A review of the book "Primal on optimal control theory" by Jason L. Speyer and David H. Jacobson, SIAM, Philadelphia, 2010.

According to the authors, the objective of this book is to make optimal control theory accessible to a large class of engineers and scientists who are not mathematicians. The technical developments are presented using only elementary mathematics, at the level of the practitioning engineer and scientist. More specifically, the authors use exclusively linear algebra, elementary differential equation theory and elementary calculus. As a corollary, optimal control in its full generality cannot be treated in detail, and the book focuses on the simplest, yet important particular cases.

A specific feature of the book is a very clear distinction made between "weak control variations" (variations which are very small, but everywhere along the path) and "strong control variations" (variations which are zero over most of the path, but arbitrary large along a very short section). Indeed, after a Chapter 2 recalling elementary facts on finite-dimensional optimization (first order and second order optimality conditions, without and with constraints), Chapter 3 introduces weak control perturbation to come up with a weak form of first order necessary conditions of optimality, first for linear control systems, and then for nonlinear control systems. After that, strong control perturbations are introduced, culminating with the strong form of first order optimality conditions, which are referred to as Pontryagin's maximum principle (PMP). The Hamilton-Jacobi-Bellman equation providing sufficient conditions for optimality is then described, and solved for a valued function which is assumed to be continuously differentiable. In particular, it is explained how the gradient of the value function can be interpreted as a Lagrange multiplier in the PMP.

Chapter 4 describes a weak form of the PMP with linear terminal equality constraints, and linear dynamics, and it is argued that attempts to generalize this to nonlinear systems lead to technical difficulties which are out of the scope of this book. Chapter 5 focuses more on linear quadratic optimal control, where the dynamics and the terminal constraints are linear, and the performance criterion is a quadratic function of the state and the control. Chapter 6 then describes an extension of Chapter 5 to H_2 and H_∞ optimal control, with a classical game-theoretical interpretation: two competing policies must be designed, a first policy (a control signal) aiming at minimizing the cost criterion, and a second policy (a disturbance signal) aiming at maximizing it.

The merit of the book, as well as its main limitation, lies in the simplicity of the technical tools which are used. Unavoidably, this provides a limited account of the power and generality of optimal control theory. On the other hand, it gives a good idea of the technical difficulties to be expected to generalize further. As a complementary reading, this reviewer suggests the book [D. Liberzon. Calculus of variations and optimal control theory. A concise introduction. Princeton University Press, Princeton, NJ, 2012] which offers an alternative, maybe broader, but still elementary treatment of optimal control theory.

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