

Erratum for the paper

D. Henrion. Semidefinite geometry of the numerical range.
Electronic Journal of Linear Algebra, Vol. 20, pp. 322-332, 2010.

Stephan Weis from the University of Erlangen-Nürnberg, Germany, kindly pointed out an error in the proof of Theorem 1. It is stated there that whenever $p(y) = \prod_k(1 + y_1 a_{1k}) \prod_k(1 + y_2 a_{2k})$ then the corresponding numerical range is a rectangle with vertices $(\min_k a_k, \min_k b_k)$, $(\min_k a_k, \max_k b_k)$, $(\max_k a_k, \min_k b_k)$, $(\max_k a_k, \max_k b_k)$. This statement is wrong.

What is a rectangle, or better said a polyhedron with edges parallel to the main axes, is the set $\mathcal{F}(A)$. The corresponding numerical range $\mathcal{W}(A)$ is not a rectangle, it is either a quadrilateral or a triangle, with some edges not parallel to the main axes. More precisely, $\mathcal{W}(A)$ is a polygon with vertices $(0, \min_k a_{2k})$, $(0, \max_k a_{2k})$, $(\min_k a_{1k}, 0)$, $(\max_k a_{1k}, 0)$. In general this is a quadrilateral, but it can degenerate to a triangle.

When $\mathcal{F}(A)$ is unbounded in one direction (i.e. when either the a_{1k} share the same sign or the a_{2k} share the same sign, but not both), then $\mathcal{W}(A)$ is a triangle since one of the four vertices becomes the origin $(0, 0)$. When $\mathcal{F}(A)$ is unbounded in both directions, $\mathcal{W}(A)$ is a quadrilateral excluding the origin. When $\mathcal{F}(A)$ is bounded, $\mathcal{W}(A)$ is a quadrilateral containing the origin.

To summarize, in the proof of Theorem 1 the word “rectangle” must be replaced by “quadrilateral or triangle”. Note that there is no impact on the statement and correctness of the theorem itself.

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