Reduced LMIs for fixed-order polynomial controller design

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Abstract

A reduction procedure based on semidefinite programming duality is applied to LMI conditions for fixed-order scalar linear controller design in the polynomial framework. It is namely shown that the number of variables in the reduced design LMI is equal to the difference between the open-loop plant order and the desired controller order. A standard linear system of equations must then be solved to retrieve controller parameters. Therefore high computational load is not necessarily expected when the number of controller parameters is large, but rather when a large number of plant parameters are to be controlled with a small number of controller parameters. Tailored interior-point algorithms dealing with the specific structure of the reduced design LMI are also discussed.

Keywords: Linear systems, Fixed-order controller design, Polynomials, Linear matrix inequalities (LMI), Semidefinite programming (SDP), Interior-point methods.

1 Introduction

Recently, sufficient LMI conditions were described for fixed-order scalar linear controller design in a pure polynomial, or algebraic framework, based on polynomial positivity conditions [9]. It was shown that the fixed-order controller design conditions, which are non-convex in general, can be formulated as convex LMI conditions as soon as a design parameter, called central polynomial, is fixed. The central polynomial plays the role of a

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reference, or target closed-loop characteristic polynomial around which design is carried out. Extensive numerical experiments revealed that in most cases, the choice of the central polynomial is easy, depending both on the open-loop dynamics (e.g. non-minimum phase zeros) and the required closed-loop dynamics (e.g. damping of flexible modes).

The fixed-order design LMI conditions described in [9] have a particular structure which is reminiscent of the Kalman-Yakubovich-Popov (KYP) lemma. LMI decision variables can be gathered into two categories:

- a small number of controller parameters, which are coefficients of the numerator and denominator controller polynomials;
- a large number of additional parameters, which are entries of a Lyapunov-like matrix proving positivity of the LMI, and hence acting as a closed-loop stability certificate.

Controller parameters and additional parameters appear in a decoupled fashion in the LMI. As shown in [9] this decoupling can be exploited to reduce conservatism when performing robust controller design, but this is out of the scope of this note.

In the sequel we will show that the decoupling between controller and additional parameters can also be exploited to reduce significantly the number of decision variables in the LMI fixer-order controller design conditions. We use ideas on SDP duality and the KYP lemma originally described in [4, 5, 6, 13, 12]. We obtain dual LMI design conditions where the number of decision variables is considerably reduced, and is equal to the difference between the open-loop plant order and the desired controller order. Tailored interior-point algorithms are described and discussed for solving the reduced LMI problems and recovering corresponding multipliers. After solving the LMI, controller parameters can be retrieved from the multipliers by solving a standard linear system of equations.

2 LMI formulation of fixed-order controller design

We are seeking a controller $y(s)/x(s)$ of fixed order stabilizing a given open-loop plant $b(s)/a(s)$.

More specifically, given polynomials $a(s)$ and $b(s)$ of degree n, we must find polynomials $x(s)$ and $y(s)$ of degree m such that the closed-loop denominator polynomial

$$c(s) = a(s)x(s) + b(s)y(s)$$

has all its roots in a given stability region $\mathcal{D} = \{s : d_{11} + d_{21}s + d_{21}^\star s^\star + d_{22}s^\star s < 0\}$. The stability region is characterized by matrix

$$D = \begin{bmatrix} d_{11} & d_{21}^\star \\ d_{21} & d_{22} \end{bmatrix}.$$

Choose $d_{11} = 0$, $d_{21} = 1$, $d_{22} = 0$ for continuous-time, or Hurwitz stability, and $d_{11} = -1$, $d_{21} = 0$, $d_{22} = 1$ for discrete-time, or Schur stability. Other values correspond to arbitrary half-planes and disks, see [9]. When all roots of a polynomial lie within region $\mathcal{D}$, we say that the polynomial is $\mathcal{D}$-stable.

Let $p = n + m$ be the closed-loop system order, let

$$\Pi = \left[\begin{array}{cc} I_p & 0_{p\times 1} \\ 0_{p\times 1} & I_p \end{array}\right]$$

and define

$$D(P) = \Pi^\star (D \otimes P)\Pi = \sum_{j=1}^{p(p+1)/2} p_j D_j$$

where P is a Hermitian matrix of dimension p, with individual entries p_j.

In [9] a sufficient LMI condition for designing a fixed-order $\mathcal{D}$-stabilizing controller is given, based on the idea of a central polynomial. Central polynomial $d(s)$ is a reference, or target closed-loop denominator polynomial around which $c(s)$ is designed.

Let

$$\begin{array}{rcl} c & = & \left[\begin{array}{cccc} c_0 & c_1 & \cdots & c_p \end{array}\right] \\ & = & \underbrace{\left[\begin{array}{cccc|cccc} x_1 & x_2 & \cdots & x_{m+1} & x_{m+2} & x_{m+3} & \cdots & 1 \end{array}\right]}_{\left[\begin{array}{cc} x & 1 \end{array}\right]} \underbrace{\left[\begin{array}{ccccccc} b_0 & b_1 & \cdots & b_n & & & \\ 0 & b_0 & \cdots & b_{n-1} & b_n & & \\ & & \ddots & & & \ddots & \\ & & & b_0 & b_1 & \cdots & b_n \\ \hline a_0 & a_1 & \cdots & a_n & & & \\ 0 & a_0 & \cdots & a_{n-1} & a_n & & \\ & & \ddots & & & \ddots & \\ & & & a_0 & a_1 & \cdots & a_n \end{array}\right]}_{S} \end{array}$$

be the coefficient vector of polynomial $c(s) = c_0 + c_1 s + \cdots + c_p s^p$, an affine function of coefficients of controller polynomials $y(s) = x_1 + x_2 s + \cdots + x_{m+1} s^m$ and $x(s) = x_{m+2} + x_{m+3} s + \cdots + s^m$ (assumed to be monic). The $2m + 1$ controller coefficients are gathered into vector x, and matrix S is referred to as the Sylvester matrix of order m, with size $2(m + 1)$-by-$(p + 1)$. Let $d = [d_0 \quad d_1 \ \cdots \ d_p]$ be the coefficient vector of central polynomial $d(s) = d_0 + d_1 s + \cdots + d_p s^p$ and define

$$F(x) = F_0 + \sum_{i=1}^{2m+1} x_i F_i = d^\star c + c^\star d$$

as an affine map acting on controller coefficient vector x.

With these notations, the fixed-order controller design LMI conditions of [9] can be formulated as follows.

Lemma 1 *Given open-loop plant polynomials $a(s)$, $b(s)$ of degree n and a $\mathcal{D}$-stable central polynomial $d(s)$ of degree $p = n + m$, if LMI*

$$F(x) + D(P) \succ 0 \tag{1}$$

is feasible for vector x and Hermitian matrix P, then the corresponding controller polynomials $x(s)$, $y(s)$ of degree m ensure $\mathcal{D}$-stability of closed-loop polynomial $c(s) = a(s)x(s)+b(s)y(s)$.

It is observed that two categories of decision variables appear in a decoupled fashion in LMI (1), namely

- a small number $2m + 1$ of controller parameters, and
- a larger number $p(p + 1)/2$ of parameters in a KYP-like term ensuring $\mathcal{D}$-stability.

3 Dual LMI formulation

We reformulate LMI feasibility problem (1) as an LMI optimization problem

$$\begin{array}{rcll} \lambda^\star & = & \max & \lambda \\ & & \text{s.t.} & F(x) + D(P) \succeq \lambda I \end{array} \tag{2}$$

whose optimum $\lambda^\star$ is strictly positive if and only if LMI (1) is strictly feasible. There is always a strictly feasible point for LMI problem (2), so the duality gap is zero [1] and we can derive a dual formulation for LMI problem (2).

Lemma 2 *Primal LMI (1) is feasible if and only if $\lambda^\star > 0$ in dual LMI*

$$\begin{array}{rcll} \lambda^\star & = & \min & \text{trace}(F_0 Z) \\ & & \text{s.t.} & \text{trace}(F_i Z) = 0, \quad i = 1, \ldots, 2m + 1 \\ & & & \text{trace}(D_j Z) = 0, \quad j = 1, \ldots, p(p + 1)/2 \\ & & & \text{trace}\, Z = 1 \\ & & & Z \succeq 0. \end{array} \tag{3}$$

Hermitian matrix Z has dimension $p + 1$, so it has $(p + 1)(p + 2)/2$ individual entries.

4 Reduced dual LMI formulation

In dual LMI (3), the null-space constraints

$$\begin{array}{ll} \text{trace}(F_i Z) = 0, & i = 1, \ldots, 2m + 1 \\ \text{trace}(D_j Z) = 0, & j = 1, \ldots, p(p + 1)/2 \end{array} \tag{4}$$

can be written equivalently as

$$Z = \sum_{k=0}^{q} z_k N_k$$

where basis matrices N_k are Hermitian and span a linear subpsace of dimension $q+1$.

If poynomials $a(s)$ and $b(s)$ are coprime, Sylvester matrix S has full row rank, and there is generically no redundant equation in system (4). Then the dimension of the null-space is equal to $q+1$ where

- $q = (p+1)(p+2)/2 - p(p+1)/2 - (2m+1) - 1 = n - m - 1 \quad \text{if} \quad m \leq n$
- $q = 0 \quad \text{if} \quad m > n.$

It can be checked that N_0 can always be chosen as a matrix with all zeros but the bottom right entry which is equal to one, so that trace $Z = z_0 + \sum_{k=1}^{q} z_k \text{trace}\, N_k = 1$ and hence $\text{trace}(F_0 Z) = z_0 \text{trace}(F_0 N_0) + \sum_{k=1}^{q} z_k \text{trace}(F_0 N_k) = f_0 + \sum_{k=1}^{q} f_k z_k$ where

$$f_0 = \text{trace}(F_0 N_0), \quad f_k = \text{trace}(F_0 N_k) - \text{trace}(F_0 N_0)\text{trace}\, N_k, \quad k = 1, \ldots, q.$$

Similarly, we denote $Z = G_0 + \sum_{k=1}^{q} z_k G_k$ where

$$G_0 = N_0, \quad G_k = N_k - (\text{trace}\, N_k) N_0, \quad k = 1, \ldots, q.$$

We have shown our main result.

Theorem 1 *Primal LMI (1) is feasible if and only if $\lambda^\star > 0$ in reduced dual LMI*

$$\begin{array}{rcll} \lambda^\star & = & \min & f_0 + \sum_{k=1}^{q} f_k z_k \\ & & \text{s.t.} & G_0 + \sum_{k=1}^{q} z_k G_k \succeq 0. \end{array} \qquad (5)$$

5 Retrieving controller parameters

Suppose that reduced dual LMI (5) is solved with an interior-point algorithm, returning $\lambda^\star > 0$. In order to retrieve original controller parameters x solving LMI (1), it is assumed that the dual variable to LMI (5) is also available. In a later section we describe tailored interior-point algorithms to solve LMI (5) efficiently and to recover the multipliers.

Since the dual to LMI (5) is the reduced primal LMI

$$\begin{array}{rcll} \lambda^\star & = & \max & f_0 + \text{trace}(G_0 X) \\ & & \text{s.t.} & \text{trace}(G_k X) = f_k, \quad k = 1, \ldots, q \\ & & & X \succeq 0 \end{array} \qquad (6)$$

we assume that we are given the Hermitian positive semidefinite (PSD) matrix X.

Conic complementarity ensures that $XZ = 0$ and

$$X = F(x) + D(P) - \lambda I \succeq 0$$

for some x (to be found) and P. Therefore

$$\begin{array}{rcl} \mathrm{trace}(D_j(X + \lambda I)) & = & \mathrm{trace}(D_j F(x)) + \mathrm{trace}(D_j D(P)) \\ & = & \mathrm{trace}(D_j F_0) + \sum_{i=1}^{2m+1} x_i \, \mathrm{trace}(D_j F_i) \end{array} \quad (7)$$

for all index j since by duality $\mathrm{trace}(D_j D(P)) = 0$ for all P.

Denoting $A_{ji} = \mathrm{trace}(D_j F_i)$ and $b_j = \mathrm{trace}(D_j(X + \lambda I - F_0))$ for $i = 1, \ldots, 2m+1$ and $j = 1, \ldots, p(p+1)/2$ as the entries of a matrix A and vector b, respectively, equations (7) form an overdetermined consistent linear system of equations

$$Ax = b \quad (8)$$

from which we can retrieve vector x.

6 Case $m \geq n - 1$

When $m \geq n-1$, the only null-space basis matrix N_0 is the zero matrix with a one in its bottom right entry. Hence $z_0 = 1$ and $Z = N_0$ in dual LMI (3), yielding $\lambda^\star = 2$, the right bottom entry in matrix F_0. As a result, primal LMI (1) is always feasible.

This is not surprising, since it is well-known that a linear system of order n can always be stabilized with a controller of order $m = n - 1$ or higher. This result is coined out by Dorato as the fundamental theorem of feedback control, see [3, Section 3.2].

In this case any PSD matrix X with zero last row and column is a valid solution for LMI (6). Using results of [8], it can be shown that (in continuous-time) matrix A is related to the Hurwitz matrix of central polynomial $d(s)$, which is guaranteed to be non-singular if $d(s)$ is stable.

Consequently, for a given central polynomial $d(s)$, a whole set of stabilizing controllers can be parametrized from the top left block in X, which describes the whole cone of PSD matrices of dimension p. Since matrix A in linear system (8) is independent of X, controller coefficients can be parametrized by varying right hand-side vector b and letting $x = A^\dagger b$ where $A^\dagger$ is the pseudo-inverse of matrix A.

7 Numerical concerns

Modern SDP solvers strongly exploit problem sparsity, so it is crucial to preserve sparsity and structure when applying the reduction procedure. The key step is the extraction of null-space basis matrices N_i satisfying linear equations (4). We are currently investigating algebraic properties of these equations, so that matrices N_i can be derived without the singular value decomposition or the QR factorization, which do not preserve sparsity in general. The two step null-space extraction algorithm described in [7] can also useful: first the matrix is reduced to echelon form via Householder transformations; then its null-space is derived via Gaussian elimination with pivoting and backward substitution. Both steps use backward stable numerical operations, and if the original matrix is sparse then it is expected that some sparsity is preserved in the obtained null-space basis, as well as in the reduced dual LMI (5).

Further efforts must be dedicated to studying numerical properties of reduced dual LMI (5), which has a very particular structure. When working with continuous-time polynomials, numerical problems are expected since the reduced null-space basis is made of exponentially ill-conditioned Hankel matrices. Even though there is no satisfying tractable measure of the conditioning of an LMI problem, it is expected that the standard conditioning of linear system of equations (8) obtained when deriving controller parameters from dual variables plays a key role in the design algorithm. Being able to distinguish between well-conditioned and ill-conditioned formulations of control problems would certainly save a significant amount of time to designers, and would also allow improving the overall numerical behavior of currently available LMI solvers, which is still unsatisfying for a lot of physically meaningful instances (e.g. flexible modes).

8 Dedicated interior-point algorithms

Primal-dual Interior-Point (IP) algorithms for solving SDP problems, such has the one implemented in SeDuMi [11], have become very popular. The reason is that for small to medium size problems they produce very efficient search directions and outperform pure primal or pure dual methods. However, the cost of computing and storing the dual iterate might be costly. Moreover, for large scale problems the cost of forming the equations for the search directions becomes very high unless some structure is exploited. Many large-scale problems are sparse and of low rank, and for this type of problems it is possible to utilize the structure to form the equations for the search directions in a clever way in order to reduce the cost. When other types or additional types of structure than sparsity is present it is sometimes possible to develop specialized algorithms. For so-called Kalman-Yakubovich-Popov (KYP) SDPs this has been presented in e.g. [4, 5, 6, 13, 12]. When pure primal (resp. dual) IP methods are used, the dual (resp. primal) solution (multipliers) is not obtained for free. Sometimes it is desirable to compute this solution. Methods to recover the multipliers are implemented in the SDP solvers DSDP3 [2] and also PENNON [10].

9 Example

We consider the standard fourth-order ($n = 4$) two-mass-spring benchmark system

$$\frac{b(s)}{a(s)} = \frac{1}{s^4 + 2s^2}$$

described e.g. in [3, Example 3.3], that we would like to stabilize with a second-order ($m = 2$) controller $y(s)/x(s)$. Note that it can be proved that there exists no static or first-order controller stabilizing this system. We consider continuous-time stability, i.e.

$$D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and we choose

$$d(s) = (s+1)^6$$

as a central polynomial.

Null-space extraction (4) is carried out with the numerically stable algorithm described in [7], and we obtain

$$N_0 = \begin{bmatrix} 0_6 & \\ & 1 \end{bmatrix}$$

and

$$N_1 = \begin{bmatrix} 33.8333 & 0 & -3.6667 & 0 & 1.5000 & 0 & -1.3333 \\ 0 & 3.6667 & 0 & -1.5000 & 0 & 1.3333 & 0 \\ -3.6667 & 0 & 1.5000 & 0 & -1.3333 & 0 & 1.1667 \\ 0 & -1.5000 & 0 & 1.3333 & 0 & -1.1667 & 0 \\ 1.5000 & 0 & -1.3333 & 0 & 1.1667 & 0 & 1.0000 \\ 0 & 1.3333 & 0 & -1.1667 & 0 & -1.0000 & 0 \\ -1.3333 & 0 & 1.1667 & 0 & 1.0000 & 0 & 0 \end{bmatrix}$$

as null-space basis matrices. Note that matrix N_1 (after some row and column permutations) is Hankel.

Reduced dual LMI (5) reads

$$\begin{array}{rcll} \lambda^\star & = & \min & f_0 + f_1 z_1 \\ & & \text{s.t.} & Z = G_0 + z_1 G_1 \succeq 0. \end{array}$$

where $f_0 = 2$, $f_1 = -18.6667$, $G_0 = N_0$ and $G_1 = N_1 - 40.5000 N_0$. This reduced dual LMI features $q = n - m - 1 = 1$ variable, versus $2m + 1 + (m+n)(m+n+1)/2 = 26$ variables for the original primal LMI (1).

Solving the above LMI amounts to solving a mere generalized eigenvalue problem (standard linear algebra), and it is easy to show that $z_1 = 0$ is the optimum value, hence $\lambda^\star = 2$.

The reduced primal LMI (6), dual to the above LMI, is given by

$$\begin{array}{rcll} \lambda^\star & = & \max & f_0 + \mathrm{trace}(G_0 X) \\ & & \text{s.t.} & \mathrm{trace}(G_1 X) = f_1 \\ & & & X \succeq 0. \end{array}$$

A trivial PSD matrix X satisyfing the linear constraints $\mathrm{trace}(G_0 X) = X_{77} = 0$ and $\mathrm{trace}(G_1 X) = -X_{66} = -18.6667$ is

$$X = \begin{bmatrix} 0_5 & & \\ & 18.6667 & \\ & & 0 \end{bmatrix}.$$

With this choice, we obtain the following matrices in linear system of equations (8):

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 30 & -12 & 2 & 4 & 0 \\ 30 & -40 & 30 & 62 & -24 \\ 2 & -12 & 30 & 90 & -92 \\ 0 & 0 & 2 & 34 & -64 \\ 0 & 0 & 0 & 2 & -12 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -2 \\ -2 \\ -64 \\ -88 \\ -54.6667 \\ 0 \end{bmatrix}.$$

Solving this system we obtain the second-order controller

$$\frac{y(s)}{x(s)} = \frac{1.0000 + 4.1185s - 10.5443s^2}{9.6276 + 6.1602s + s^2}$$

stabilizing plant $b(s)/a(s)$. Indeed, the roots of closed-loop denominator polynomial $c(s) = a(s)x(s) + b(s)y(s)$ are located at $-0.1206 \pm i0.7255$, $-0.6647 \pm i0.4357$, -0.7652 and -3.8246.

Now seeking a third-order ($m = 3$) controller with central polynomial

$$d(s) = (s+1)^7$$

we obtain as explained in section 6 the trivial dual matrix

$$Z = \begin{bmatrix} 0_7 & \\ & 1 \end{bmatrix}$$

solving reduced dual LMI (5), instead of solving an original primal LMI (1) with $2m + 1 + (m+n)(m+n+1)/2 = 35$ variables.

Any matrix X with zero last row and column and a PSD upper left block of dimension 7 works as a solution to reduced primal LMI (6). With the trivial choice

$$X = 0_8$$

we obtain the following matrices in linear system (8):

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 42 & -14 & 2 & 0 & 4 & 0 & 0 \\ 70 & -70 & 42 & -14 & 86 & -28 & 4 \\ 14 & -42 & 70 & -70 & 182 & -154 & 86 \\ 0 & 2 & -14 & 42 & -98 & 154 & -182 \\ 0 & 0 & 0 & 2 & -14 & 46 & -98 \\ 0 & 0 & 0 & 0 & 0 & 2 & -14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -2 \\ 2 \\ 26 \\ -156 \\ -152 \\ -48 \\ 0 \end{bmatrix}.$$

The resulting third-order controller

$$\frac{y(s)}{x(s)} = \frac{1.0000 + 4.5781s + 0.1094s^2 - 1.9687s^3}{4.9688 + 8.0469s + 4.5781s^2 + s^3}$$

stabilizes plant $b(s)/a(s)$ and the roots of closed-loop denominator polynomial $c(s) = a(s)x(s)+b(s)y(s)$ are located at $-0.0869 \pm i0.9962$, $-0.7799 \pm i0.6259$, $-0.9222 \pm i0.3866$ and -1.0000.

10 Conclusion

In this note we have shown that, in the polynomial framework, the sufficient LMI fixed-order controller design conditions of [9] have a specific structure that can be exploited to reduce significantly the number of dual LMI decision variables. We use SDP duality results described in [6, 12, 13] to come up with a reduced LMI where the number of decision variables is equal to the difference between the open-loop plant order and the desired controller order. Dedicated interior-point algorithms similar to those used for solving KYP-SDP problems can then be applied. Controller parameters are retrieved afterwards from primal multipliers by solving a standard linear system of equations.

As an obvious consequence, no LMI problem must be solved when designing a controller whose order is equal to or greater than the order of the plant minus one. As with standard pole placement, design merely amounts to solving a linear system of equations.

Another observation is that the computational effort increases when the order of the controller to be designed decreases. This could be expected, since it is well-known that PID or static output feedback controller design, when carried out systematically and rigorously, can be very computationally demanding. Conversely, it is also known that in the state-space framework, designing a stabilizing output feedback dynamical controller boils down to resolving a convex LMI problem as soon as the controller has the same order than the plant. As a conclusion, high computational load is not necessarily expected when the number of controller parameters is large, but rather when a large number of plant parameters are to be controlled with a small number of controller parameters.

Finally, we are currently studying applications of the reduction technique to robust analysis and design problems in the scalar or matrix polynomial frameworks. For example, in the case of polytopic or interval parametric uncertainty, it is expected that the particular structure of the LMI conditions of [9], where a different Lyapunov certificate appears in each vertex LMI condition, can be exploited to reduce significantly the computational effort.

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