State Regulation and Estimation Using Polynomial Toolbox

Vladimír Kučera [1]
Michael Šebek [2]
Didier Henrion [3]

State feedback is a basic tool to design control systems. It can be used whenever the state of the system is available for measurement. Furthermore, in some situations, an estimate of the state can be used in lieu of the true state to achieve the design specifications like stabilization, eigenstructure assignment, and norm minimization.

For linear systems of the form $\dot{x} = Fx + Gu$, one can apply state feedback $u = -Lx$ to achieve a desired eigenstructure of the closed loop system $\dot{x} = (F - GL)x$.

The dual of state feedback is output injection. Given a linear system $\dot{x} = Fx$, $y = Hx$, one can apply output injection $\dot{x} = Fx - Ky$ to obtain the closed loop system $\dot{x} = (F - KH)x$ with a prescribed eigenstructure.

A number of software tools have been developed to facilitate computer-aided design. The best known among these is MATLAB and its associated toolboxes. The state feedback and/or output injection design is achieved by working with real matrices $F, G,$ and H using well developed and reliable algorithms of linear algebra, as manifested by the Control System Toolbox.

---

[1] Czech Technical University in Prague, CZ
[2] Institute of Information Theory and Automation, Prague, CZ
[3] Laboratoire d´analyse et d´architecture des systèmes CNRS, Toulouse, F

Supported by the European Commission under project CP97-7010, Grant Agency of the Czech Republic under project 102/99/1368, and Ministry of Education of the Czech Republic under project VS97-034.

There is an alternative approach that is based on transfer functions and their polynomial factorizations. Write $(sI-F)^{-1}G = B_R(s)A_R^{-1}(s)$, where A_R, B_R is a pair of right coprime polynomial matrices. Similarly, one can write $H(sI-F)^{-1} = A_L^{-1}(s)B_L(s)$, where A_L, B_L is a pair of left coprime polynomial matrices. Then state feedback or output injection can be designed using the polynomial matrices A_R, B_R and A_L, B_L in lieu of the real matrices F, G, and H.

This alternative design has been hindered in the past by the lack of reliable algorithms to perform numerical calculations with polynomials or polynomial matrices. However, a software tool that meets the requirements is now available. Called Polynomial Toolbox, it is a MATLAB toolbox for polynomials, polynomial matrices, and their applications in control system design and signal processing.

The aim of the presentation is to describe the use of Polynomial Toolbox in the design of state feedback regulators and output injection estimators. These designs are special cases of eigenstructure assignment, a convenient reference design problem. The design procedure consists in calculating a spectral factor and a constant solution of a polynomial matrix equation. This solution directly provides the feedback or injection gains desired.