Fixed-Order H_∞ Decentralized Control with Model Based Feedforward for Elastic Web Winding Systems

Dominique Knittel
ERT-Enroulement
University of Strasbourg I
Strasbourg, France
dominique.knittel@ipst-ulp.u-strasbg.fr

Marc Vedrines
ERT-Enroulement and LGECO
Institut National des Sciences Appliquées de Strasbourg,
Strasbourg, France
Marc.vedrines@insa-strasbourg.fr

Didier Henrion
LAAS-CNRS, Toulouse, France
and Faculty of Electrical Engineering
Czech Technical University in Prague, Czech Republic
henrion@laas.fr

Prabhakar Pagilla
School of Mechanical and Aerospace Engineering
Oklahoma State University, Stillwater
Oklahoma, USA
pagilla@ceat.okstate.edu

***Abstract*—This paper presents centralized and decentralized fixed order $\mathcal{H}_\infty$ controller designs with model based feedforward for web winding systems which provide improved web tension and velocity regulation. First, mathematical models of the fundamental elements in a web process line are presented. An improved state space model is then derived for the entire system; the model enables calculation of the phenomenological model feedforward signals and helps in the synthesis of $\mathcal{H}_\infty$ controllers around the set points given by the reference signals. Different $\mathcal{H}_\infty$ control strategies with additive feedforward have been synthesized, especially $\mathcal{H}_\infty$ based PI controllers which are relevant for industrial applications. The strategies have been validated on a nonlinear simulator identified on a 3-motor winding test bench.**

Keywords*: *winding systems; large scale systems; decentralized control; fixed order H_∞ control, feedforward control.

I. INTRODUCTION

The systems handling web material such as textile, paper, polymer or metal are common in the manufacturing industry. Modeling and control of web handling systems have been studied for several decades. However, increasing requirement on control performance and better handling of elastic web material have led to continued search for more sophisticated robust control strategies. One of the objectives in such systems is to improve decoupling between web tension and speed, so that constant web tension can be maintained during process speed changes. So far, many industrial web transport systems have used decentralized PI-type controllers. However, for better control performance and robustness to uncertainties, more efficient control strategies such as LQG or $\mathcal{H}_\infty$ should be used. Most modern control law designs require the construction and validation of a precise plant model. In this work, we use a 3-motor nonlinear simulation model resulting from modeling and identification of an experimental bench. The detailed description of the nonlinear model is given in [14, 22]; the most important laws on which it is based will be recalled here. The model of a large scale web winding system is then deduced from the experimentally verified model. A new state space description has been elaborated [28] and is presented in the Appendix.

Robust control has already been applied to web handling for reduced-scale systems, containing not more than 3 motors, with multivariable $\mathcal{H}_\infty$ centralized controllers and LPV structures [14]. Nevertheless, web processing lines are generally large scale systems, i.e. with a large number of actuators and sensors. Therefore, it is not suitable to use a centralized controller for such processes. An alternative solution is to use semi-decentralized control with or without overlapping: the global system is split into several subsystems controlled independently by its own controller [12, 2, 5].
Recently, multivariable decentralized control strategies have been proposed for industrial metal transport systems [8, 9], and for elastic web with $\mathcal{H}_\infty$ controllers [7, 12]. Decentralized control with overlapping of adjacent subsystems, as in [15, 12, 3, 25], can be useful to reduce the coupling between two consecutive subsystems. Such a control strategy has already given good results in the case of a vehicle platoon [16]. Several improvements of multivariable $\mathcal{H}_\infty$ controllers with one or two degrees of freedom [12, 2] and $\mathcal{H}_\infty$ state feedback control with full or partial integral actions [3, 4] have been applied to web winding systems.

A number of issues related to modeling and control of continuous web processing lines were investigated in [18, 19, 20, 28]. A new model of the unwind/rewind roll that explicitly includes the time-varying inertia and radius of the roll was developed in [23]. A decentralized adaptive controller with model based feedforward was developed and experimentally verified in [24].

A major drawback of standard H_∞ design algorithms, as implemented in currently available computer-aided control system design (CACSD) software is the high order of the computed controllers. Indeed, the order of the controller is typically equal to the order of the plant plus the order of the frequency weighting functions. With current model reduction techniques, the controller order cannot always be reduced *a posteriori* while preserving stability and satisfying performance. It is therefore highly relevant, especially for industrial applications, to develop design algorithms producing fixed-order (e.g. static output feedback, or multivariable PID) controllers from the outset. After more than four decades of intensive research efforts, it turns out that, deceptively, efficient software for designing fixed-order controllers is not available. The underlying mathematical problem seems to be difficult since fixed-order controller design can be formulated as a typically nonsmooth (nondifferentiable) affine problem in the nonconvex cone of stable matrices (or, equivalently, stable polynomials). However, recent progress in nonlinear variational analysis, tailored towards solving H_∞ fixed-order control problems [29] paved the way for the development of nonsmooth optimization algorithms based on quasi-Newton (BFGS), bundling and gradient sampling. A MATLAB software called HIFOO (H-Infinity Fixed-Order Optimization) has been released in late 2005, see [30], and uses local optimization techniques. This software has been used in this work for the reduced-order controller calculation.

This paper presents fixed order $\mathcal{H}_\infty$ control strategies coupled with model based feedforward control, applied to web transport systems. It is shown that such a strategy leads to much improved web tension and velocity regulation performance. The outline of the paper is as follows. Section II gives the main physical laws used to build a nonlinear model which was also identified on an experimental bench composed of three motors (Fig. 1). Linearization of the model around fixed web tension and velocity reference values gives the state space model that is useful for modern controller synthesis. Section III is dedicated to centralized full and fixed reduced order $\mathcal{H}_\infty$ control design with physical model based feedforward. The decentralized fixed order design is then described in section IV. Finally, section V gives conclusions of this work and indicates some future research directions.

II. PLANT MODELING

The nonlinear model [14] of a web transport system is built from the equations describing the web tension behavior between two consecutive rolls and the velocity of each roll. This model was identified on a 3-motor experimental bench represented in Fig. 1.

A. Web Tension Calculation

The computation of the tensions between two rolls of web transport systems is based on three laws.

1) Hooke's law:

The tension T of an elastic web is a function of the web strain ε:

$$T = E\,S\,\varepsilon = E\,S\,(L - L_0)/L_0\,, \qquad (1)$$

where E is the modulus of elasticity, S the web cross section, L and L_0 are stretched and unstretched web lengths, respectively.

2) Coulomb's law:

The study of a web tension on a roll can be considered as a problem of friction between solids [13, 14].

3) Equation of Continuity:

This equation, applied to the web, yields [13, 14]:

$$L\frac{dT_k}{dt} = E\,S(V_{k+1} - V_k) + T_{k-1}V_k - T_k(2V_k - V_{k+1})\,. \qquad (2)$$

where k is the span number (see Fig. 2).

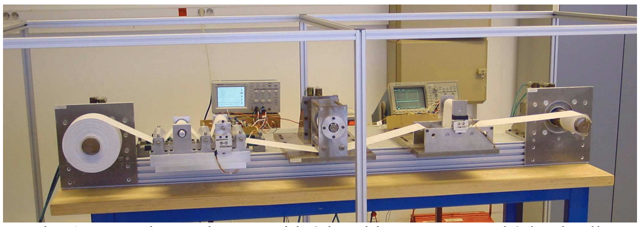

Fig. 1 : Experimental setup with 3 brushless motors and 2 load cells

B. Web Velocity Calculation

The linear velocity V_k of roll k is obtained from the torque balance [13, 14]:

$$\frac{d}{dt}\left(J_k\frac{V_k}{R_k}\right) = R_k(T_k - T_{k-1}) + K_kU_k + C_f\,, \qquad (3)$$

where K_kU_k is the motor torque and C_f is the friction torque. The inertia J_k and the radius R_k of unwind and winder are time dependent and vary substantially during processing.

A large scale web handling system of any number of driven rolls can be built from the equations (1), (2) and (3). A schematic representation of a multi-motor transport system is shown in Fig. A.2 in the Appendix.

C. State Space Representation

A scheme of a 3-motor setup with an industrial control strategy based on PI controllers is represented in Fig. 2. The inputs to the system are the torque control signals (u_u, u_v, u_w) of the brushless motors; the measurements are the unwinder and winder web tensions T_u and T_w and the web velocity $V = V_3$. The web velocity is set by the master traction motor whereas the web tensions in the spans are controlled by the unwind and rewind motors. The nonlinear state-space model is composed of (2) for the different web spans and (3) for the different rolls.

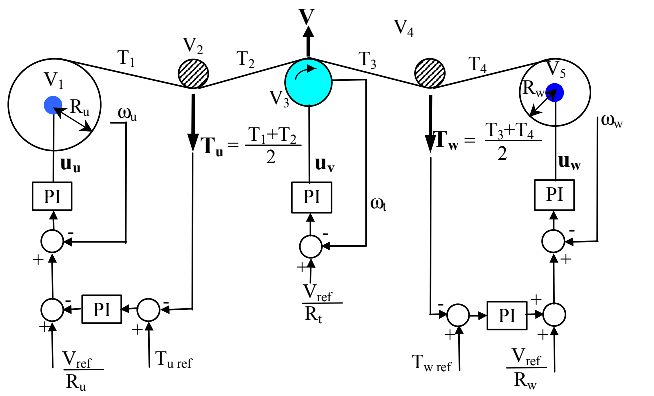

Fig. 2. Distributed industrial control scheme for a winding process

In [14] a global three-motor state space model is presented using a first order linearization and under the assumption that J_k/R_k is slowly varying. In this work, a more precise state space model is used by decomposing the nonlinear equations as follows [28]. Define

$$V_i = V_0 + v_i \quad T_i = T_0 + t_i \quad U_i = U_{si_0} + u_{si} \tag{4}$$

where v_i, t_i, u_{si} are signal variations around the reference values. The three-motor system variational dynamics can be presented in the following form:

Subsystem 1, with $\underline{x}_1^T = [v_1 \quad t_1 \quad v_2 \quad t_2]$:

$$E_1 \dot{\underline{x}}_1 = A_1 \underline{x}_1 + B_1 u_{s1} + H_1 + A_{12} \underline{x}_2 \tag{5}$$

Subsystem 2, with $\underline{x}_2^T = [v_3]$:

$$E_2 \dot{\underline{x}}_2 = A_2 \underline{x}_2 + B_2 u_{s2} + H_2 + A_{21} \underline{x}_1 + A_{23} \underline{x}_3 \tag{6}$$

Subsystem 3, with $\underline{x}_3^T = [t_3 \quad v_4 \quad t_4 \quad v_5]$:

$$E_3 \dot{\underline{x}}_3 = A_3 \underline{x}_3 + B_3 u_{s3} + H_3 + A_{32} \underline{x}_2 \tag{7}$$

A description of the first subsystem is given in the appendix. The constant values and the non-linear terms are included in the H_i vectors whereas the matrices A_{ij} describe the coupling effect of subsystem j on subsystem i.

III. CENTRALIZED CONTROL WITH FEEDFORWARD

Centralized control design for a 3-motor plant is the subject of the next subsection.

A. *Full order centralized H_∞ control design for a 3-Motor Plant*

Strong coupling between web velocity and tension makes the control of web transport systems inherently difficult. Several methods for suppressing this coupling in a system with two driven rolls have been studied [22].

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbance rejection. In unwinding-winding applications, the synthesis should be done using a linear model corresponding to the starting phase, i.e., an empty roller at the winder. The starting phase is very important: if a problem occurs in this phase, most likely, the rewind roll will be badly wound.

Due to a wide variation of the roller radius during the unwinding-winding process, the dynamic behavior of the system is considerably modified with time. With quasi-static assumption on radius variations, the static gains between the control signals and web tensions appear to be proportional to the inverse of the radius [14]:

$$Gain_{DC}\left(\frac{T_u}{u_u}\right) = \frac{1}{R_u} \quad and \quad Gain_{DC}\left(\frac{T_w}{u_w}\right) = \frac{1}{R_w} \tag{8}$$

We therefore multiply the control signals by the corresponding radius measurement or estimation and controller synthesis is done using the plant which includes the radii multiplication (gain scheduling). This approach allows us to reduce web tension variations significantly despite velocity changes during processing [14], [12].

We synthesized a centralized H_∞ controller with output weighting and model matching (Fig. 3) for the system composed of equations (5), (6) and (7) without the vectors H_i. Model M_o gave the desired transfer function T_{yr}. In our case, M_o was a second order transfer function.

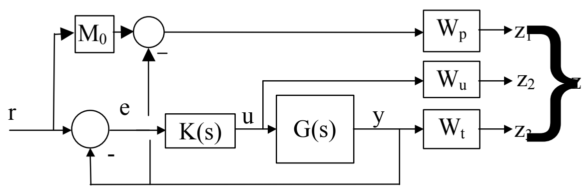

Fig. 3. S/KS/T weighting scheme with model matching for the design of H_∞ controllers

The weighting functions W_p, W_u, and W_t appear in the closed loop transfer matrix:

$$T_{zr} := \begin{bmatrix} W_p(M_0 - T_{yr}) \\ W_u S_{er} K \\ W_t T_{yr} \end{bmatrix} \tag{9}$$

where S_{er} is the sensitivity function and T_{yr} is the complementary sensitivity function. The controller $K(s)$ is calculated using LMI's (Linear Matrix Inequalities) with the LMI toolbox of the MATLAB Software in order to minimize the H_∞ norm of the transfer function T_{zr} between input vector r and weighted outputs z:

$$K = \operatorname{argmin} \| T_{zr} \|_\infty \tag{10}$$

where $\|T_{zr}\|_\infty = \sup_\omega \sigma_{\max}(T_{zr}(j\omega))$ and $\sigma_{\max}$ denotes the maximum singular value. The weighting function W_p has a high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as follows [14]:

$$W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0} \tag{11}$$

where M is the maximum peak magnitude of the sensitivity S_{er}, ω_B is the required frequency bandwith, and ε_0 is the steady-state error allowed. The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies.

To specify independently the tracking performance and robustness to perturbations, a two degree of freedom controller (for example a 2DOF H_∞ controller) can be used as described in [11, 2].

To take into account the inherent system nonlinearities and some constant values (such as static friction in roller bearings) in the control strategy, model based feedforward signals have been added to the control signals. Thus, the control signals U_{si0}, which depend on the reference values of web tension and velocity and on the system state, are added to the H_∞ controller outputs u_i (Fig. 4). These feedforward signals are calculated online with the feedforward controller called C_{ff} in Fig. 4: U_{si0} cancels the H_i element where it appears (all elements of H_i can not be cancelled using input U_{si0} , see Appendix)

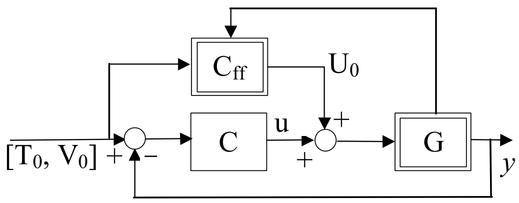

Fig. 4: Control strategy with feedforward signals

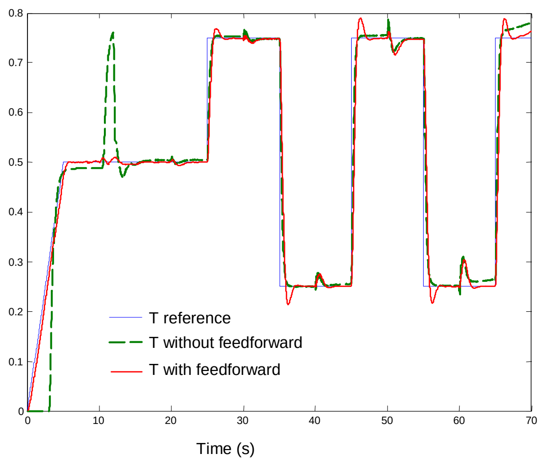

Fig. 5. Simulated web tensions (kg) for H_∞ centralized controller with and without additive feedforward signals

Figures 5 and 6, respectively, present the simulated web unwinder tension and web velocity (by assuming no slipage between the web and master speed roller) for H_∞ controller with and without additive feedforward signals. As expected, the centralized controller with additive feedforward signals not only improves the starting phase but also cancels the static errors to the web reference tension and velocity.

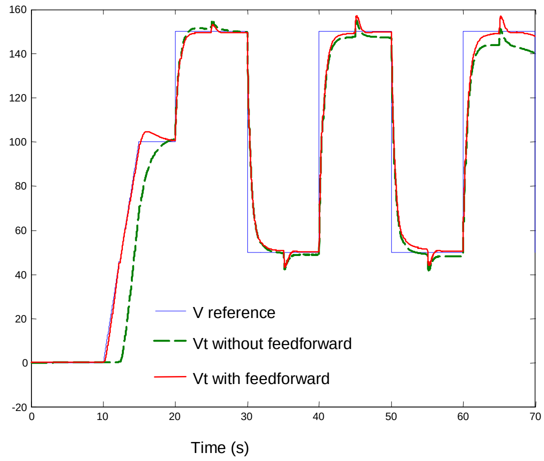

Fig. 6. Simulated web velocity (m/min) for H_∞ centralized controller with and without additive feedforward signals

In Fig. 7, G is changed and thus the feedforward controller C_{ff} and the feedback controller C have been calculated for the initial system G_{init}. Moreover, C_{ff} neglects now the static and viscous friction effects. We can observe some tension oscillations in the starting phase. This chatter is cancelled by adding a constant parameter (used as a tuning parameter) to C_{ff} .

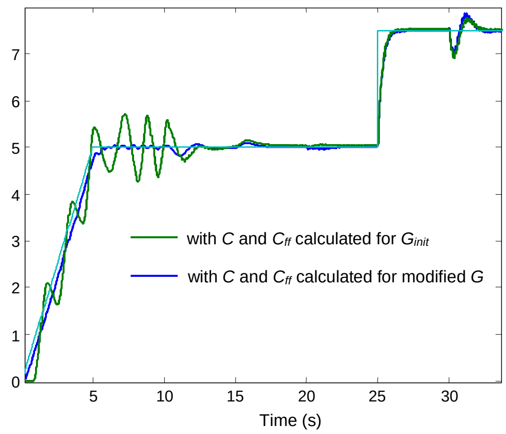

Fig. 7. Simulated web tensions (N) for H_∞ centralized controller with additive feedforward signals

B. *Reduced order centralized H_∞ control design for a 3-motor plant*

The approach we follow to design a fixed-order linear controller ensuring closed-loop $\mathcal{H}_\infty$ performance is as follows. First, the closed-loop system must be stabilized. The closed-loop system matrix is affine in the controller state-space matrices, see relation (3) in [30]. However, this matrix is nonsymmetric, hence the stabilization problem is nonconvex when formulated in the controller parameter space. Stabilization is ensured when the spectral abscissa (the largest real part of the eigenvalues) of the closed-loop system matrix is strictly negative. Hence, a direct way to ensure stabilization is to minimize the spectral abscissa. It turns out that this function can be nonsmooth, or even non-Lipschitz. Second, the $\mathcal{H}_\infty$ norm of the transfer function between a specified set of inputs and output must be minimized. In this case also, the underlying optimization problem is typically nonconvex and nonsmooth.

In order to overcome the nonconvexity and nonsmoothness of the fixed-order $\mathcal{H}_\infty$ design, various techniques have been developped:

- Convex approximations (polytopes, ellipsoids or LMI) of nonconvex stability regions, introducing an amount of conservatism which is sometimes difficult to assess.
- LMI formulations introducing lifting variables (e.g. Lyapunov matrices), which has the drawback of introducing many aritificial variables (typically of the order of the square of the system dimension).
- Nonconvex programming (global optimization, BMI solvers, nonsmooth optimization), mostly with a guarantee of local convergence only (since finding and certifying global optima is generally too expensive).

In this paper, we follow the latter approach. After several years of fundamental research in nonlinear variational analysis, Burke, Lewis and Overton recently designed a nonsmooth, nonconvex, hybrid optimisation algorithm implemented in a public-domain MATLAB package called HANSO (Hybrid Algorithm for Non-Smooth Optimization). The algorithm mixes in a parametrizable, user-friendly way, several optimization techniques, namely quasi-Newton updating, bundling and gradient sampling [31].

HANSO is at the core of another public-domain MATLAB package called HIFOO ($\mathcal{H}_\infty$ Fixed Order Optimization) which is tailored at solving fixed-order controller design problems. See [30] for a brief introduction to HIFOO, with some simple numerical examples. HIFOO can be downloaded at http://www.cs.nyu.edu/overton/software/hifoo

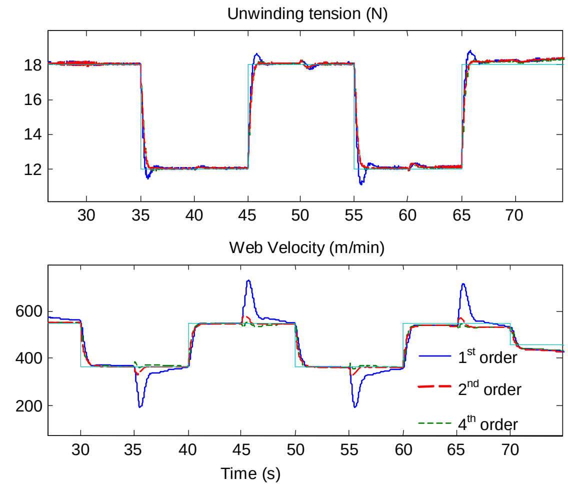

Fig. 8. Simulated web unwinding tension (N) and web velocity (m/min) for reduced order H_∞ centralized controller without additive feedforward signals

Simulation results, using the non-linear simulator, are given in Fig. 8 for different orders of the centralized controller applied on the 3 motor plant.

IV. Decentralized Control with Feedforward

The focus of this part is to design completely decentralized controllers for web processing lines. Therefore, in industrial applications, the global system is divided into several subsystems with each subsystem containing exactly one actuator: one subsystem is only under velocity control (master speed roller) whereas the other subsystems are under web tension control.

To improve the dynamic behavior, additive measures can be included in the controller synthesis (Fig. 9); similar to the PI strategy presented in Fig. 2. In our case, the web velocity measured in each subsystem is used as the second controller input. The synthesis scheme is represented on Fig. 10.

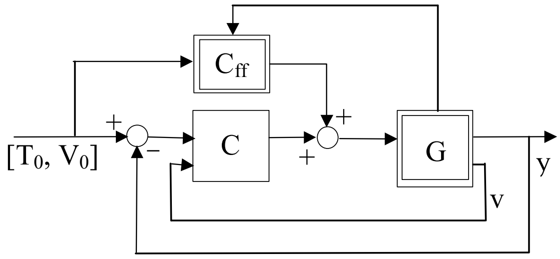

Fig. 9: Control strategy with additive measures and feedforward signals

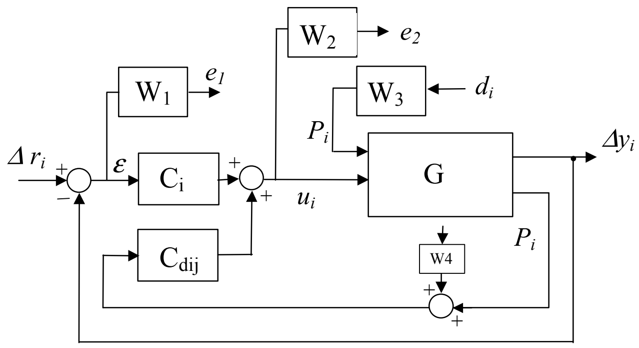

Fig. 10. H_∞ control design with additional measure

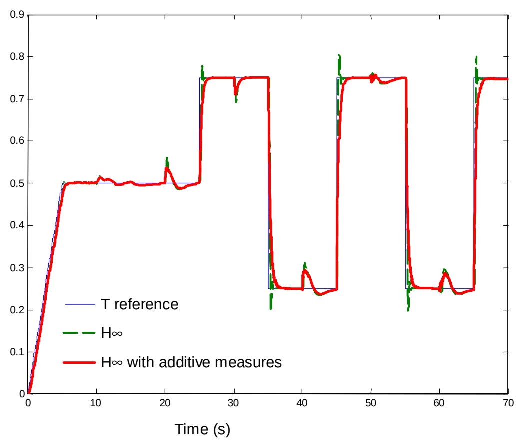

Fig. 11. Simulated unwinding web tension (kg) for completely decentralized controllers with feedforward signals: H_∞ with and without additive measures

Fig. 11 shows the unwind tension simulation result for decentralized H_∞ controller (with feedforward signals) and with/without additive velocity measures. The two-input controller improves reference tracking while reducing velocity-tension coupling.

For calculating H_∞ based PI controllers for each subsystem, the adopted control strategy is represented in Fig. 12. The zero-order controller $\underline{k}^T$ (constant vector) has to be calculated in order to minimize the H_∞ norm of the transfer function between the inputs and the weighted outputs, by using the HIFOO software. This synthesis problem now consists of computing an H_∞ static output feedback controller for an augmented system.

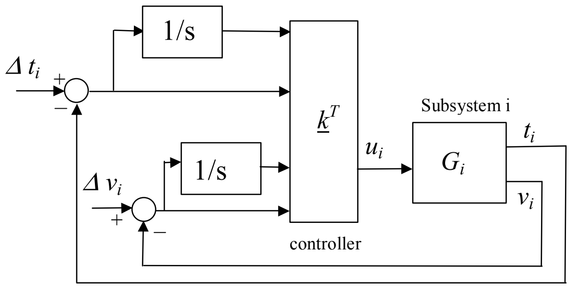

Fig. 12 : PI control of subsystem i

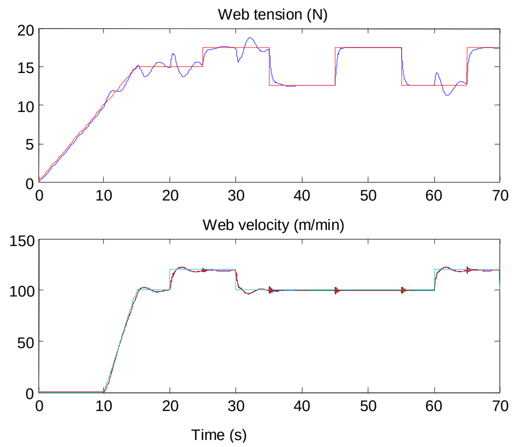

Fig. 13 : H_∞ PI decentralized control

In Fig. 13, H_∞ based PI decentralized controllers give satisfactory tension and velocity reference tracking. Nevertheless, the influence of a velocity step on web tension is not negligible. By replacing the zero-order controller $\underline{k}^T$ with a one-order, the results are improved.

V. Conclusion

Web processing lines are generally large scale systems and therefore it is not suitable to use a centralized controller for such processes. In this paper, a decentralized state space model for web processing lines is presented which leads to online calculation of feedforward control action. Decentralized fixed-order H_∞ control strategies with feedforward actions are synthesized and validated on a realistic nonlinear web handling simulator. The synthesis of H_∞ based PI decentralized controllers is relevant for industrial applications. Future work will deal with fixed structure controller synthesis to improve the decoupling between consecutive subsystems.

Acknowledgment

The authors wish to thank the French Ministry of Research for financial support through the project "Winding and high velocity handling of flexible webs" (ERT 8, contract number 01 B 0395). We are grateful to J. V. Burke, A. S. Lewis and M. L. Overton for showing interest in solving control problems. Our special thanks go to M. L. Overton for spending so much time and energy in coding, developing and debugging HANSO and HIFOO.

References

[1] B. D. O. Anderson, J. B. Moore, *Optimal Control: Linear Quadratic Techniques*. New Jersey: Englewood Cliffs-Prentice Hall, 1990.

[2] A. Benlatreche, D. Knittel, E. Ostertag, "Robust decentralized control strategies for large scale web handling systems," in *Proc. IFAC Large scale systems conference, July 26-28, 2004, Osaka, Japan.*

[3] A. Benlatreche, D. Knittel, E. Ostertag, "State feedback controllers synthesis using BMI optimisation for large scale web handling systems," *16th IFAC World Congress, July 4-8, 2005, Prague, Czech Republic*.

[4] A. Benlatreche, D. Knittel, E. Ostertag, "State feedback control with full or partial integral action for large scale winding systems," *40th Annual General Meeting of Industry Applications Society, October 2-6, 2005, Hong Kong*.

[5] A. Benlatreche, D. Knittel, E. Ostertag, "Robust decentralized control strategies for large scale web handling systems," *IFAC J. Control Engineering Practice*, to appear.

[6] S. Boyd, C. Barrat, *Linear controller design. Limits of performance.* Prentice Hall, 1991.

[7] J.E. Geddes, M. Postlethwaite, "Improvements in Product Quality in Tandem Cold Rolling Using Robust Multivariable Control", 1998 *IEEE Trans. On Control Systems*, vol 6, No. 2.

[8] J. E. Geddes, M. Postlethwaite, "Improvements in Product Quality in Tandem Cold Rolling Using Robust Multivariable Control," 1998 *IEEE Trans. on Control Systems*, vol. 6, No 2.

[9] M. J. Grimble, G. Hearns, *Advanced control for hot rolling mills*: *Advances in control*, *Highlights of ECC'99*. Springer, pp. 135-169.

[10] D. Knittel, D. Gigan, E. Laroche, "Robust decentralized overlapping control of large scale winding systems," in *Proc*. 2002 *American Control Conference, Anchorage*.

[11] D. Knittel, E. Laroche, D. Gigan, H. Koç, "Tension control for winding systems with two degrees of freedom $\mathcal{H}_\infty$ controller," 2003 *IEEE Trans. on Industry Applications,* 39(1), pp. 113-120.

[12] D. Knittel, "Robust control design using $\mathcal{H}_\infty$ methods in large scale web handling systems," in *Proc. of 7th Int. Conf. on Web Handling, IWEB2003,* Stillwater (Oklahoma).

[13] H. Koç, "Modélisation et commande robuste d'un système d'entraînement de bande flexible," *Ph.D thesis (in French)*. University Louis Pasteur (Strasbourg I University), France, 2000.

[14] H. Koç, D. Knittel, M. de Mathelin, G. Abba, "Modelling and robust control of winding systems for elastic webs," 2002 *IEEE Trans. on Control Systems Technology* 10(2), pp. 197-208.

[15] D. D. Siljak, *Decentralized control of complex systems*. Academic press, 1991.

[16] S. S. Stankovic, M. J. Stanojevic, D. D. Siljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Trans. on Control Systems Technology*, vol.8, N° 8, pp. 816-832, September, 2000.

[17] K. Zhou, J. Doyle, K. Glover, *Robust and Optimal Control*. New Jersey, *Prentice Hall*, 1995. Upper Saddle River.

[18] P. R. Pagilla, E. O. King, L. Dreinhoefer and S. Garimella, “Robust observer-based control of an aluminum strip processing line”, IEEE Transactions on Industry Applications, vol. 36, no. 3, pp. 835-840, 2000

[19] P. R. Pagilla, S. S.. Garimella, L. H. Dreinhoefer and E. O. King “Dynamics and control of accumulators in continuous strip processing lines”, IEEE Transactions on Industry Applications, vol. 37, no. 3, pp. 934-940, 2001.

[20] P.R. Pagilla, I.P. Singh and R.V. Dwivedula, “A Study on Control of Accumulators in Web Processing Lines,’’ ASME Journal of Dynamic Systems, Measurement, and Control, vol. 126, pp. 453-461, September 2004.

[21] P. R. Pagilla, R. V. Dwivedula, Y. Zhu and L. P. Perera “ Periodic tension disturbance attenuation in web process lines using active dancers” ASME Journal of Dynamic Systems, Measurement, and Control, vol. 125, PP. 361-371, September 2003.

[22] R. V. Dwivedula, Y. Zhu and P. R. Pagilla, “Characteristics of active and passive dancers: A comparative study”, Control Engineering Practice, 2005, vol. 14, no. 3, pp. 409-423, April 2006.

[23] P. R. Pagilla, N. B. Siraskar and R. V. Dwivedula, “Decentralized control of web processing lines”, in Proc. of the 2005 IEEE Conf. on Control Applications, Toronto, Canada.

[24] P. R. Pagilla, N. B. Siraskar and R. V. Dwivedula, “Decentralized adaptive control of large-scale systems with applications to web processing lines”, 16th IFAC World Congress, Prague, Czech Republic, July 2005.

[25] P. R. Pagilla, D. Knittel, “Recent advances in web longitudinal control,” in *Proc.* of 8*th* *Inlt. Conf. on Web Handling, IWEB2005,* Stillwater (Oklahoma).

[26] G. Duc and S. Font, “Commande $\mathcal{H}_\infty$ et μ analyse”, Edition Hermes, France, 1999.

[27] T. Sakamoto and T. Kobayashi, “Decomposition and decentralized controller design of web transfer system”, in *Proc. IFAC Large scale systems conference, July 26-28, 2004, Osaka, Japan*

[28] D. Knittel, A. Arbogast, M. Vedrines, P. Pagilla, “Decentralized robust control strategies with model based feedforward for elastic web winding systems”, *American Control Conference*, Minnesota, 2006

[29] J. V. Burke, A. S. Lewis, M. L. Overton. A nonsmooth, nonconvex optimization approach to robust stabilization by static output feedback and low-order controller. *Proc. IFAC Symp. Robust Control Design, Milan, Italy, 2003*.

[30] J. V. Burke, D. Henrion, A. S. Lewis, M. L. Overton. HIFOO - a MATLAB package for fixed-order controller design and H-infinity optimization. *Submitted to IFAC Symp. Robust Control Design, Toulouse, France, 2006*

[31] J. V. Burke, A. S. Lewis, M. L. Overton. A robust gradient sampling algorithm for nonsmooth, nonconvex optimization. SIAM J. Optim. 15:751-779, 2005.

Appendix

A1- Modeling of subsystem 1:

Mechanical equation of the unwind roll:

Nomenclature :

K_1	motor torque constant
U_{s1}	motor torque
C_{fsunw}	static friction coefficient
$f_{v1u}, f_{v1u}, f_{v1u}$	viscous friction coefficient
J_1	unwinder inertia
R_1	wound roll radius
l	web width
ρ	web mass density
h	web thickness

$$\frac{d}{dt}\left(J_1\Omega_1\right) = R_1T_1 - K_1U_{s1} - C_{fs_{unw}} - f_{v_{1u}}\Omega_1 + f_{v_{2u}}\Omega_1^2 - f_{v_{3u}}\Omega_1^3$$

and with the inertia calculation : $J_1 = \frac{\pi}{2} l \rho \left(R_1^4 - R_{1_0}^4\right)$

the result is :

$$\frac{d}{dt}\left(J_1\frac{V_1}{R_1}\right) = -2\pi l \rho R_1^3 \frac{h}{2\pi}\frac{V_1}{R_1}\frac{V_1}{R_1} + J_1\frac{\dot{V}_1}{R_1} + J_1\frac{V_1}{R_1^2}\frac{h}{2\pi}\frac{V_1}{R_1}$$

with : $V_1 = V_0 + v_1;\quad T_1 = T_0 + t_1;\quad U_{s1} = U_{s1_0} + u_{s1}$ the non-linear equation for small signals leads to :

$$J_1\dot{v}_1 = \left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right)V_0^2 - \frac{f_{v_{3u}}}{R_1^2}V_0^3 - R_1C_{fs_{unw}} - f_{v_{1u}}V_0 + R_1^2T_0 - R_1K_1U_{s1_0}\right)$$
$$+\left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right) + 3\frac{f_{v_{3u}}}{R_1^2}V_0\right)v_1^2 - \frac{f_{v_{3d}}}{R_1^2}v_1^3$$
$$+\left(\left(l\rho h R_1^2 - \frac{J_1 h}{2\pi R_1^2} + \frac{f_{v_{2u}}}{R_1}\right)2V_0 - 3\frac{f_{v_{3u}}}{R_1^2}V_0^2 - f_{v_{1u}}\right)v_1 + R_1^2 t_1 - R_1K_1u_{s1}$$

Mechanical equation of the roller with unwind tension sensor:

f_2	viscous friction coefficient	$J_2\dot{V}_2 = -R_2^2\left(T_2 - T_1\right) - f_2V_2$

with: $V_2 = V_0 + v_2;\quad T_1 = T_0 + t_1;\quad T_2 = T_0 + t_2$ the result is : $J_2\dot{v}_2 = -f_2V_0 - R_2^2t_1 - f_2v_2 + R_2^2t_2$

Web span tension models:

Nomenclature :

E	Young modulus
S	web section

$$L_1\dot{T}_1 = V_2\left(ES + T_1\right) - V_1\left(ES + 2T_1 - T_b\right)$$

S web section Tb unwinding wound internal tension	$L_2\dot{T}_2 = V_3(ES - T_2) - V_2(ES + 2T_2 - T_1)$
With $V_1 = V_0 + v_1;\ V_2 = V_0 + v_2;\ T_1 = T_0 + t_1$ and $V_2 = V_0 + v_2;\ V_3 = V_0 + v_3;\ T_2 = T_0 + t_2$ the result is : $L_1\dot{t}_1 = V_0T_b - V_0T_0 - (E_0 + T_0 - T_b)v_1 - V_0t_1 + E_0v_2 + v_2t_1 - 2v_1t_1$ $L_2\dot{t}_2 = V_0t_1 - E_0v_2 - 3V_0t_2 + (E_0 - 2T_0)v_3 - 2V_0T_0 + t_1v_2 - 2v_2t_2 - v_3t_2$ with $E_0 = ES + T_0$	

State space representation of subsystem 1:

$$E_1\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1u_{s1} + H_1 + A_{12}\underline{x}_2 \quad \text{with} \quad \underline{x}_1^T = [v_1 \quad t_1 \quad v_2 \quad t_2]$$

$$E_1 = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 \\ 0 & 0 & J_2 & 0 \\ 0 & 0 & 0 & L_2 \end{bmatrix} \quad A_1 = \begin{bmatrix} a_1 & R_1^2 & 0 & 0 \\ -(E_0 + T_0 - T_b) & -V_0 & -E_0 & 0 \\ 0 & -R_2^2 & -f_2 & R_2^2 \\ 0 & V_0 & -E_0 & -3V_0 \end{bmatrix} \quad B_1 = \begin{bmatrix} -K_1R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E_0 - 2T_0 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \left(\rho SR_1^2 - \dfrac{J_1h}{2\pi R_1^2} + \dfrac{f_{v_{2u}}}{R_1}\right)V_0^2 - \dfrac{f_{v_{3u}}}{R_1^2}V_0^3 - R_1C_{fs_{unw}} - f_{v_{1u}}V_0 + R_1^2T_0 - R_1K_1U_{s1_0} + \left(\rho SR_1^2 - \dfrac{J_1h}{2\pi R_1^2} + \dfrac{f_{v_{2u}}}{R_1} + 3\dfrac{f_{v_{3u}}}{R_1^2}V_0\right)v_1^2 - \dfrac{f_{v_{3u}}}{R_1^2}v_1^3 \\ V_0T_b - V_0T_0 + v_2t_1 - 2v_2t_1 \\ -f_2V_0 \\ -2V_0T_0 + t_1v_2 - 2v_2t_2 - v_3t_2 \end{bmatrix}$$

$$\text{where } a_1 = 2V_0\left(\rho SR_d^2 - \frac{J_dh}{2\pi R_d^2} + \frac{f_{v_{2d}}}{R_d}\right) - 3\frac{f_{v_{3d}}}{R_d^2}V_0^2 - f_{v_{1d}}$$

A2 : decentralized control strategy for a large scale web transport system