**A review of the book "Approximation methods for polynomial optimization: models, algorithms and applications" by Zhening Li, Simai He, Shuzhong Zhang, Springer Briefs in Optimization, Springer, New York, 2012.**

The book is an outgrowth of the first author's PhD thesis, defended in 2011. It is a well-written timely collection of state-of-the-art approximation algorithms for polynomial optimization problems $\max_{x \in X} p(x)$ where a real multivariate polynomial objective function $p$ is maximized on a (possibly nonconvex or discrete) set $X$ described by a finite number of polynomial equations and/or inequalities.

Approximation algorithms are potentially useful to obtain suboptimal solutions of difficult optimization problems for which optimal or almost optimal solutions cannot be found at reasonable computational cost. The authors are interested in polynomial-time algorithms (i.e. for which the running time is a polynomial function of the number of variables) yielding an approximation ratio, defined as a real number $\tau \in (0,1]$ such that a feasible solution $\hat{x} \in X$ satisfying $p(\hat{x}) \geq \tau \max_{x \in X} p(x) \geq 0$ can be found in polynomial time. In the case no approximation ratio can be found, it can still be useful to resort to approximation algorithms yielding a relative approximation ratio, defined as a real number $\tau \in (0,1]$ such that a feasible solution $\hat{x} \in X$ satisfying $p(\hat{x}) - \min_{x \in X} p(x) \geq \tau(\max_{x \in X} p(x) - \min_{x \in X} p(x))$ can be found in polynomial time. If it follows from the problem structure that $\min_{x \in X} p(x) \geq 0$, knowledge of a relative approximation ratio readily implies knowledge of an approximation ratio. As explained in section 1.4.2, an approximation ratio is not necessarily a relevant measure of performance of an approximation algorithm, or a measure of the practical difficulty of solving a given optimization problem. It conveys however some information on the problem at hand.

Chapter 2 deals with approximation algorithms when set $X$ is a Euclidean ball, with objective functions $p$ of increasing degree of complexity, starting with multilinear forms, then homogeneous forms, and eventually general inhomogeneous polynomials. Chapter 3 extends these results to more general sets $X$, namely hypercubes, Euclidean spheres, the intersection of centered ellipsoids, and convex sets for which outer and inner ellipsoidal approximations are available. As in the previous chapter, objective functions of increasing complexity are considered. Some motivating applications are briefly described in chapter 4, with references to the technical literature: approximation and decomposition of tensors in signal processing, approximation of densities in quantum physics, portfolio selection with higher moments, sensor network localization, and combinatorial optimization problems of graph theory.

The approximation algorithms rely on basic routines of randomized and deterministic eigenvalue decomposition of matrices, and, in the most intricate cases, on semidefinite programming (SDP) relaxations. All of the approximation results of the book are conveniently summarized and listed in table 5.1 for quick reference, with a unified nomenclature introduced in sections 1.3.1 and 1.3.2.

As mentioned by the authors in the conclusion, most of the approximation algorithms described in the book are new, and they can be considered as an alternative to the systematic hierarchy of SDP relaxations introduced in the early 2000s by Jean-Bernard Lasserre (for general polynomial optimization). Whereas the rate of convergence of Lasserre's hierarchy is still poorly understood (even for very specific problem classes) and its guarantee of performance is only asymptotic, the quality of the approximations produced by the algorithms described in the book can be quantified.

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