Robust H_∞ Fixed Order Control Strategies for Large Scale Web Winding Systems

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Abstract— **Demand for improved performance under a wide variety of dynamic conditions and web materials is placing additional emphasis on developing new advanced control strategies. This paper presents centralized and decentralized fixed order $\mathcal{H}_\infty$ controller results with model based feedforward for web winding systems which provide improved web tension and velocity regulation. First, mathematical models of fundamental elements in a web process line are presented. A state space model is developed which enables calculation of the phenomenological model feedforward signals and helps in the synthesis of $\mathcal{H}_\infty$ controllers around the set points given by the reference signals. Different $\mathcal{H}_\infty$ control strategies with additive feedforward have been validated on a nonlinear simulator identified on a 3-motor winding test bench.**

I. Introduction

THE systems handling web material such as textile, paper, polymer or metal are common in the manufacturing industry. Modeling and control of web handling systems have been studied for several decades. However, increasing requirement on control performance and better handling of elastic web material have led to search for more sophisticated robust control strategies. One of the objectives in such systems is to improve decoupling between web tension and speed, so that constant web tension can be maintained during process speed changes. So far, many industrial web transport systems have used decentralized PI-type controllers. However, more efficient control strategies such as LQG or $\mathcal{H}_\infty$ are expected to provide design flexibility in the presence of various uncertainties as well as better performance. Most modern control law designs require the construction and validation of a reasonable plant model. In this article, we use a 3-motor nonlinear simulation model resulting from modeling and identification of an experimental bench. The detailed description of the nonlinear model is given in [14, 22]; the most important laws on which it is based will be recalled here. The model of a large scale web winding system is then deduced from the experimentally verified model. A new state space description has been elaborated [28] and is presented in the Appendix.

Robust control has already been applied to web handling for reduced-size systems, containing not more than 3 motors, with multivariable $\mathcal{H}_\infty$ centralized controllers and LPV structures [14]. Nevertheless, web processing lines are generally large scale systems, i.e. with a high number of actuators and sensors, and it is not suitable to use a centralized controller for such processes. An alternative solution is to use semi-decentralized control: the global system is split into several subsystems controlled independently by its own controller [12, 2, 5].

Recently, multivariable decentralized control strategies have been proposed for industrial metal transport systems [8, 9], and for elastic web with $\mathcal{H}_\infty$ controllers [7, 12]. Decentralized control with overlapping of adjacent subsystems, as in [15, 12, 3, 25], can be useful to reduce the coupling between two such subsystems. Such a control strategy has already given good results in the case of a vehicle platoon [16]. Several improvements of multivariable $\mathcal{H}_\infty$ controllers with one or two degrees of freedom [12, 2] and $\mathcal{H}_\infty$ state feedback control with full or partial integral actions [3, 4] have been applied to web winding systems.

A number of issues related to modeling and control of continuous web processing lines were investigated in [18, 19, 20, 28]. A new model of the unwind/rewind roll that explicitly includes the time-varying inertia and radius of the roll was developed in [23]. A decentralized adaptive controller with model based feedforward was developed and experimentally verified in [24].

A major drawback of standard H_∞ design algorithms, as implemented in currently available computer-aided control system design (CACSD) software is the high order of the computed controllers. Indeed, the order of the controller is typically equal to the order of the plant plus the order of the

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frequency weighting functions. With current model reduction techniques, the controller order cannot always be reduced a posteriori while preserving stability and a satisfying performance. In any case, it is an additional computational burden. It is therefore highly relevant, especially for industrial applications, to develop design algorithms producing fixed-order (e.g. static output feedback, or multivariable PID) controllers from the outset. After more than four decades of intensive research efforts, it turns out that, deceptively, efficient software for designing fixed-order controllers is still not readily available. The underlying mathematical problem seems to be difficult since fixed-order controller design can be formulated as a typically nonsmooth (nondifferentiable) affine problem in the nonconvex cone of stable matrices (or, equivalently, stable polynomials). However, recent progress in nonlinear variational analysis, tailored at solving H_∞ fixed-order control problems [29] paved the way for the development of nonsmooth optimization algorithms based on quasi-Newton (BFGS), bundling and gradient sampling. A MATLAB software called HIFOO (H-Infinity Fixed-Order Optimization) has been released in late 2005, see [30], and uses local optimization techniques. This software has been used in this work for the reduced order controller calculation.

This paper presents fixed order $\mathcal{H}_\infty$ control strategies coupled with model based feedforward control, applied to web transport systems (Other strategies with full order $\mathcal{H}_\infty$ control are presented in [28]). It is shown that such a strategy leads to much improved web tension and velocity regulation performance. The outline of the paper is as follows. Section II gives the main physical laws used to build a nonlinear model which was also identified on an experimental bench composed of three motors (Fig. 1). Scaling of the experimentally identified model to a large scale simulation model is also explained. Linearization of the model around a fixed web tension and velocity reference values gives the state space model that is useful for modern controller synthesis. Section III is dedicated to centralized full and fixed reduced order $\mathcal{H}_\infty$ control design with physical model based feedforward. The decentralized fixed order design is then described in section IV. Finally, section V gives conclusions of this work and indicates some future research directions.

II. Plant Modeling

The nonlinear model [14] of a web transport system is built from the equations describing the web tension behavior between two consecutive rolls and the velocity of each roll. This model was identified on a 3-motor experimental bench given in Fig. 1.

A. Web Tension Calculation

The dynamics of web tension between two rolls of web transport systems is based on three laws.

1) Hooke's law:

The tension T of an elastic web is a function of the web strain ε:

$$T = E\,S\,\varepsilon = E\,S\,(L - L_0)/L_0 \qquad (1)$$

where E is the modulus of elasticity, S the web cross section, L and L_0 are stretched and unstretched web lengths, respectively.

2) Coulomb's law:

The study of a web tension on a roll can be considered as a problem of friction between solids [13, 14].

3) Equation of Continuity:

This equation, applied to the web, yields [13, 14]:

$$L\frac{dT_k}{dt} = E\,S(V_{k+1} - V_k) + T_{k-1}V_k - T_k(2V_k - V_{k+1}) \qquad (1)$$

where k is the span number and takes values from 0 to N-1.

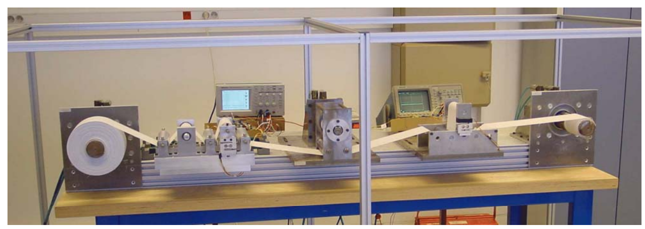

Fig. 1 : Experimental setup with 3 brushless motors and 2 load cells

B. Web Velocity Calculation

The linear velocity V_k of roll k is obtained from the torque balance [13, 14]:

$$\frac{d}{dt}\left(J_k\frac{V_k}{R_k}\right) = R_k(T_k - T_{k-1}) + K_k U_k + C_f \qquad (2)$$

where $K_k U_k$ is the motor torque and C_f is the friction torque. In the case of unwind and rewind rolls (k = 1, k = N), the inertia and radius are time dependent and can substantially vary during processing.

A large scale web handling system of any number of driven rolls can be built from the equations , (1) and (2). A schematic representation of a multi-motor transport system is shown in Fig. A.2 in the Appendix.

C. *State Space Representation*

A scheme of a 3-motor setup with PI controllers is represented in Fig. 2. The inputs to the system are the torque control signals (u_u, u_v, u_w) of the brushless motors; the measurements are the unwinder and winder web tensions T_u and T_w and the web velocity $V = V_3$. The web velocity is set by the master traction motor whereas the web tensions in the spans are controlled by the unwind and rewind motors.

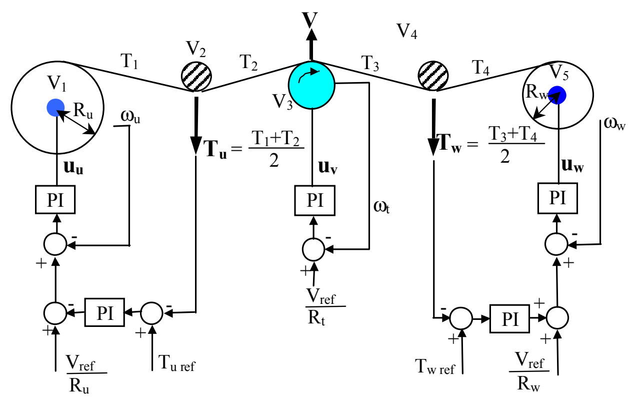

Fig 2 : distributed industrial control scheme for a winding process

The nonlinear state-space model is composed of (1) for the different web spans and (2) for the different rolls. In [14] a global three-motor state space model is presented using a first order linearization and under the assumption that J_k/R_k is slowly varying. In this work, a more precise state space model is used by decomposing the nonlinear equations as follows [28]: define

$$V_i = V_0 + v_i \quad T_i = T_0 + t_i \quad U_i = U_{si_0} + u_{si} \tag{4}$$

where v_i, t_i, u_{si} are signal variations around the reference values. The three-motor system variational dynamics can be presented in the following form:

Subsystem 1 with $\underline{x}_1^T = [v_1 \quad t_1 \quad v_2 \quad t_2]$:

$$E_1\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1u_{s1} + H_1 + A_{12}\underline{x}_2 \tag{5}$$

Subsystem 2 with $\underline{x}_2^T = [v_3]$:

$$E_2\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2u_{s2} + H_2 + A_{21}\underline{x}_1 + A_{23}\underline{x}_3 \tag{6}$$

Subsystem 3 with $\underline{x}_3^T = [t_3 \quad v_4 \quad t_4 \quad v_5]$:

$$E_3\dot{\underline{x}}_3 = A_3\underline{x}_3 + B_3u_{s3} + H_3 + A_{32}\underline{x}_2 \tag{7}$$

A description of the first subsystem is given in the appendix. More details can be found in [28]. The constant values and the non-linear terms are included in the H_i vectors whereas the matrices A_{ij} describe the effect (coupling) of subsystem j on subsystem i.

III. Centralized Control with Feedforward for Winding Systems

The controller design for a 3-motor plant is the subject of the next subsection.

A. Full order Centralized H_∞ control design for a 3-Motor Plant

Coupling between web velocity and tension makes the control of web transport systems inherently difficult. Several methods suppressing this coupling in a system with two driven rolls have been studied [22].

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbance rejection. The synthesis should be done using a linear model corresponding to the starting phase, i.e., an empty roller at the winder. The starting phase is very important: if a problem occurs in this phase, most likely, the rewind roll will be badly wound.

Due to a wide variation of the roller radius during the unwinding-winding process, the dynamic behavior of the system is considerably modified with time. To analyze this modification, let us consider the unwinder and winder separately. With quasi-static assumption on radius variations, the static gains between the control signals and web tensions appear to be proportional to the inverse of the radius [14]:

$$Gain_{DC}\left(\frac{T_u}{u_u}\right) = \frac{1}{R_u} \quad and \quad Gain_{DC}\left(\frac{T_w}{u_w}\right) = \frac{1}{R_w} \tag{8}$$

We therefore multiply the control signals by the corresponding radius measurement or estimation and controller synthesis is done using the plant which includes the radii multiplication (gain scheduling). This approach allows us to reduce web tension variations significantly despite velocity changes during processing [14], [12].

We synthesized a centralized H_∞ controller with output weighting and model matching (Fig. 3) for the system composed of equations (5), (6) and (7) without the vectors H_i. Model M_o gave the desired transfer function T_{yr}. In our case, M_o was a second order transfer function.

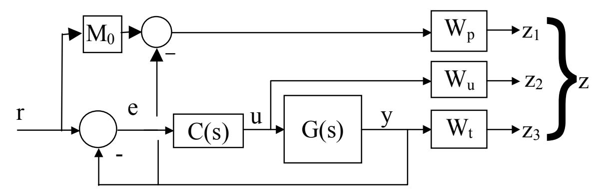

Fig. 3. S/KS/T weighting scheme with model matching for the design of H_∞ controllers

The weighting functions W_p, W_u, and W_t appear in the closed loop transfer matrix T_{zr} which is to be minimized. The weighting function W_p has a high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as follows [14]:

$$W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B\varepsilon_0} \tag{9}$$

where M is the maximum peak magnitude of the sensitivity S, ω_B is the required frequency bandwith ε_0 is the steady-state error allowed. The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies. The controller $C(s)$ is calculated using LMI's (Linear Matrix Inequalities) with the LMI toolbox of the Matlab Software.

To specify independently the tracking performance and robustness to perturbations, a two degree of freedom controller (for example a 2DOF H_∞ controller) can be used [11, 2].

To take into account the inherent system nonlinearities and some constant values (such as static friction in roller bearings) in the control strategy, model based feedforward signals have been added to the control signals. Thus, the control signals U_{si0} , which depend on the reference values of web tension and velocity and on the system state, are added to the H_∞ controller outputs u_i (Fig. 4). These feedforward signals are calculated online with the feedforward controller called C_{ff} in Fig. 4: U_{si0} cancels the H_i element where it appears (all elements of H_i can not be cancelled using input U_{si0} , see Appendix)

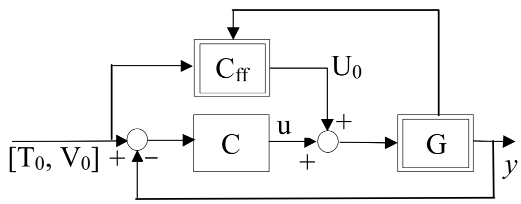

Fig. 4: Control strategy with feedforward signals

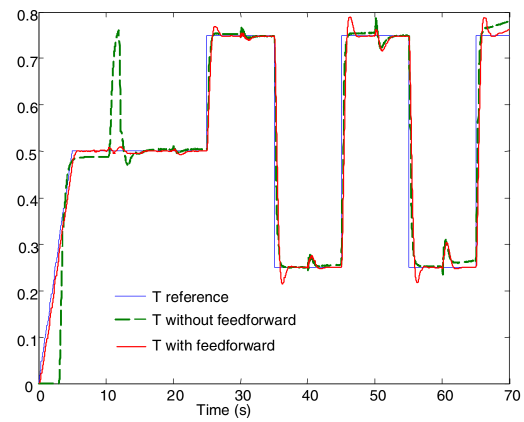

Fig. 5. Simulated web tensions (kg) for H_∞ centralized controller with and without additive feedforward signals

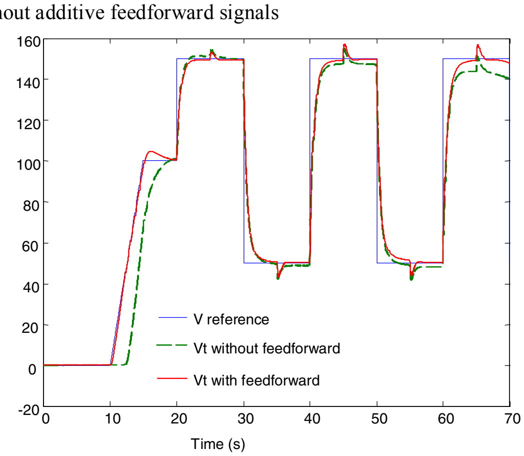

Fig. 6. Simulated web velocity (m/mn) for H_∞ centralized controller with and without additive feedforward signals

Figures 5 and 6 present, respectively, the simulated web unwinder tension and web velocity (by assuming no sliping between web and master roll) for H_∞ controller with and without additive feedforward signals. As expected, the centralized controller with additive feedforward signals not only improves the starting phase but also cancels the static errors to the web reference tension and velocity.

In Fig. 7, G is changed and thus the feedforward controller C_{ff} and the feedback controller C have been calculated for the initial system G_{init}. Moreover, C_{ff} neglects now the static and viscous friction effects. We can observe a little tension chatter in the starting phase due to the feedforward action C_{ff} which is now too small. This chatter is cancelled by adding a constant (tuning parameter) to C_{ff} .

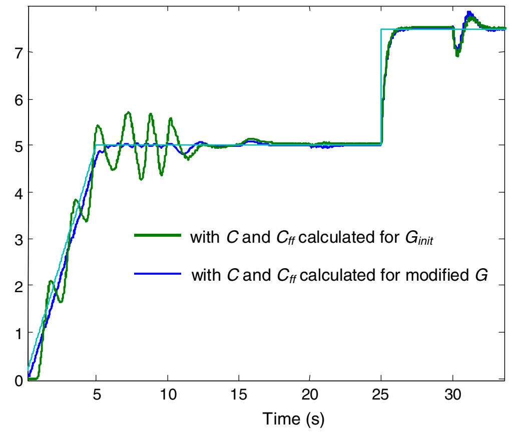

Fig. 7. Simulated web tensions (N) for H_∞ centralized controller with additive feedforward signals

B. Reduced order Centralized H_∞ control design for a 3-Motor Plant

The approach we follow to design a fixed-order linear controller ensuring closed-loop $\mathcal{H}_\infty$ performance is as follows:

- first, the closed-loop system must be stabilized. The closed-loop system matrix is affine in the controller state-space matrices, see e.g.relation (3) in [30]. However, this matrix is nonsymmetric, hence the stabilization problem is nonconvex when formulated in the controller parameter space. Stabilization is ensured when the so-called spectral abscissa (the largest real part of the eigenvalues) of the closed-loop system matrix is strictly negative. Hence a direct way to ensure stabilization is to minimize the spectral abscissa. It turns out that this function can be nonsmooth, or even non-Lipschitz;

- second, the $\mathcal{H}_\infty$ norm of the transfer function between a specified set of inputs and output must be minimized. Here too, the underlying optimization problem is typically nonconvex and nonsmooth.

In order to overcome or address nonconvexity and nonsmoothness of fixed-order $\mathcal{H}_\infty$ design, researchers have developed various techniques:

- convex approximations (polytopes, ellipsoids or LMI) of nonconvex stability regions, introducing an amount of conservatism which is sometimes difficult to assess;

- LMI formulations introducing lifting variables (e.g. Lyapunov matrices), which has the drawback of introducing

many aritificial variables (typically of the order of the square of the system dimension);
- nonconvex programming (global optimization, BMI solvers, nonsmooth optimization), mostly with a guarantee of local convergence only (since finding and certifying global optima is generally too expensive).
In this paper, we follow the latter approach. After several years of fundamental research in nonlinear variational analysis, Burke, Lewis and Overton recently designed a nonsmooth, nonconvex, hybrid optimisation algorithm implemented in a public-domain MATLAB package called HANSO (Hybrid Algorithm for Non-Smooth Optimization). The algorithm mixes in a parametrizable but user-friendly way several optimization techniques, namely quasi-Newton updating, bundling and gradient sampling [31].
HANSO is at the core of another public-domain Matlab package called HIFOO ($\mathcal{H}_\infty$ Fixed Order Optimization) which is tailored at solving fixed-order controller design problems. See [30] for a brief introduction to HIFOO, with some simple numerical examples. HIFOO can be downloaded at http://www.cs.nyu.edu/overton/software/hifoo

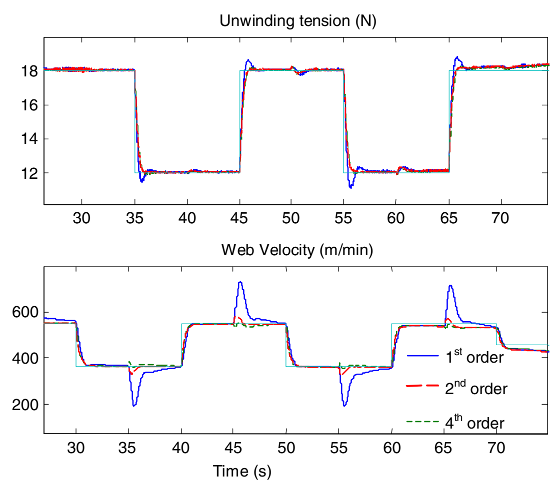

Fig. 8. Simulated web unwinding tension and web velocity for reduced order H_∞ centralized controller without additive feedforward signals

Simulation results, using the non-linear simulator, are given in Fig. 8 for different orders of the centralized controller applied on the 3 motor plant. Satisfactory tracking and decoupling behavior have been obtained using the second-order controller.

IV. Decentralized Control with Feedforward

The focus of this part is to design a completely decentralized controller for web processing lines. Therefore, in industrial applications, the global system is divided into several subsystems with each subsystem containing only one actuator: one subsystem is under velocity control (master speed roll) whereas the other subsystems are under web tension control. Simulation results obtained by 1st order controller are illustrated in Fig. 9.
To improve the dynamic behavior, additive measures can be included in the controller synthesis (Fig. 10); similar to the PI strategy presented in Fig. 2. In our case, the web velocity measured or estimated in each subsystem is used as the second controller input. The synthesis scheme is represented on Fig. 11.

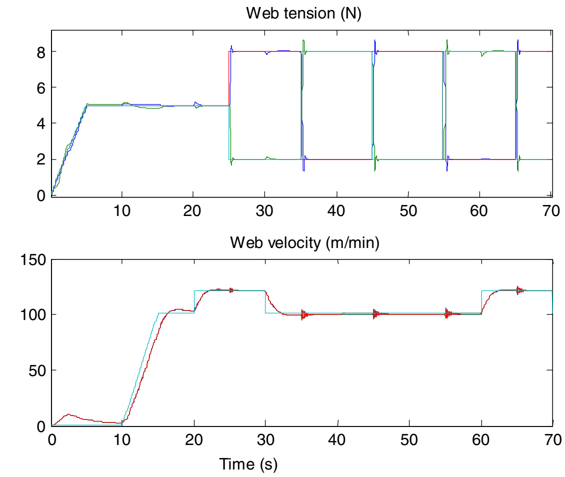

Fig.9: 1st order SISO decentralized control with feedforward

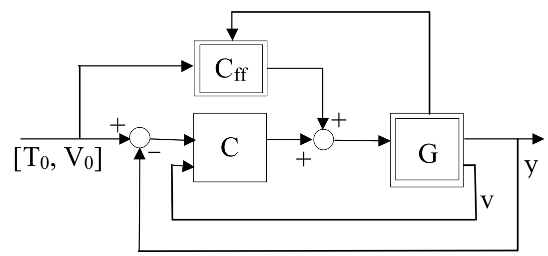

Fig. 10: Control strategy with additive measures and feedforward signals

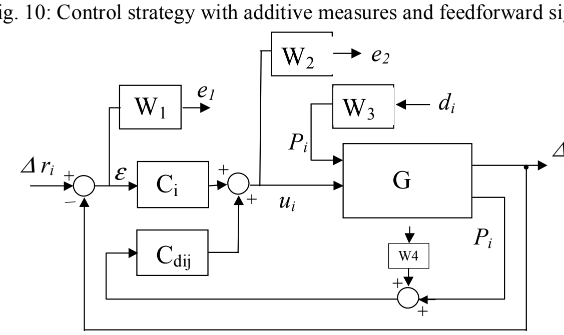

Fig. 11. H_∞ control design with additional measure

Fig. 12 shows the unwinding and winding web tension and web velocity simulation results for decentralized H_∞ 5th order controller with feedforward signals and with additive velocity measures: the two-input controller improves reference tracking while reducing velocity-tension coupling. In Fig. 13, H_∞ based PI decentralized controllers gives satisfactory tension and velocity references tracking. Nevertheless, the influence of a velocity step to the web tension is not negligible.

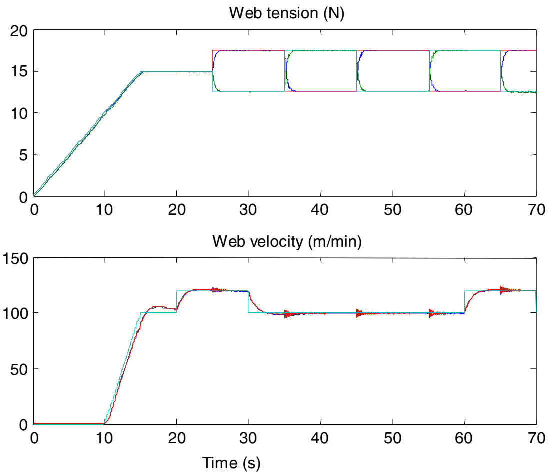

Fig. 12 : 5th order H_∞ decentralized control

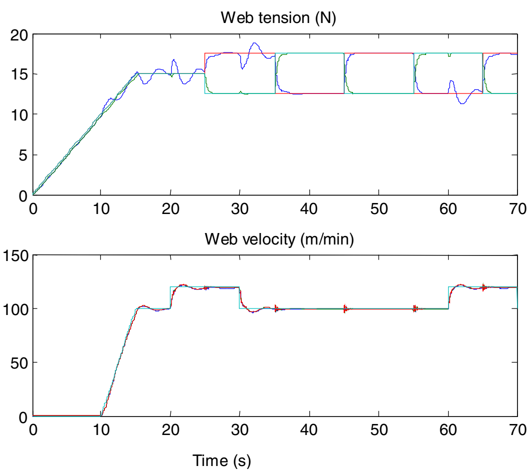

Fig. 13 : H_∞ PI decentralized control

V. CONCLUSION

Compared to decentralized PID controllers classically used in industrial web winding systems, multivariable H_∞ controllers had already shown improved web tension and speed decoupling. Web processing lines are generally of large scale and therefore it is not suitable to use a centralized controller for such processes. In this paper, a decentralized state space model for web processing lines is presented which leads to online calculation of feedforward control action. Different decentralized fixed order H_∞ control strategies with feedforward are then synthesized and validated on a nonlinear web handling simulator. Future work will deal with fixed structure controller synthesis to improve the decoupling between consecutive subsystems.

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Appendix A1 : State space representation of Subsystem 1 :

K_1 : motor torque constant; U_{s1} : motor torque; C_{fsunw} : static friction coefficient; f_{v1u}, f_{v1u}, f_{v1u} : viscous friction coefficients; J_1: unwinder inertia; R_1 : wound roll radius; l : web width; ρ : web mass density; h : web thickness; f_2 : viscous friction coefficient; E : Young modulus; S : web section; Tb : unwinding wound internal tension

$$E_1 \dot{\underline{x}}_1 = A_1 \underline{x}_1 + B_1 u_{s1} + H_1 + A_{12} \underline{x}_2 \quad \text{with} \quad \underline{x}_1^T = \begin{bmatrix} v_1 & t_1 & v_2 & t_2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 \\ 0 & 0 & J_2 & 0 \\ 0 & 0 & 0 & L_2 \end{bmatrix} \quad A_1 = \begin{bmatrix} a_1 & R_1^2 & 0 & 0 \\ -(E_0 + T_0 - T_b) & -V_0 & -E_0 & 0 \\ 0 & -R_2^2 & -f_2 & R_2^2 \\ 0 & V_0 & -E_0 & -3V_0 \end{bmatrix} \quad B_1 = \begin{bmatrix} -K_1 R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E_0 - 2T_0 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \left(\rho S R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right) V_0^2 - \frac{f_{v_{3u}}}{R_1^2} V_0^3 - R_1 C_{fs_{unw}} - f_{v_{1u}} V_0 + R_1^2 T_0 - R_1 K_1 U_{s1_0} + \left(\rho S R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1} + 3\frac{f_{v_{3u}}}{R_1^2} V_0\right) v_1^2 - \frac{f_{v_{3u}}}{R_1^2} v_1^3 \\ V_0 T_b - V_0 T_0 + v_2 t_1 - 2 v_2 t_1 \\ -f_2 V_0 \\ -2 V_0 T_0 + t_1 v_2 - 2 v_2 t_2 - v_3 t_2 \end{bmatrix}$$

Appendix A2 : decentralized control strategy for a large scale web transport system :