LMIs and Polynomial Methods in Control: Illustrative Examples[1]

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Abstract

Linear Matrix Inequalities (LMIs) are now widely used as a convenient tool for solving a significant number of control problems. Probably for historical reasons, LMIs have mainly been used conjointly with state-space methods. Several recent works however tend to prove that polynomial (or algebraic) methods can also rely on LMIs. With the help of simple illustrative examples, the purpose of this paper is to show how the simultaneous use of LMIs and polynomial methods can shed new insights into several important control problems. The algorithms described in this paper will be part of the next version of the Polynomial Toolbox for Matlab, to be released soon[5].

Keywords

Linear Systems, Polynomial Methods, Linear Matrix Inequalities.

1 Introduction

Linear Matrix Inequalities (LMIs) are now widely recognized as a useful tool for studying dynamical systems. In [1], the authors show that most of the analysis and design problems for linear systems can be naturally cast as LMIs. Based on recent achievements on interior point methods for addressing convex optimization problems [13], efficient numerical tools were developed to solve LMIs.

Probably for historical reasons, LMI techniques have so far most notably been used conjointly with state-space methods. A very few works concentrate on the use of LMIs in the so-called polynomial (or algebraic) approach to linear systems control, described in [9, 10]. However, recent works tend to prove that polynomial methods can also rely on LMIs, see [5] for a recent survey. As a consequence, the relevance of LMIs in polynomial methods paves the way for the development of several new numerical tools for dealing with polynomial matrices and solving standard control problems.

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The objective of this tutorial paper is to illustrate some previously developed techniques mixing polynomial methods and LMIs. The emphasis is not on the formulation of systems control problems using polynomial methods and LMIs. The reader interested in this topic is referred to the original technical papers and references pointed out in the companion survey paper [5]. The emphasis here is rather on very simple illustrative numerical examples aimed at showing how one can practically solve control problems using polynomial methods and LMIs.

Three typical problems arising in control are addressed in this paper: spectral factorization of a polynomial matrix (Section 2), stability analysis of a polytope of polynomial matrices (Section 3) and simultaneous stabilization of a family of linear systems (Section 4).

The algorithms described in this paper will soon be added to the already existing and quite extensive set of numerical routines available in the POLYNOMIAL TOOLBOX for MATLAB [15, 16, 14].

2 Spectral Factorization

Let B be a para-Hermitian polynomial matrix positive definite on the stability boundary. Then there exists a polynomial matrix A such that A is stable and $A^\star A = B$, where the star denotes the adjoint. Given B, the problem of finding spectral factor A is referred to as the spectral factorization problem. Spectral factorization of polynomial matrices is a fundamental ingredient in the polynomial approach to LQ, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control [11, 4].

In [17, Prop. 5.1], it is shown that the whole set of spectral factors of a continuous-time polynomial matrix positive definite on the real axis can be parametrized through an LMI in some symmetric matrix P. Once the LMI is solved for some P, the spectral factor is recovered by extracting the Cholesky factor of a constant Toeplitz matrix arising in the LMI. Moreover, the maximization of the trace of matrix P over this LMI yields a Hurwitz stable spectral factor, see [17, Prop. 5.2]. Similar results were achieved in the discrete-time case [3]. In this section, we will illustrate these results with the help of two simple numerical examples.

2.1 Continuous-time Spectral Factorization

Let

$$B(s) = \begin{bmatrix} 16 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 24 \\ -24 & 0 \end{bmatrix} s + \begin{bmatrix} -2 & -8 \\ -8 & -49 \end{bmatrix} s^2 + \begin{bmatrix} 0 & 9 \\ -9 & 0 \end{bmatrix} s^3 + \begin{bmatrix} 1 & -5 \\ -5 & 26 \end{bmatrix} s^4$$

be a continuous-time para-Hermitian matrix, i.e. $B(s) = B^\star(s) = B^T(-s)$. One can check that $B(s)$ is positive definite on the stability boundary. In other words $B(jw) \succ 0$ for all real w.

Following [17, Algo. 5.1], we first have to solve the LMI optimization problem

$$\begin{array}{ll} \max & \mathrm{Trace} P \\ \text{s.t.} & H(P) = H_0 + \mathcal{Q}^T P \mathcal{P} + \mathcal{P}^T P \mathcal{Q} \succ 0 \\ & P = P^T \end{array}$$

$$H_0 = \left[\begin{array}{rr|rr|rr} 16 & 4 & 0 & -12 & -1 & -4 \\ 4 & 2 & 12 & 0 & -4 & -24.5 \\ \hline 0 & 12 & 0 & 0 & -0 & -4.5 \\ -12 & 0 & 0 & 0 & 4.5 & -0 \\ \hline -1 & -4 & -0 & 4.5 & 1 & -5 \\ -4 & -24.5 & -4.5 & -0 & -5 & 26 \end{array}\right]$$

is a matrix built from coefficients of $B(s)$ and

$$\mathcal{P} = \left[\begin{array}{cc|cc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathcal{Q} = \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

are projection matrices. Using the LMI Toolbox for Matlab [12] with a relative accuracy of 10^{-8}, we obtain

$$P_{\rm opt} = \left[\begin{array}{rr|rr} 12 & -8 & 5 & -16 \\ -8 & -1 & 5 & 20.5 \\ \hline 5 & 5 & 3 & -9.5 \\ -16 & 20.5 & -9.5 & 29 \end{array}\right].$$

The optimality of the solution ensures that matrix

$$H(P_{\rm opt}) = \left[\begin{array}{rr|rr|rr} 16 & 4 & 12 & -20 & 4 & -20 \\ 4 & 2 & 4 & -1 & 1 & -4 \\ \hline 12 & 4 & 10 & -11 & 3 & -14 \\ -20 & -1 & -11 & 41 & -5 & 29 \\ \hline 4 & 1 & 3 & -5 & 1 & -5 \\ -20 & -4 & -14 & 29 & -5 & 26 \end{array}\right]$$

has rank 2, see [18] for further details.

Second, we must find a factorization of the above constant matrix. With the help of the Cholesky factorization, we get $H(P_{\rm opt}) = G^T G$ with

$$G = \left[\begin{array}{rr|rr|rr} 4 & 1 & 3 & -5 & 1 & -5 \\ 0 & 1 & 1 & 4 & 0 & 1 \end{array}\right].$$

The desired spectral factor is then retrieved from coefficients in G. We obtain

$$A(s) = \left[\begin{array}{rr} 4 & 1 \\ 0 & 1 \end{array}\right] + \left[\begin{array}{rr} 3 & -5 \\ 1 & 4 \end{array}\right] s + \left[\begin{array}{rr} 1 & -5 \\ 0 & 1 \end{array}\right] s^2.$$

One can check that indeed $A(s)$ is Hurwitz stable and $A^\star(s)A(s) = B(s)$, see Figure 1.

2.2 Discrete-time Spectral Factorization

The discrete-time spectral factorization using LMIs follows exactly along the same lines as in the continuous-time case, see [3].

Let

$$B(z) = \left[\begin{array}{rr} -6 & 32 \\ 3 & 0 \end{array}\right] z^{-2} + \left[\begin{array}{rr} 2 & -20 \\ -6 & -32 \end{array}\right] z^{-1} + \left[\begin{array}{rr} 66 & -11 \\ -11 & 84 \end{array}\right] + \left[\begin{array}{rr} 2 & -6 \\ -20 & -32 \end{array}\right] z + \left[\begin{array}{rr} -6 & 3 \\ 32 & 0 \end{array}\right] z^2$$

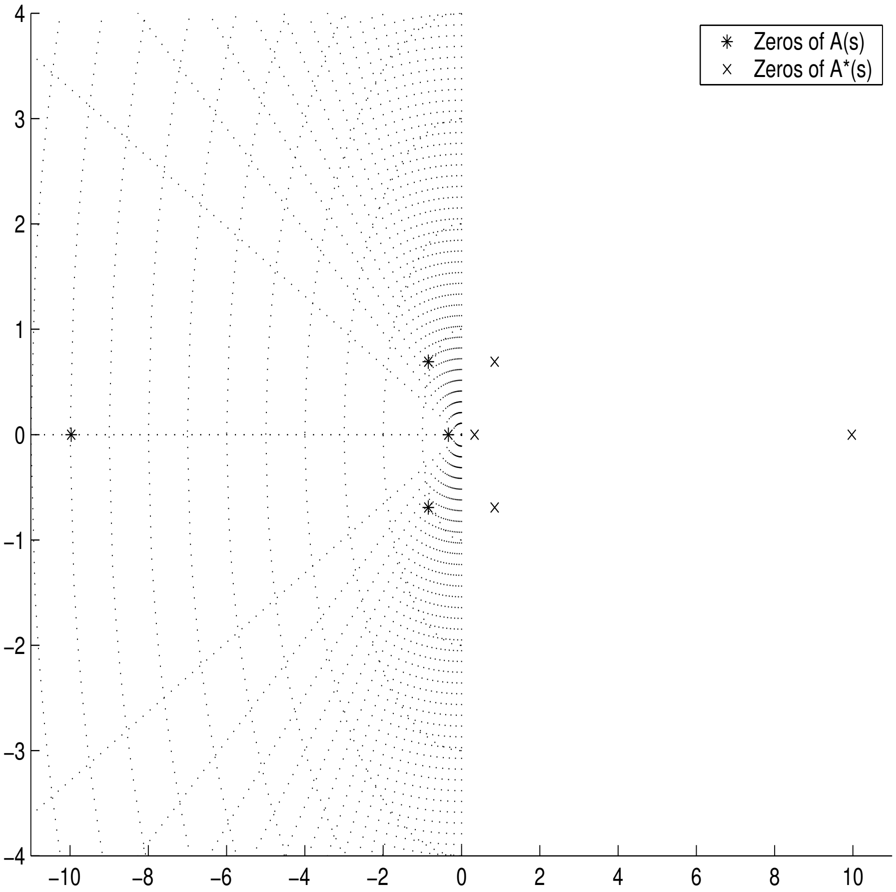

Figure 1: Zeros of $B(s) = A^\star(s)A(s)$

be a discrete-time para-Hermitian two-sided polynomial matrix, i.e. $B(z) = B^\star(z) = B^T(z^{-1})$. One can check that $B(z)$ is positive definite on the stability boundary. In other words $B(e^{j\theta}) \succ 0$ for all real θ.

Following [3], we first have to solve the LMI optimization problem

$$\begin{array}{ll} \max & \mathrm{Trace} P \\ \text{s.t.} & H(P) = H_0 - \mathcal{Q}^T P \mathcal{Q} + \mathcal{P}^T P \mathcal{P} \succ 0 \\ & P = P^T \end{array}$$

where P is a matrix to be found,

$$H_0 = \left[\begin{array}{rr|rr|rr} 0 & 0 & 0 & 0 & -6 & 3 \\ 0 & 0 & 0 & 0 & 32 & 0 \\ \hline 0 & 0 & 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & 0 & -20 & -32 \\ \hline -6 & 32 & 2 & -20 & 66 & -11 \\ 3 & 0 & -6 & -32 & -11 & 84 \end{array}\right]$$

is a matrix built from coefficients of $B(z)$ and $\mathcal{P}$ and $\mathcal{Q}$ are the projection matrices as defined in Paragraph 2.1. Using the LMI Toolbox for Matlab [12] with a relative accuracy of 10^{-8}, we obtain

$$P_{\mathrm{opt}} = \left[\begin{array}{rr|rr} -0.6432 & 2.7659 & 0.5062 & -0.9717 \\ 2.7659 & -16.4661 & -0.5424 & 12.6449 \\ \hline 0.5062 & -0.5424 & -1.6256 & 0.5045 \\ -0.9717 & 12.6449 & 0.5045 & -33.6112 \end{array}\right].$$

The optimality of the solution ensures that matrix

$$H(P_{\mathrm{opt}}) = \left[\begin{array}{rr|rr|rr} 0.6432 & -2.7659 & -0.5062 & 0.9717 & -6.0000 & 3.0000 \\ -2.7659 & 16.4661 & 0.5424 & -12.6449 & 32.0000 & 0.0000 \\ \hline -0.5062 & 0.5424 & 0.9824 & 2.2614 & 2.5062 & -6.9717 \\ 0.9717 & -12.6449 & 2.2614 & 17.1450 & -20.5424 & -19.3551 \\ \hline -6.0000 & 32.0000 & 2.5062 & -20.5424 & 64.3744 & -10.4955 \\ 3.0000 & 0.0000 & -6.9717 & -19.3551 & -10.4955 & 50.3888 \end{array}\right]$$

has rank 2, see [18] for further details.

Second, we must find a factorization of the above constant matrix. With the help of the singular value decomposition, we get $H(P_{\mathrm{opt}}) = G^T G$ with

$$G = \left[\begin{array}{rr|rr|rr} 0.7295 & -4.0254 & -0.2566 & 2.7470 & -8.0095 & 0.8961 \\ -0.3332 & -0.5123 & 0.9574 & 3.0982 & 0.4712 & -7.0417 \end{array}\right].$$

The desired spectral factor is then retrieved from coefficients in G. We obtain

$$A(z) = \left[\begin{array}{rr} 0.7295 & -4.0254 \\ -0.3332 & -0.5123 \end{array}\right] + \left[\begin{array}{rr} -0.2566 & 2.7470 \\ 0.9574 & 3.0982 \end{array}\right] z + \left[\begin{array}{rr} -8.0095 & 0.8961 \\ 0.4712 & -7.0417 \end{array}\right] z^2.$$

One can check that indeed $A(z)$ is Schur stable and $A^\star(z)A(z) = B(z)$, see Figure 2.

3 Stability of Polynomial Matrix Polytopes

Stability of polynomial matrix polytopes is directly related to robust stability of linear systems described by matrix fraction descriptions and affected by polytopic uncertainty, see [6] for a short survey. Polytopic uncertainty is probably the most general representation of the parametric uncertainty affecting a linear system. However, this generality has recently led to the somehow disappointing conclusion that most of the stability analysis problems involving polytopic uncertainty cannot be solved efficiently.

In [6], it is shown that sufficient robust stability conditions can be derived via parameter dependent Lyapunov functions. These conditions are expressed as convex LMI problems, whereas the original necessary and sufficient stability conditions unavoidably lead to a non-convex problem. In this section, we will illustrate these results with the help of a simple numerical example.

Consider the continuous-time interval polynomial matrix

$$\left[\begin{array}{cc} [7.7 - 2.3s + 4.3s^2, 3.7 + 2.7s + 4.3s^2] & [-3.1 - 6s - 2.2s^2, -4.1 - 7s - 2.2s^2] \\ 3.6 + 6.4s + 4.3s^2 & [3.2 + 11s + 8.2s^2, 16 + 12s + 8.2s^2] \end{array}\right].$$

Associated to the above interval polynomial matrix is a polynomial matrix polytope with $2^3 = 8$ vertices denoted by $A_1(s), \ldots, A_8(s)$.

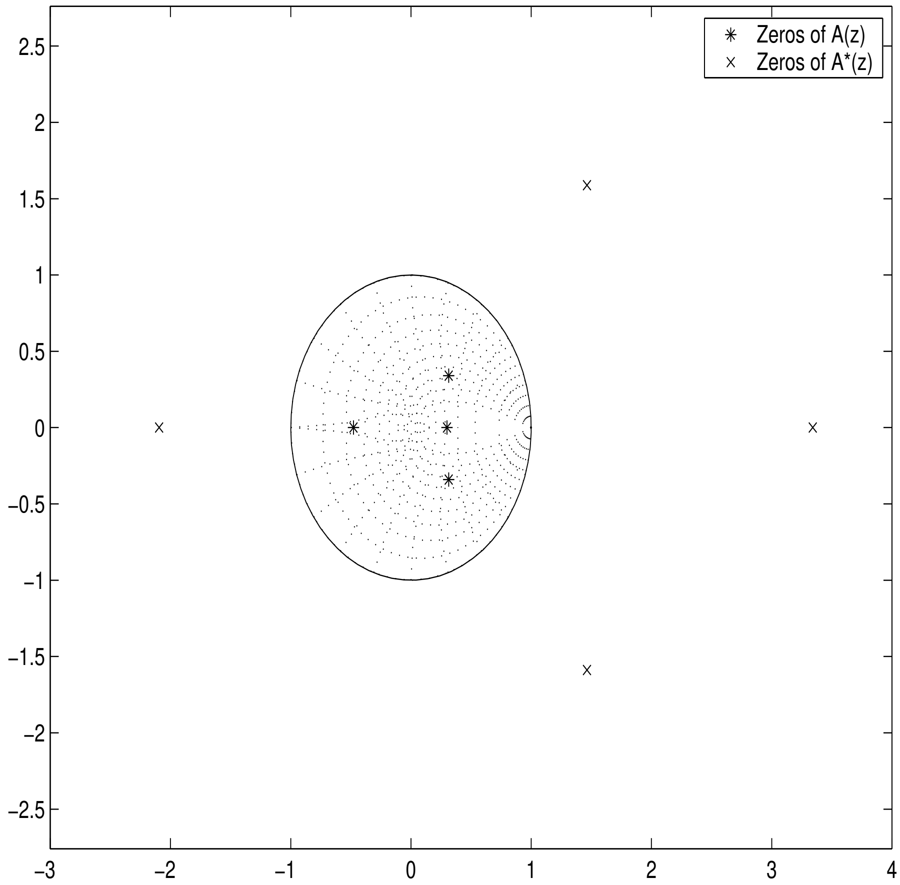

Figure 2: Zeros of $B(z) = A^\star(z)A(z)$

As described in [6], the polytope of polynomial matrices with vertices $A_i(s)$ is robustly stable if there exist matrices $P_1, \ldots, P_8$ and F, G solution to the LMI feasibility problem

$$\begin{bmatrix} \mathcal{Q} \\ \mathcal{P} \\ \mathcal{A}_i \end{bmatrix}^T \begin{bmatrix} 0 & P_i & F \\ P_i & 0 & G \\ F^T & G^T & 0 \end{bmatrix} \begin{bmatrix} \mathcal{Q} \\ \mathcal{P} \\ \mathcal{A}_i \end{bmatrix} \prec 0 \qquad i = 1, \ldots, 8$$

$$P_i = P_i^T \succ 0$$

where

$$\mathcal{P} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathcal{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

are projection matrices, and

$$\mathcal{A}_i = \begin{bmatrix} A_i^0 & A_i^1 & A_i^2 \end{bmatrix}$$

are constant matrices built from coefficients of polynomial matrix vertices

$$A_i(s) = A_i^0 + A_i^1 s + A_i^2 s^2.$$

Using the LMI Toolbox for Matlab [12], the above problem was found feasible for

$$P_1 = \begin{bmatrix} 3.7354 & 0.2711 & 0.1767 & 0.0364 \\ 0.2711 & 2.2550 & 0.4162 & 1.0318 \\ 0.1767 & 0.4162 & 2.7773 & 1.2283 \\ 0.0364 & 1.0318 & 1.2283 & 2.8336 \end{bmatrix} P_2 = \begin{bmatrix} 2.7504 & -0.0435 & 1.0393 & 0.3271 \\ -0.0435 & 2.5026 & -0.0255 & 1.0368 \\ 1.0393 & -0.0255 & 2.9039 & 0.7286 \\ 0.3271 & 1.0368 & 0.7286 & 2.8959 \end{bmatrix} \cdots$$

and

$$F = \begin{bmatrix} -0.1267 & -0.1034 \\ 0.1395 & -0.0547 \\ 213.0715 & -17906.3902 \\ 4445.8848 & 11925.2280 \end{bmatrix} \quad G = \begin{bmatrix} -213.4209 & 17906.1930 \\ -4445.7996 & -11925.3861 \\ -0.0465 & -0.0906 \\ 0.1415 & -0.0472 \end{bmatrix}.$$

The root locus of the interval polynomial matrix, obtained by a brute-force three-dimensional sweeping, is represented in Figure 3. One can check that the interval polynomial is indeed robustly stable.

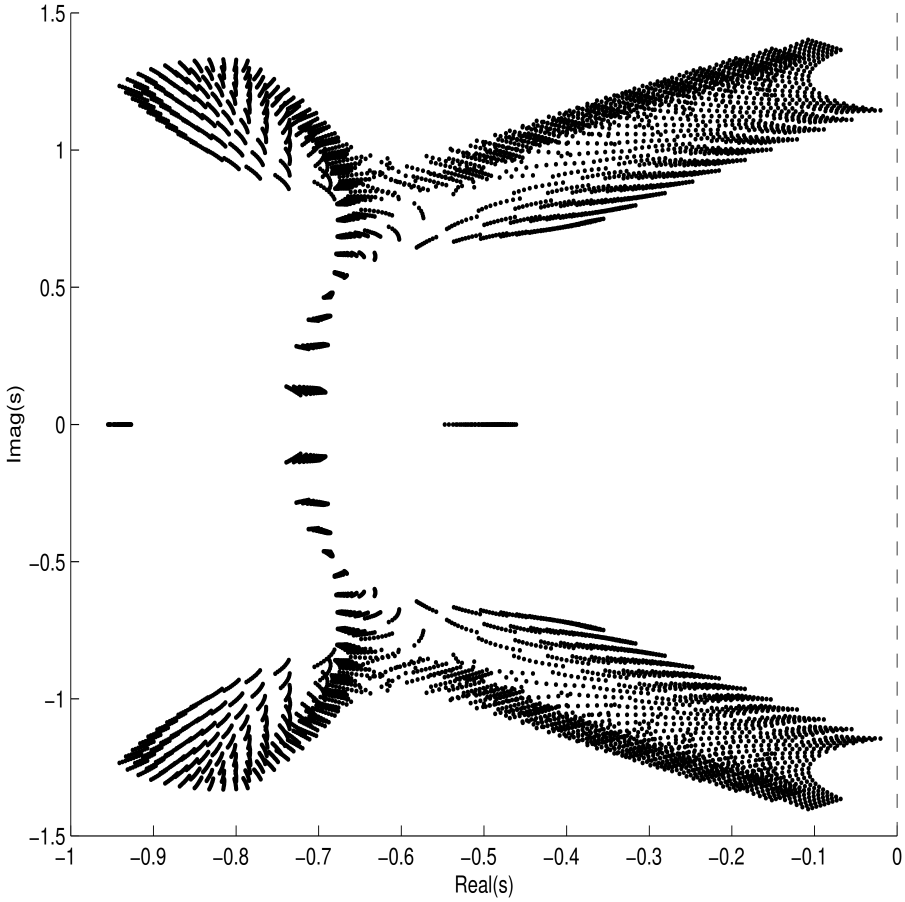

Figure 3: Root Locus of the Interval Polynomial Matrix

4 Simultaneous Stabilization

The problem of simultaneously stabilizing a family of SISO linear systems by one fixed controller of given order is recognized as one of the difficult open issues in linear system theory. This problem arises for instance when trying to preserve stability of a control system under the failure of sensors, actuators or processes.

A new approach to simultaneous stabilization was recently proposed in [8], putting emphasis on polynomial methods and LMIs. Using Hermite-Fujiwara matrices, the simultaneous stabilization problem is formulated as a Bilinear Matrix Inequality (BMI) problem. A heuristic based on LMIs is then used to solve this non-convex problem. In this section, we illustrate this approach on a simple numerical example.

Consider the parameter-varying continuous-time system of transfer function

$$\frac{1+\sqrt{\theta}+s}{3\theta+11+(\theta^2-10)s+s^2}, \quad 1 \le \theta \le 11.$$

We aim at simultaneously stabilizing with a first order controller

$$\frac{x_0+x_1 s}{x_2+x_3 s}$$

four plants at operating points $\theta_1 = 1$, $\theta_2 = 3$ and $\theta_3 = 7$ and $\theta_4 = 11$.

In [8] it is shown that the problem of finding controller coefficients x_i amounts to solving the following non-convex BMIs

$$\begin{bmatrix} \begin{pmatrix} 4x_0^2 + 8x_0x_1 - 8x_0x_2 \\ +56x_0x_3 + 56x_1x_2 - 252x_2^2 \\ +392x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 4x_0x_3 \\ +28x_2x_3 \end{pmatrix} \\ 0 & \begin{pmatrix} -22x_0x_3 + 2x_0x_1 + 4x_1^2 \\ -14x_1x_2 - 8x_1x_3 + 2x_0x_2 \\ -18x_2^2 + 162x_2x_3 - 252x_3^2 \end{pmatrix} & 0 \\ \begin{pmatrix} 4x_0x_3 \\ +28x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 2x_1x_3+ \\ +2x_2x_3 \\ -18x_3^2 \end{pmatrix} \end{bmatrix} \succ 0$$

$$\begin{bmatrix} \begin{pmatrix} 5.4641x_0^2 + 14.9282x_0x_1 \\ +34.5359x_0x_2 + 109.2820x_0x_3 \\ +109.2820x_1x_2 - 40x_2^2 \\ +800x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 5.4641x_0x_3 \\ +40x_2x_3 \end{pmatrix} \\ 0 & \begin{pmatrix} -7.4641x_0x_3 + 2x_0x_1 \\ +5.4641x_1^2 + 3.4641x_1x_2 \\ +34.5359x_1x_3 + 2x_0x_2 \\ -2x_2^2 + 2x_2x_3 \\ -40x_3^2 \end{pmatrix} & 0 \\ \begin{pmatrix} 5.4641x_0x_3 \\ +40x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 2x_1x_3 \\ +2x_2x_3 \\ -2x_3^2 \end{pmatrix} \end{bmatrix} \succ 0$$

$$\begin{bmatrix} \begin{pmatrix} 7.2915x_0^2 + 26.5830x_0x_1 \\ +348.3686x_0x_2 + 233.3281x_0x_3 \\ +233.3281x_1x_2 + 2496x_2^2 \\ +2048x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 7.2915x_0x_3 \\ +64x_2x_3 \end{pmatrix} \\ 0 & \begin{pmatrix} 70.7085x_0x_3 + 2x_0x_1 \\ +7.2915x_1^2 + 85.2915x_1x_2 \\ +348.3686x_1x_3 + 2x_0x_2 \\ +78x_2^2 + 3042x_2x_3 \\ +2496x_3^2 \end{pmatrix} & 0 \\ \begin{pmatrix} 7.2915x_0x_3 \\ +64x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 2x_1x_3 \\ +2x_2x_3 \\ +78x_3^2 \end{pmatrix} \end{bmatrix} \succ 0$$

$$\begin{bmatrix} \begin{pmatrix} 8.6332x_0^2 + 37.2665x_0x_1 \\ +1046.2907x_0x_2 + 379.8630x_0x_3 \\ +379.8630x_1x_2 + 9768x_2^2 \\ +3872x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 8.6332x_0x_3 \\ +88x_2x_3 \end{pmatrix} \\ 0 & \begin{pmatrix} 213.3668x_0x_3 + 2x_0x_1 \\ +8.6332x_1^2 + 230.6332x_1x_2 \\ +1046.2907x_1x_3 + 2x_0x_2 \\ +222x_2^2 + 24642x_2x_3 \\ +9768x_3^2 \end{pmatrix} & 0 \\ \begin{pmatrix} 8.6332x_0x_3 \\ +88x_2x_3 \end{pmatrix} & 0 & \begin{pmatrix} 2x_1x_3 \\ +2x_2x_3 \\ +222x_3^2 \end{pmatrix} \end{bmatrix} \succ 0.$$

In order to solve this problem, we use a heuristic. Because of the non-convexity of the problem, we have no guarantee that the heuristic will converge to a solution. The BMIs are first reformulated as LMIs with a rank constraint, see [8]. The rank constraint is then handled via a linearization scheme based on cone complementarity, see [2, 7].

Using an LMI Toolbox implementation of the linearization scheme, we obtain the following solution to the above BMIs

$$x_0 = 0.07121 \quad x_1 = 0.08621 \quad x_2 = 0.02839 \quad x_3 = 0.007077$$

corresponding to the normalized controller

$$\frac{10.0611 + 12.1815s}{4.0108 + s}$$

simultaneously stabilizing the set of four plants. Indeed, one can check that the four associated closed-loop characteristic polynomials

$$\begin{array}{c} 76.2736 + 12.3269s + 7.1923s^2 + s^3 \\ 107.7037 + 59.3309s + 15.1923s^2 + s^3 \\ 165.0263 + 242.8935s + 55.1923s^2 + s^3 \\ 219.9057 + 551.8440s + 127.1923s^2 + s^3 \end{array}$$

are all Hurwitz stable.

5 Conclusion

We have illustrated the simultaneous use of polynomial methods and LMIs on three key control problems: spectral factorization in both continuous-time and discrete-time, robust stability analysis and robust design via simultaneous stabilization. The algorithms described in this paper will be part of the next version of the POLYNOMIAL TOOLBOX for MATLAB [14].

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