem, the Nevanlinna-Pick interpolation, and their application in H_∞ optimization. These are building blocks of an operator theoretical approach to systems control.

In summary, this book motivates the introduction of modern concepts of functional analysis in systems engineering, and it gives a nice overview and accessible account of the key results. The focus is preferably put on the ideas and the intuition, not so much on the (sometimes tedious) technical details, making this book a valuable reference for graduate students, scientists and engineers willing to reinforce their knowledge of basic functional analysis.

Didier Henrion, Toulouse, June 2013.