LMIs for Linear Systems Control by Polynomial Methods[1]

Didier Henrion[2,3]

Abstract

Linear matrix inequalities (LMIs) prove to be a useful tool for solving several analysis and design problems in linear systems control. However, LMI techniques have mainly been used conjointly with state-space methods. The aim of this paper is to survey several recent achievements that tend to prove that LMI feasibility and optimization problems also arise when dealing with control problems formulated in the algebra of polynomial matrices.

Keywords

Linear Systems, Polynomial Matrices, Linear Matrix Inequalities.

1 Introduction

Linear matrix inequalities (LMIs) are now widely recognized as a useful tool for studying dynamical systems. In [9], the authors show that most of the analysis and design problems for linear systems can be naturally cast as LMIs. LMI problems belong to the class of convex optimization problems, and can be viewed as a generalization of linear programming to the convex cone of positive semidefinite matrices. For this reason, optimization over LMIs is sometimes also referred to as semidefinite programming, see [47] and references therein.

Following the pioneering theoretical work of Nesterov and Nemirovski on interior point methods for addressing convex optimization problems [38], several numerical tools were developed to solve LMIs in an efficient way [37, 13]. An impressive amount of work has been devoted recently either to the study of new algorithms for solving LMI problems or to showing that a given problem actually reduces to an LMI, see [14] for a recent overview. For example, the sophisticated algebraic techniques proposed in [11] to solve the suboptimal $\mathcal{H}_\infty$ problem were shown to reduce, perhaps in a deceptively simple way, to solving a mere LMI problem [15].

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[2]Laboratoire d'Architecture et d'Analyse des Systèmes, Centre National de la Recherche Scientifique, 7 Avenue du Colonel Roche, 31 077 Toulouse, cedex 4, France. FAX: +33 5 61 33 69 69. E-mail: `henrion@laas.fr`

[3]Institut National des Sciences Appliquées, Complexe Scientifique de Rangueil, 31 077 Toulouse, cedex 4, France.

It must however be underlined that LMI techniques have so far most notably been used conjointly with state-space methods [49, 43]. A very few works concentrate on the use of LMIs in the so-called polynomial approach to linear systems control, described extensively in [34, 35]. Unsurprisingly, this has precluded the use of powerful LMI optimization tools for solving control issues formulated in the ring of polynomial matrices. Following our previous research endeavor that consisted in showing that reliable and efficient numerical tools can also be designed for dealing with polynomial matrices [19], the aim of this paper is precisely to demonstrate that LMI problems similarly arise in the polynomial approach to linear systems control.

As an immediate byproduct, the relevance of LMIs in polynomial methods paves the way for the development of a whole bunch of new numerical tools for dealing with polynomial matrices and solving standard control problems. These new routines are to be inserted to the already existing and quite extensive set of numerical routines available in the Polynomial Toolbox for Matlab [41, 42, 40].

The outline of this survey paper is as follows. In Section 2 we first mention standard and "easy" control problems that can be approached using polynomial methods and convex LMI optimization. In Section 3, we enumerate a series of inherently "difficult" control problems that can be dealt with using LMIs conjointly with polynomial methods. These problems are non-convex, but we show that convex LMI optimization generally provides useful approximation techniques or heuristics. Finally, in Section 4 we mention several extensions of the previously described achievements and we point out directions for further research.

2 Convex problems

Several problems involving polynomial matrices can be cast as a standard convex LMI optimization problem for which efficient and widely spread numerical tools are now available. We survey two of them in this section.

2.1 Positive definite polynomial matrices

Relationships between polynomial methods and LMIs were actually described in the famous visionary paper by J. C. Willems [48] as early as in 1971. It was then shown that a given rational para-Hermitian matrix is non-negative on the imaginary axis if there exists a Hermitian matrix solution to a certain LMI.

Very recently, this result has been extended to the ring of polynomial matrices. In [46, Prop. 5.1], it is shown that the whole set of spectral factors of a continuous-time polynomial matrix positive definite on the real axis can be parametrized through an LMI in some symmetric matrix P. Once the LMI is solved for some P, the spectral factor is recovered by extracting the Cholesky factor of a constant Toeplitz matrix arising in the LMI. Moreover, the maximization of the trace of matrix P over this LMI yields a Hurwitz stable spectral factor, see [46, Prop. 5.2]. Polynomial matrices that are sign definite on the imaginary axis play an important role in systems and control theory. They can represent spectral density function of stochastic processes and show up in spectral factorization, a fundamental ingredient in the polynomial approach to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimization [36, 17].

In the same vein, it is shown in [16] that the set of continuous-time (resp. discrete-time) two-sided polynomial matrices non-negative on the real axis (resp. on the unit circle) is described by a convex LMI. Two-sided polynomial matrices arise when studying linearization schemes for solving spectral factorization, see [27].

2.2 $\mathcal{D}$-stability of polynomial matrices

Fundamental linear system features are elegantly captured by polynomial matrices. For example, second-order matrix polynomials naturally arise when studying highly damped systems or vibrational systems such as oil derricks, space stations, regional power system models [30]. More generally, the zeros of the denominator polynomial matrix in a matrix fraction description [29] characterizes system dynamics and performance. Satisfactory transient time response can be ensured as soon as the zeros are located in some specific region of the complex plane [1]. Following a terminology introduced in [2], the problem of checking whether the zeros of a given polynomial matrix belong to a given region $\mathcal{D}$ is referred to as the $\mathcal{D}$-stability problem.

Following an interesting idea exposed in [3], it is shown in [22] that checking $\mathcal{D}$-stability of a polynomial matrix can be cast as a convex LMI optimization or feasibility problem. As an outcome, this precludes the preliminary computation of the determinant of the polynomial matrix and provides an alternative to the direct computation of the zeros. Moreover, rectangular, non-reduced and complex polynomial matrices can be handled on a unified basis. Covered stability regions include arbitrary half-planes, disks, ellipses, parabolas or their non-convex complements or possibly non-connected unions, not necessarily symmetric with respect to the real axis. Most importantly, the stability conditions can be freely combined with other LMI conditions on coefficients of the polynomial matrix. Finally, it is also shown that in the case of monic pencil matrices, the stability conditions reduce to the standard Lyapunov theorem, one of the most basic LMI arising in control theory [9]. This sheds light into the relationships between the Lagrange multiplier matrix stemming from semidefinite programming duality [47] and the standard Lyapunov matrix ubiquitous in linear system control [29].

3 Non-convex problems

In the previous section, we have described problems involving polynomial matrices that can be solved using standard LMI techniques. Because of convexity, such problems can be solved efficiently with the help of widely spread tools. Unfortunately, several problems in control systems do not automatically lead to such nice and well-studied convex formulations. Deceptively simple analysis or design issues can reveal strong non-convexities. In general, no efficient algorithms are available for solving these non-convex problems, and one has to resort to more or less sophisticated heuristics (without guarantee of convergence), or to time-consuming global optimization techniques. The aim of this section is to report some of these control problems that lead to non-convex formulations when approached with polynomial methods.

3.1 Simultaneous stabilization

The problem of simultaneously stabilizing a family of SISO linear systems by one fixed controller of given order is recognized as one of the difficult open issues in linear system theory. This problem arises for instance when trying to preserve stability of a control system under the failure of sensors, actuators or processes [6]. Several approximate design methods were designed to solve this problem. Most of them make use of state-space arguments, see e.g. [10] and the references therein.

A new approach was recently proposed in [20], putting emphasis on polynomial methods. Using Hermite-Fujiwara matrices, the authors derive a formulation of the simultaneous stabilization problem as an optimization problem where all the non-convexity is concentrated into the constraint that a matrix subject to an LMI has rank one. Geometric properties of the intersection of a set of ellipsoids [18] are first exploited to obtain necessary or sufficient LMI conditions for simultaneous stabilization. A heuristic based on potential reduction is then described to address directly the non-convex rank-constrained LMI problem.

3.2 Stability of 2-D polynomial matrices

Studying stability of multi-dimensional systems appears of central importance in systems theory and signal processing. Complex industrial and physical processes are frequently of multi-dimensional nature, such as multi-pass processes in manufacturing, traffic control problems, image processing and information processes [28]. Two-dimensional (2-D) systems (e.g. with time and space coordinates) arise in several applications, such as mining and steel processing industry or repetitive industrial processes [8]. As pointed out in [24], in spite of the relevance of 2-D systems, there is still a need for unified methods for checking stability of such systems.

Following an approach similar to that described in [22], it is shown in [24] that checking stability of a 2-D polynomial matrix in various regions of the complex plane can be cast as a non-convex rank-one LMI optimization or feasibility problem. Convex LMI relaxations can readily be derived from this formulation. Most importantly, the stability condition can be freely combined with other LMI conditions on coefficients of the 2-D polynomial matrix.

3.3 Stability of polynomial matrix polytopes

In Paragraphs 2.2 and 3.2, we have mentioned that stability of one- or two-indeterminate polynomial matrices in various regions of the complex plane can be checked via convex LMI optimization. This holds when coefficients of the polynomial matrices are perfectly known. However, in real world some uncertainty may affect these coefficients. Uncertainty may arise from neglected dynamics, approximate modeling or unavoidable variations of physical parameters. As far as polynomial methods are concerned, this leads to polynomial matrices with uncertain coefficients.

Several analysis and design tools have emerged to cope with these uncertainties. They were coined out as robust control techniques, see e.g. the recent textbooks [2, 5, 49]. It must be however pointed out that most the results available so far concern robust stability of constant matrices or scalar polynomials. A very few works deal with robust stability of polynomial matrices. Some first results in this direction were obtained in [25], where stability of polynomial matrix polytopes is studied. Polytopic uncertainty is probably the most general representation of the parametric uncertainty affecting a linear system [2, 5]. However, this generality has recently led to the somehow disappointing conclusion that most of the stability analysis problems involving polytopic uncertainty cannot be solved efficiently [7]. The aim of the authors in [25] was then to show that sufficient robust stability conditions can however be derived through special relaxation techniques based on parameter dependent Lyapunov functions [39]. These conditions are expressed as convex LMI problems, whereas the original necessary and sufficient stability conditions unavoidably lead to a non-convex problem. Interestingly enough, numerical experiments tend to show that the gap between sufficiency and necessity of such stability conditions is kept relatively narrow for problems of low dimensions.

3.4 Fixed-order controller design

Similarly to the simultaneous stabilization problem mentioned in the preceding paragraph, the problem of stabilizing a system by a controller of fixed order belongs to the family of difficult and open control problems. This is because the set of stable polynomials is not convex in the space of polynomial coefficients. Several approximate techniques were devised to address the fixed-order stabilization problem. Most of them consist in relaxing the non-convex stability constraint on closed-loop characteristic polynomial coefficients into less restrictive but potentially conservative linear constraints, see e.g. [33] and [31] for recent developments and related references.

When pursuing a polynomial approach for studying fixed-order design, we face the closely related problem of deciding whether an affine family of polynomials contains a stable element [26]. Here also, it is shown that finding a stable polynomial in an affine polynomial family can be formulated as an LMI feasibility problem with a non-convex rank-one constraint. In [26], a linearization technique based on cone complementarity and successive LMI optimization problems is described to overcome this non-convexity. This technique, originally proposed in [12], is a heuristic in the sense that convergence to a rank-one solution of the LMI is not guaranteed. Note that this technique still differs from the relaxation methods described in [20] to address the simultaneous stabilization problem.

3.5 Control of systems with input constraints

The presence of input constraints is an ubiquitous and unavoidable feature in control systems. Input constraints may represent physical limitations of the actuators. They may also account for saturation non-linearities, see [4, 44].

When using polynomial matrix factor descriptions for stabilizing input constrained linear systems, most of the results found in the literature belong to the class of so-called anti-windup techniques, see [32] for an overview. A controller is first designed without considering input constraints. It is validated afterwards in the presence of input constraints. If the controller is not satisfactory, then it is designed again. A more systematic approach is pursued in [44] using state-space techniques. It relies on the concept of local stabilization, which basically means that closed-loop stability is ensured by the existence of a quadratic Lyapunov function valid only in some neighborhood of the origin in the state-space.

In [21], the authors show that polynomial methods can also be useful for studying local stabilization of SISO input constrained linear systems. LMI techniques proved instrumental to this achievement. This work has been extended to MIMO systems in [23]. In these references, it is proved that both a stability region and a controller valid within this region can be found upon solving a convex LMI feasibility problem. Considered stability regions in the state-space are convex polytopes or ellipsoids. Their size can be maximized via convex LMI optimization. However, it is underlined that the stability regions obtained through these methods are only convex approximations of the real stability region. The actual stability region can be a very intricated, non-convex set in general. Moreover, there is a trade-off between the size of the stability region and the order of the controller. Generally speaking, large stability regions can only be obtained at the price of high order controllers that may prove impractical.

4 Extensions and further developments

We have thus shown that LMI techniques can prove particularly useful for solving exactly (Section 2) or approximately (Section 3) several important control problems formulated in the algebra of polynomials and polynomial matrices. We believe that further work must be devoted to the simultaneous use of LMIs and polynomial methods in control. First, this may provide new insights into notoriously difficult problems. Second, this may result in a new unified approach to linear systems control via polynomial methods. The distinguished feature of this approach would then be its systematic reliance on LMI optimization techniques. Third and perhaps most importantly, this may lead to the development of novel numerical routines for dealing with standard control problems. In this section, we enumerate possible extensions of the results surveyed in Sections 2 and 3 and we mention directions for further research.

4.1 Robust stability of polynomial matrices

The approach described in Paragraphs 2.2 and 3.2 for studying stability of one- or two-indeterminate polynomial matrices in given regions of the complex plane can be extended to address robust stability of uncertain polynomial matrices. Covered uncertainty model include norm-bounded, sector-bounded, positive real, parametric or interval real or complex uncertainties. This would extend the results presented in 3.3, only valid for polytopic uncertainties. It can be shown that addressing robust stability of polynomial matrices amounts to solving a non-convex rank-constrained LMI problem of the same kind of that arising in simultaneous stabilization (see Paragraph 3.1) or fixed-order controller design (see Paragraph 3.4). As in μ-analysis [49], non-convexity of the corresponding optimization problem is the price one has to pay for such a level of generality in the uncertainty model.

4.2 Hard design problems

Rank-constrained LMIs can also be used for solving design problems formulated with the help of the results exposed in Paragraphs 2.2 and 3.2. Hard design problems, such as robust pole placement into a disk or static-output feedback stabilization, can be shown to reduce to a rank-constrained LMI. Usually, these hard design problems are addressed via a relaxation of the non-convex closed-loop stability constraints into convex linear constraints, see [33, 31]. It can be proved that convex LMI constraints can also be used for this purpose. The advantages in using LMI relaxations of stability conditions is twofold. First, the approach is systematic. Second, since a linear constraint is a special case of an LMI, LMI relaxations are presumably more general that linear programming relaxations.

4.3 Global optimization

So far, we have only mentioned convex LMI optimization as useful relaxation techniques for addressing rank-constrained LMI problems. It must however be pointed out that global optimization techniques can be used for solving non-convex problems, especially when the non-convexity has special structure. Several interesting related results have recently been achieved using state-space arguments, see [45]. It can be shown that these kind of algorithms can also be applied in a polynomial setting.

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