**A review of the book "Homogeneous polynomial forms for robustness analysis of uncertain systems" by Graziano Chesi, Andrea Garulli, Alberto Tesi and Antonio Vicino. Lecture Notes in Control and Information Sciences, vol. 390, Springer, Berlin, 2009.**

The book describes convex optimization techniques to deal with stability and performance evaluation for linear dynamical systems subject to parametric uncertainty. It summarizes approximately a decade of research activities, and most of the material in the book has been gathered from conference and journal publications by the authors. The originality of the authors' approach to system robustness is in the use of polynomial positivity conditions together with linear matrix inequalities (LMI). The reviewer and the second author of the book under review edited the first book on the topic [Positive polynomials in control, Lecture Notes in Control and Inform. Sci., 312, Springer, Berlin, 2005], and similar techniques have been used in the context of signal processing in [B. Dumitrescu, Positive trigonometric polynomials and signal processing applications, Springer, Dordrecht, 2007]. LMI formulations of polynomial positivity conditions can be seen as dual to LMI formulations of generalized problems of moments, and this primal/dual point of view, together with applications in many areas of engineering and applied mathematics, has been comprehensively surveyed in [J. B. Lasserre, Moments, positive polynomials and their applications, Imp. Coll. Press, London, UK, 2009].

The book starts with two mathematically oriented chapters on positivity of algebraic forms (multivariate homogeneous polynomials) and the existence of nonnegative forms which cannot be expressed as the sums of squares of other forms, known already to D. Hilbert at the beginning of the twentieth century. It is recalled that knowing whether a polynomial is the sum of squares (SOS) boils down to solving an LMI, or equivalently, a convex semidefinite programming problem, a generalization of linear programming to the cone of positive semidefinite matrices. Off-the-shelf solvers are available for that purpose, the most successful of which are implementations of primal-dual interior-point methods using logarithmic barrier functions.

The remaining chapters of the book focus on applications of the LMI formulation of polynomial SOS conditions to stability and performance assessment of linear systems subject to parametric (either affine or rational polytopic real-valued) uncertainty. The approach is mostly dual, in the sense that an energy-type certificate is expected in the form of a positive, coercive Lyapunov function whose time derivative is negative along systems trajectories. Homogeneous polynomial Lyapunov functions of increasing degree are sought by solving a hierarchy of embedded LMI relaxations. The primal (moment) formulation is used to guarantee optimality of a given LMI relaxation.

Chapter 3 deals with stability of uncertain time-varying linear systems and Lyapunov functions depending polynomially on the state, but not depending on the uncertain parameters. This allows for an arbitrarily fast time-variation of the uncertain parameters, such as, for example, fast discontinuous switching. Chapter 4, in contrast, deals with time-invariant uncertainty, and the relevant class of certificates is Lyapunov functions depending quadratically on the state and polynomially on the parameters. In both chapters, these choices are motivated by the availability of converse Lyapunov theorems showing that the robust stability implies the existence of such certificates with the expected dependence on the state and the parameters. The degree of dependence is generally unknown, and it can be very large. Chapter 5 combines the techniques of Chapters 3 and 4 by considering time-varying parameters with limited rates of variation, and therefore Lyapunov function being polynomial both in the state and in the parameters. Extensions to H-infinity performance and discrete-time systems are also described. The final chapter, Chapter 6, collects results on parametric stability margin computation and related nonconvex quadratic distance problems solved with convex LMI relaxations.

As already mentioned, the book is mostly a collection of results which were already published in the systems and control literature. The authors exploit the large combinatorics of problems (time-varying vs. time-invariant uncertainty, continuous vs. discrete, stability vs. performance, etc.) to enumerate, sometimes mechanically, many similar applications of LMI techniques. As a result, many sections of the book are repetitive and notationally dense. Moreover, the authors introduce their own terminology and notations to refer to concepts which are sometimes standard in other domains (e.g. in algebraic geometry). As far as I know, the authors are the only ones using these sometimes awkward conventions, and this does not ease the reading of the book. For example, the "complete square matricial representation" of a quadratic form is often called the Gram matrix, and a "homogeneous polynomial form" is often called an algebraic form, or simply a form in the mathematical literature. Most importantly, in the reviewer's opinion, most of the attention is focused on the dual problem (LMI formulation of polynomial positivity conditions) whereas the primal problem (LMI formulation of generalized problems of moments) is not discussed. However, it could have been instrumental to understanding the linear algebra procedure of Section 1.9 (used to certify optimality and/or exactness of an LMI relaxation, and already implemented in the Matlab toolboxes GloptiPoly and YALMIP). See the above-cited book by Lasserre for a more balanced treatment of these primal and dual aspects in the general setting of polynomial optimization.

In summary, the book is a welcome and timely addition to the technical literature, written by leading experts in the field. It is aimed at systems and control researchers willing to explore further than mainstream LMI Lyapunov analysis techniques, with the possibility of trading off between conservatism and computational burden.

Didier Henrion, Toulouse, January 2010.