Fixed-Order Robust H_∞ Controller Design with Regional Pole Assignment

Fuwen Yang, Mahbub Gani, and Didier Henrion

Abstract

In this paper, the problem of designing fixed-order robust H_∞ controllers is considered for linear systems affected by polytopic uncertainty. A polynomial method is employed to design a fixed-order controller that guarantees that all the closed-loop poles reside within a given region of the complex plane. In order to utilise the freedom of the controller design, an H_∞ performance specification is also enforced by using the equivalence between robust stability and H_∞ norm constraint. The design problem is formulated as a Linear Matrix Inequality (LMI) constraint whose decision variables are controller parameters. An illustrative example demonstrate the feasibility of the proposed design methods.

Index Terms

Fixed-order controller; regional pole assignment; robust H_∞ control; polytopic uncertain system; LMI

I. Introduction

The controllers obtained from standard design procedures usually have same order as the McMillan order of the generalised plant derived from a physical plant and some frequency weighting functions [18]. Since the order of the generalised plant may be high, designing full-order controllers narrows the scope of use in practical applications, such as embedded control systems for the space and aeronautics industry. Hence there has been an increasing and considerable interest in designing low-, fixed-order controllers, see e.g. [10], [17], [9] and references therein for a sample. However, there are fundamental difficulties inherent to the fixed-order controller design. Many researchers have attempted to tackle these issues in the last three decades. For example, in [5], [6], the fixed-order controller

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design problem was formulated in a state-space framework as an LMI minimisation problem subject to an additional nonconvex, matrix rank constraint.

Recently, an LMI formulation was developed for fixed-order controller design in a polynomial framework, based on polynomial positivity conditions [9]. The method can assign the closed-loop poles in a given region of the complex plane, solving the regional pole assignment problem [3]. Nonconvexity of the fixed-order controller design problem is overcome by choosing a particular tuning parameter, the so-called central polynomial. The limitation and conservatism of the approach are concentrated in the choice of this polynomial, and some physically motivated rules of thumb are suggested in [9].

In a typical design setup, besides region pole assignment, we also expect that the closed-loop system satisfies some additional performance indices, such as H_2 and/or H_∞ norm bounds. Therefore, in [7], a polynomial method was introduced to solve the H_∞ control problem, as an extension of the approach of [9]. A sufficient condition has been provided via a geometric argument method. In this paper, we propose another, distinct extension of the polynomial approach of [9] for the regional pole assignment to the H_∞ design with the fixed-order controller. Contrary to the geometric argument method used in [7], we use the well-known equivalence between robust stability and H_∞ norm constraint, resulting in a perhaps simpler formulation.

We assume additionally that the systems under consideration have polytopic uncertainty, a rather general way of capturing the lack of knowledge on the physical system parameters. It includes the well-known interval parametric uncertainty [1]. Most results invoking polytopic systems are robust stability analysis results [4], [14], [8]. Only a few design techniques have been proposed, see e.g. [10], [17], [9].

Based on these motivations, in this paper, we consider the regional pole assignment and H_∞ optimisation problem with fixed-order controller for polytopic uncertain systems. A unified fixed-order robust H_∞ controller design method for both continuous and discrete-time systems is introduced. As in [9], the whole conservatism of the approach is captured by the choice of the central polynomial, but on the other hand the design conditions are provided in a flexible linear matrix inequality (LMI) framework [2], allowing the use of standard convex optimisation software.

The remainder of this paper is organised as follows. In Section II, the problem of designing fixed-order robust H_∞ controllers is formulated for polytopic uncertain systems. A polynomial LMI based condition is developed in Section III to design the fixed-order controller such that the closed-loop poles reside within a stability region. The fixed-order robust H_∞ controllers are designed in Section IV by using the equivalence between robust stability and H_∞ norm constraint. An illustrative example is presented in Section V to demonstrate the feasibility of the proposed method. Some concluding remarks and future research directions are provided in Section VI.

II. Problem Formulation

Consider the transfer function of an uncertain system

$$p(s,\lambda) = \frac{b(s,\lambda)}{a(s,\lambda)}$$

where $a(s,\lambda)$ and $b(s,\lambda)$ are polynomials in the Laplace (continuous-time) or shift (discrete-time) operator s, and parameter λ models the system uncertainty. We assume that $a(s,\lambda)$ and $b(s,\lambda)$ are affected by polytopic uncertainty, i.e., they reside in polytopes with given vertices $a^i(s)$, and $b^i(s)$, for $i = 1, 2, \cdots, N$.

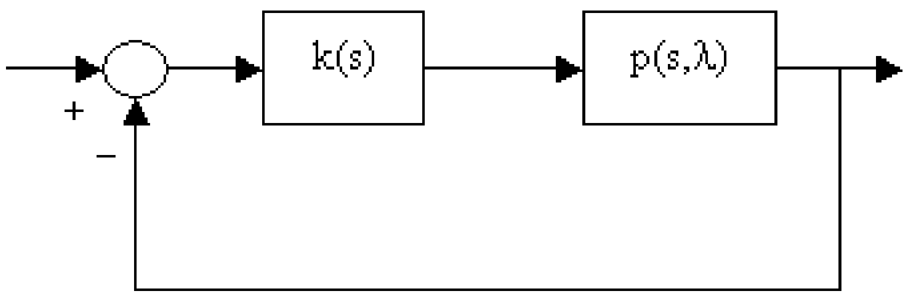

Fig. 1. Standard negative feedback configuration.

We consider a standard negative feedback configuration shown in Fig. 1, where

$$k(s) = \frac{y(s)}{x(s)}$$

is the transfer function of the fixed-order controller, with $x(s)$ and $y(s)$ the controller denominator and numerator polynomials, respectively.

For the purpose of regional pole assignment, we introduce

$$\mathcal{D} = \left\{ s \in \mathbb{C} : \begin{bmatrix} 1 \\ s \end{bmatrix}^* \underbrace{\begin{bmatrix} d_{11} & d_{12} \\ d_{12}^T & d_{22} \end{bmatrix}}_{D} \begin{bmatrix} 1 \\ s \end{bmatrix} < 0 \right\} \qquad (1)$$

as a stability region in the complex plane, where the star $*$ denotes transpose conjugate and symmetric matrix D has one strictly positive eigenvalue and one strictly negative eigenvalue. Standard choices for $\mathcal{D}$ are the left half-plane ($d_{11} = 0, d_{12} = 1, d_{22} = 0$, continuous-time) and the unit disk ($d_{11} = -1, d_{12} = 0, d_{22} = 1$, discrete-time). As explained in [8], with different choices of the scalars d_{11}, d_{12} and d_{22}, the region $\mathcal{D}$ can be used to represent many kinds of standard pole regions, such as disk, vertical strips, horizontal strips, conic sector, etc. In the sequel, we say that a polynomial is D-stable when its roots belong to the region $\mathcal{D}$ shaped by matrix D.

The problem of designing a fixed-order controller $k(s)$ such that, for all admissible uncertainty affecting system $p(s,\lambda)$, the closed-loop poles belong to the $\mathcal{D}$ stability region has been addressed in [9] by using a polynomial method and LMIs. Our objective is to extend this method so that for all admissible uncertainty, the closed-loop system achieves the following H_∞ performance:

$$\|G(s,\lambda)\|_\infty \le \gamma,$$

for a prescribed positive scalar γ, and where $G(s,\lambda)$ is any transfer function whose numerator $n(s,\lambda)$ and denominator $d(s,\lambda)$ polynomials both depend affinely in controller polynomials $x(s)$, $y(s)$. Transfer function $G(s)$ can be for example the sensitivity function or complementary sensitivity function [18].

III. Regional Pole Assignment

In this section, we consider real polynomials $c(s) = c_0 + c_1 s + ... + c_m s^m$ and $d(s) = d_0 + d_1 s + ... + d_m s^m$. We also assume that both leading coefficient scalars c_m and d_m are non-zero, so that polynomials $c(s)$ and $d(s)$ do not feature infinite zeros.

Let

$$c = \begin{bmatrix} c_0 & c_1 & \cdots & c_m \end{bmatrix}, \qquad d = \begin{bmatrix} d_0 & d_1 & \cdots & d_m \end{bmatrix}$$

respectively denote row vector coefficients of $c(s)$, $d(s)$ and let

$$\Pi = \begin{bmatrix} 1 & & & 0 \\ & \ddots & & \vdots \\ & & 1 & 0 \\ 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \end{bmatrix}$$

be a matrix of size $2m \times (m+1)$.

Lemma 1: [9] Polynomial $d(s)$ is D-stable if and only if there exist a D-stable polynomial $c(s)$ and a symmetric matrix $P = P^T$ of dimension m such that

$$c^T d + d^T c - \Pi^T (D \otimes P) \Pi \succeq 0,$$

where $\otimes$ is the Kronecker product, and $\succeq 0$ means positive semidefinite.

We notice that the LMI condition of Lemma 1 is simultaneously linear in coefficients of $d(s)$ and in matrix P. This property can be exploited to provide less conservative convex conditions than the well-known quadratic stability for assessing robust stability of polynomials, for details see [8].

Now consider that uncertain polynomial $d(s,\lambda)$ resides in a polytope with given vertices $d^i(s)$ for $i = 1, 2, \cdots, N$. Suppose that $d^i(s) = d_0^i + d_1^i s + ... + d_m^i s^m$ for $i = 1, 2, \cdots, N$ and let us denote its row vector of coefficients d^i. With these notations, we have the following result from [9].

Lemma 2: [9] The uncertain polynomial $d(s,\lambda)$ belonging to a polytope with given vertices $d^i(s)$ is robustly D-stable if there exist a D-stable polynomial $c(s)$ and N matrices $P^i = P^{iT}$ of dimension m satisfying the LMIs

$$c^T d^i + d^{iT} c - \Pi^T (D \otimes P^i) \Pi \succeq 0$$

for all $i = 1, 2, \ldots, N$.

Note that the condition of Lemma 2 is sufficient, but not necessary in general, for ensuring robust stability of a polynomial polytope, Extending Lemma 2, we can also find the controller with transfer function $y(s)/x(s)$ such that the closed-loop characteristic polynomial $d(s,\lambda) = a(s,\lambda)x(s) + b(s,\lambda)y(s)$ is robustly D-stable for all admissible uncertainties in the system $p(s,\lambda)$. Let us denote $d^i(x,y)$ the row vector of coefficients of vertex polynomial $d^i(s,\lambda)$. Note that $d^i(x,y)$ depends affinely on x and y, the row vectors of coefficients of polynomials $x(s)$ and $y(s)$, respectively.

Theorem 1: Given a D-stable central polynomial $c(s)$, for all uncertain system polynomials $a(s,\lambda)$ and $b(s,\lambda)$ which reside in polytopes with given vertices $a^i(s)$ and $b^i(s)$, there exist controller polynomials $x(s)$ and $y(s)$ such that the closed-loop system is robustly D-stable if there exist vectors x, y, and symmetric matrices $P^i = P^{iT}$ satisfying the LMIs

$$c^T d^i(x,y) + d^i(x,y)^T c - \Pi^T (D \otimes P^i)\Pi \geq 0 \tag{2}$$

for all $i = 1, 2, \ldots, N$.

Theorem 1 provides a simple fixed-order controller design method since the coefficients of the controller polynomial are explicit decision variables entering the LMI. Contrary to the design procedure outlined in [5], [6], where a full-order controller is reconstructed via tedious linear algebra, solving LMI (2) directly yields controller coefficients.

Note also that the degree of the central polynomial $c(s)$ enforces the degree of the controller polynomials $x(s)$, $y(s)$ to be found. However, we have only sufficient conditions for fixed-order controller design, the main source of conservatism being the choice of the central polynomial $c(s)$.

In the next section, we extend these results to H_∞ performance.

IV. Robust H_∞ Control

H_∞ controller design has been studied mostly in a state-space setting [5], [6], [18], but also in a polynomial setting [12], [11]. However, state-space methods generate controllers of the same order as the plant. In this section, we will use a polynomial method to design fixed-order controllers which satisfy an H_∞ performance. For that purpose, we use the equivalence between robust stability and H_∞ performance constraint.

Consider a polynomial affected by an additive uncertainty:

$$d_\delta(s) = d(s) + n(s)\delta, \quad \|\delta\| < \gamma^{-1}$$

where $d(s)$ is a given nominal polynomial, $n(s)$ is also a given polynomial and δ is a real-valued scalar whose magnitude is less than a given bound γ^{-1}. In virtue of the small-gain theorem[1], see e.g. [18, Theorem 9.1], robust stability of uncertain polynomial $d_\delta(s)$ is equivalent to the H_∞ performance constraint

$$\left\| \frac{n(s)}{d(s)} \right\|_\infty \leq \gamma \tag{3}$$

[1]The small-gain theorem is most often formulated for complex-valued uncertainty. In [15] it was shown that in the case of a single uncertainty block, it is equally valid for real-valued uncertainty.

on the rational transfer function $n(s)/d(s)$. Denoting by n the row vector of coefficients of polynomial $n(s)$, we obtain the following Corollary from Lemma 1.

Corollary 1: Given a D-stable polynomial $c(s)$ and a scalar $\gamma > 0$, the transfer function $n(s)/d(s)$ is D-stable and satisfies the H_∞ performance constraint (3) if there exist a symmetric matrix $P = P^T$ and a scalar ε such that

$$\begin{bmatrix} c^T d + d^T c - \varepsilon c^T c - \Pi^T (D \otimes P)\Pi & n^T \\ n & \varepsilon\gamma^2 \end{bmatrix} \succeq 0. \quad (4)$$

Proof: Applying LMI stability condition (2) to uncertain polynomial matrix $d_\delta(s)$ yields the uncertain LMI

$$c^T(d + \delta n) + (d + \delta n)^T c - \Pi^T (D \otimes P)\Pi \geq 0 \quad (5)$$

that must be satisfied for all real-valued δ such that

$$\gamma^{-2} - \delta^2 > 0.$$

Applying Finsler's Lemma [16] to the uncertain LMI (5) yields robust LMI (4). ■

Note that, as soon as c and γ are fixed, LMI (4) is linear in the decision variables P, ε, n and d.

The relation between the central polynomial and a characteristic polynomial achievable by pole assignment is clarified in the following lemma.

Lemma 3: If controller polynomials $x(s)$, $y(s)$ are known such that closed-loop performance constraint (3) is ensured, then upon setting $c(s) = a(s)x(s) + b(s)y(s)$ in Corollary 1, LMI (4) becomes necessarily feasible.

Proof: If controller polynomials $x(s)$, $y(s)$ ensure closed-loop stability and performance, then H_∞ constraint (3) can be expressed as the scalar polynomial positivity constraint

$$d^*(s)d(s) - \gamma^{-2} n^*(s)n(s) \geq 0$$

for all $s \in \partial\mathcal{D}$, the one-dimensional boundary of the stability region $\mathcal{D}$. Following the proof of Lemma 3 in [9], it follows that there exists a matrix P satisfying the matrix inequality

$$d^T d - \gamma^{-2} n^T n - \Pi^T (D \otimes P)\Pi \succeq 0.$$

Using a Schur complement argument, this can be written as

$$\begin{bmatrix} d^T d - \Pi^T (D \otimes P)\Pi & n^T \\ n & \gamma^2 \end{bmatrix} \succeq 0$$

which is LMI (4) upon setting $c(s) = d(s) = a(s)x(s) + b(s)y(s)$ and $\varepsilon = 1$. ■

In other words, the previous lemma indicates that a "good" choice of central polynomial $c(s)$ in LMI (4) is precisely an achievable characteristic polynomial $d(s) = a(s)x(s)+b(s)y(s)$ ensuring stability and H_∞ performance. Of course, in practice, such a polynomial is not known in advance, otherwise the design problem has been solved.

Now we consider the uncertain polynomial $n(s, \lambda)$ and $d(s, \lambda)$ which reside in polytopes with given vertices $n^i(s)$ and $d^i(s)$ $(i = 1, 2, \cdots, N)$, where $n^i(s) = n^i_0 + n^i_1 s + ... + n^i_m s^m$ $d^i(s) = d^i_0 + d^i_1 s + ... + d^i_m s^m$ for

$i = 1, 2, \ldots, N$. Define the robust H_∞ performance constraint as

$$\left\| \frac{n(s,\lambda)}{d(s,\lambda)} \right\|_\infty \leq \gamma \tag{6}$$

which must hold for all admissible values of uncertain parameter λ. Finally, assume as in Section III that coefficient row vectors $n^i(x,y)$, $d^i(x,y)$ depend affinely in coefficients of controller polynomials $x(s)$ and $y(s)$.

Theorem 2: Given a D-stable central polynomial $c(s)$ and a scalar $\gamma > 0$, then for all uncertain system polynomials $a(s,\lambda)$ and $b(s,\lambda)$ which reside in polytopes with given vertices $a^i(s)$ and $b^i(s)$, there exist controller polynomials $x(s)$ and $y(s)$ such that the closed-loop system is robustly D-stable and satisfies the H_∞ performance constraint (6), if there exist vectors x, y, symmetric matrices $P^i = P^{iT}$ and scalars ε_i such that the inequalities

$$\begin{bmatrix} c^T d^i(x,y) + d^i(x,y)^T c - \varepsilon_i c^T c - \Pi^T (D \otimes P^i) \Pi & n^i(x,y)^T \\ n^i(x,y) & \varepsilon_i \gamma^2 I \end{bmatrix} \geq 0, \tag{7}$$

are satisfied for all $i = 1, 2, \ldots, N$.

Proof: The proof is readily obtained from Theorem 1 and Corollary 1. ■

In contrast with the state-space solution to the H_∞ control problem, our polynomial method can readily accomodate controller structure requirements. The controller can be enforced any structure, such as e.g. PID. Moreover, continuous-time and discrete-time systems are treated in a unified framework.

From Theorem 2, we can see that the central polynomial plays a key role in the fixed-order controller design. It will affect the design performance and even the feasibility of the solution. Therefore, the designer should be careful when choosing the central polynomial, and this is the main limitation of the approach.

V. An Example

Consider a simple flexible structure whose transfer function is described by:

$$p(s,\lambda) = \frac{b(s,\lambda)}{a(s,\lambda)} = \frac{1}{s(s^2 + 2\xi\omega_1(\lambda)s + \omega_1^2(\lambda))(s^2 + 2\xi\omega_2(\lambda)s + \omega_2^2(\lambda))}$$

where ξ is a damping factor and pulsations $\omega_i(\lambda) = (1 + e\lambda_i)\omega_{i0}$, $|\lambda_i| \leq 1$ are known only within a given relative percentage e around nominal values ω_{i0}, due to modeling errors. For example let $\xi = 0.05$, $\omega_{10} = 1$, $\omega_{20} = 10$ and $e = 4\%$.

Our objective is to design a stabilizing second order controller $k(s) = y(s)/x(s)$ ensuring a sufficiently small H_∞ norm of the closed-loop sensitivity function

$$\|S(s,\lambda)\|_\infty = \left\| \frac{a(s,\lambda)x(s)}{a(s,\lambda)x(s) + b(s,\lambda)y(s)} \right\|_\infty \leq \gamma$$

for all possible values of the uncertain parameters λ. Typical desirable values lie around 1.4 [12]. The pole assignment region is the open left-half plane, hence $d_{11} = d_{22} = 0$ and $d_{12} = 1$ in (1). Since parameters ω_i appear multiaffinely, we overbound the uncertainty by considering that the denominator of $p(s,\lambda)$ is an interval polynomial whose coefficients vary independently in 4 distinct given intervals, generating a polytope of open-loop polynomials $a^i(s)$, $b^i(s)$ with $N = 2^4 = 16$ vertices. The performance specification is then formulated on the vertex sensitivity functions: $\|S^i(s)\|_\infty \leq \gamma$ for all $i = 1, 2, \ldots, 16$.

Following the rules of thumb outlined in [9], we choose a central polynomial mirroring the nominal stable open-loop poles, and containing additional dynamics:

$$c(s) = (s^2 + 0.1s + 1)(s^2 + s + 100)(s + 1)^3.$$

Here we add three roots at -1 corresponding to the open-loop plant integrator and the dynamics of the second-order controller. For the choice $\gamma = 2$, solving[2] the LMI problem (7) we obtain a first robust controller

$$k(s) = \frac{y(s)}{x(s)} = \frac{14.7553 - 48.6010s + 57.0964s^2}{2.94545 + 1.39576s + s^2}. \tag{8}$$

The unit step response of the open-loop plant (after normalization and without the integrator) is represented (dashed plot) on Fig. 2, together with the unit step responses of the 16 vertex closed-loop plants (solid plots). We can observe some initial undershoot and a relatively sluggish response with is otherwise overall insensitive to parameter variations. The Bode magnitude diagrams of the open-loop (dashed plot) and closed-loop vertex sensivitity functions $S^i(s)$ (solid plots) are represented on Fig. 3. The Bode magnitude diagrams of the open-loop (dashed plot) and closed-loop vertex complementary sensitivity functions $1 - S^i(s)$ (solid plots) are represented on Fig. 4.

If we desire a faster closed-loop response, we can enforce faster poles in the central polynomial:

$$c(s) = (s^2 + 0.1s + 1)(s^2 + s + 100)(s + 3)^3$$

while lowering the upper bound on the H_∞ norm. For the choice $\gamma = \sqrt{1.8}$, we obtain a second robust controller

$$k(s) = \frac{y(s)}{x(s)} = \frac{1495.67 + 844.919s + 2438.32s^2}{34.0240 + 9.48836s + s^2}$$

yielding the step responses of Fig. 5 and the frequency reponses of Figs. 6 and 7. In particular, we can see that this second controller ensures a larger closed-loop bandwidth than the first controller, and hence a faster but also more erratic time response. The norms $\|S^i(s)\|_\infty$ achieved at the 16 vertices are as follows: 1.2340, 1.2353, 1.2481, 1.2496, 1.2507, 1.2521, 1.2615, 1.2631, 1.2682, 1.2696, 1.2744, 1.2758, 1.2870, 1.2886, 1.2944, 1.2961.

VI. Conclusions

In this paper, the problem of fixed-order robust H_∞ controller design has been considered for systems with polytopic uncertainty. We have extended results previously available for regional pole assignment to H_∞ performance specifications by using the equivalence between robust stability and H_∞ norm constraint. An illustrative example has demonstrated the feasibility of the proposed design methods. The strengths and weaknesses of the approach are

[2]The LMI problem is parsed under Matlab 7.3 with YALMIP 3.0 [13] and solved with SeDuMi 1.1 in less than 1 second on a standard desktop computer.

concentrated in the choice of the central polynomial, a tuning parameter akin to weigthing functions in standard H_∞ control.

Extending this work to MIMO systems should not be difficult, as soon as we ensure that both numerator and denominator polynomial matrices depend affinely in the controller numerator and denominator polynomial matrices. We are also planning to formulate similar LMI design conditions to ensure H_2 performance. Similarly, using the same techniques we should also be able to design fixed-order linear parameter-varying (LPV) controllers, and/or to cope with rational uncertainties modeled by linear fractional transforms (LFT). Finally, much more challenging research topics should be the study of the numerical behavior of LMI solvers on this kind of problems (especially when the designed controller approaches the limits of achievable performance), as well as a quantitative evaluation of the conservatism introduced by fixing the central polynomial.

Acknowledgments

We thank Denis Arzelier for pointing out reference [15] where the equivalence between real- and complex-valued single-block uncertainty is mentioned in the context of the small-gain theorem.

References

[1] S. P. Bhattacharyya, H. Chapellat, and L. H. Keel, *Robust Control*: *The Parametric Approach*. Upper Saddle River, NJ: Prentice-Hall, 1995.

[2] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM Studies in Applied Mathematics, Philadelphia, 1994.

[3] M. Chilali and P. Gahinet, H_∞ design with pole placement constraints: An LMI approach, *IEEE Trans. Automatic Control*, vol. 41, no. 3, pp. 358-367, 1996.

[4] M. C. de Oliviera, J. Bernussou and J. C. Geromel, A new discrete-time robust stability condition, *Systems & Control Letters*, vol. 37, no. 4, pp. 261-265, 1999.

[5] P. Gahinet and P. Apkarian, A linear matrix inequality approach to H_∞ control, *Int. J. Robust and Nonlinear Control*, vol. 4, pp. 421-448, 1994.

[6] T. Iwasaki and R. E. Skelton, All controllers for the general H infinity control problem: LMI existence conditions and state space formulas, *Automatica*, vol. 30, no. 8, pp. 1307–1317, 1994

[7] D. Henrion, LMI optimization for fixed-order H_∞ controller design, in *Proceedings of the IEEE Conference on Decision and Control*, pp. 4646-4651, Maui, Hawaii, December 9-12, 2003.

[8] D. Henrion, D. Arzelier, and D. Peaucelle, Positive polynomial matrices and improved LMI robustness conditions, *Automatica*, vol. 39, pp. 1479-1485, 2003.

[9] D. Henrion, M. Šebek, and V. Kučera, Positive polynomials and robust sabilization with fixed-order controllers, *IEEE Trans. Automatic Control*, vol. 48, no. 7, pp. 1178-1186, 2003.

[10] L. H. Keel and S. P. Bhattacharyya, Robust stability and performance with fixed-order controller, *Automatica*, vol. 35, pp. 1717-1724, 1999.

[11] V. Kučera, Diophantine equations in control - a survey, *Automatica*, vol. 29, no. 6, pp. 1361-1375, 1993.

[12] H. Kwakernaak, Robust control and H_∞ optimization, *Automatica*, vol. 29, no. 2, pp. 255-273, 1993.

[13] J. Löfberg. YALMIP: a toolbox for modeling and optimization in Matlab. Proceedings of the IEEE CACSD Symposium, Taipei, Taiwan, 2004.

[14] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, A new robust D-stability condition for real convex polytopic uncertainty, *Systems & Control Letters*, vol. 40, pp. 21-30, 2000.

[15] M. A. Rotea, M. Corless, D. Da, I. R. Petersen. Systems with structured uncertainty: relations between quadratic and robust stability. *IEEE Transactions on Automatic Control*, vol. 38, no. 5, pp. 799-803, 1993.

[16] C. W. Scherer and S. Weiland, *LMIs in Control*. Lecture notes at the Dutch Institute of Systems and Control, The Netherlands, 2000.

[17] S. Wang and J. H. Chow, Low-order controller design for SISO systems using coprime factors and LMI, *IEEE Trans. Automatic Control*, vol. 45, no. 6, pp. 1166-1169, 2000.

[18] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.

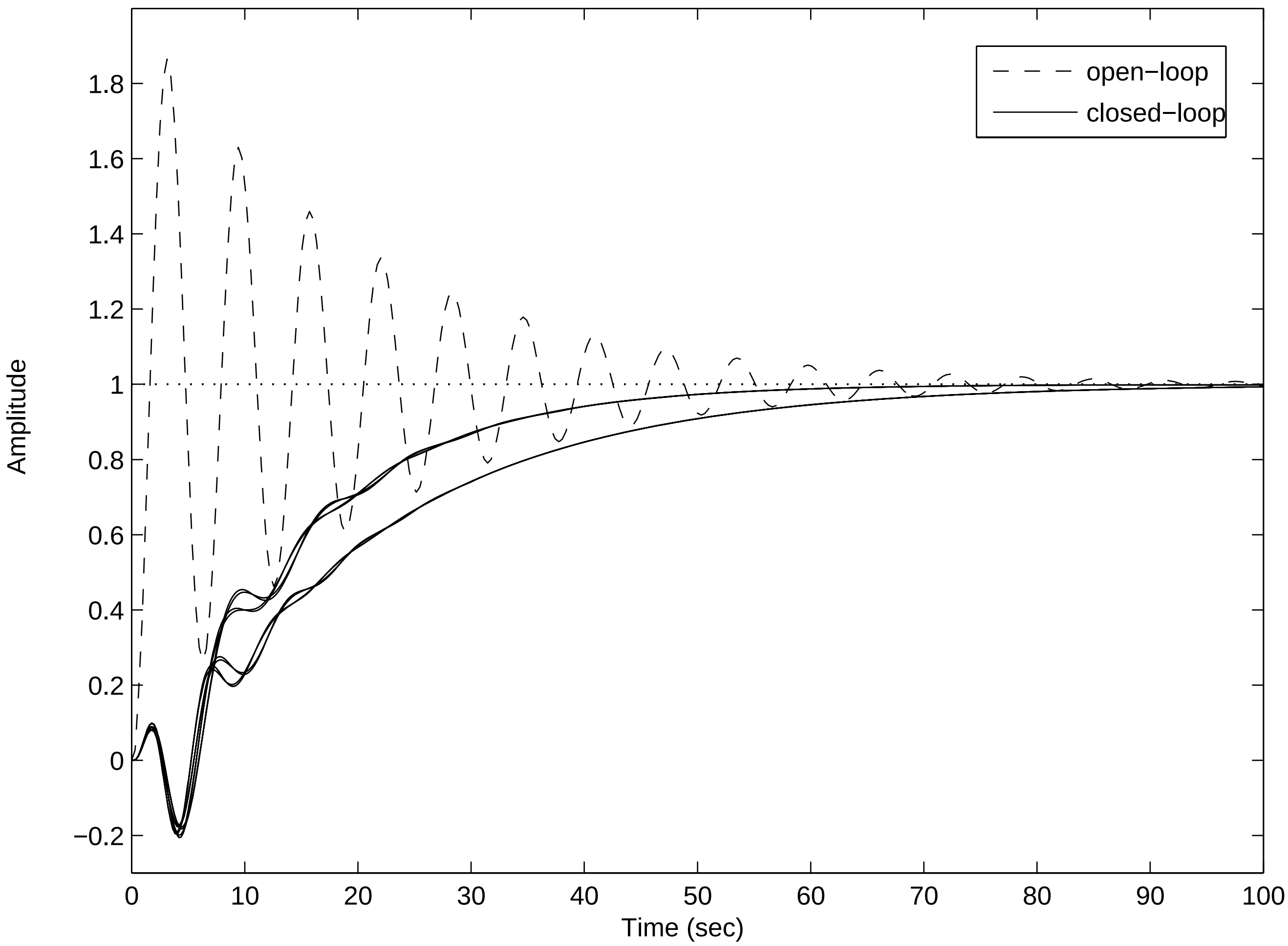

Fig. 2. Unit step responses for the first controller.

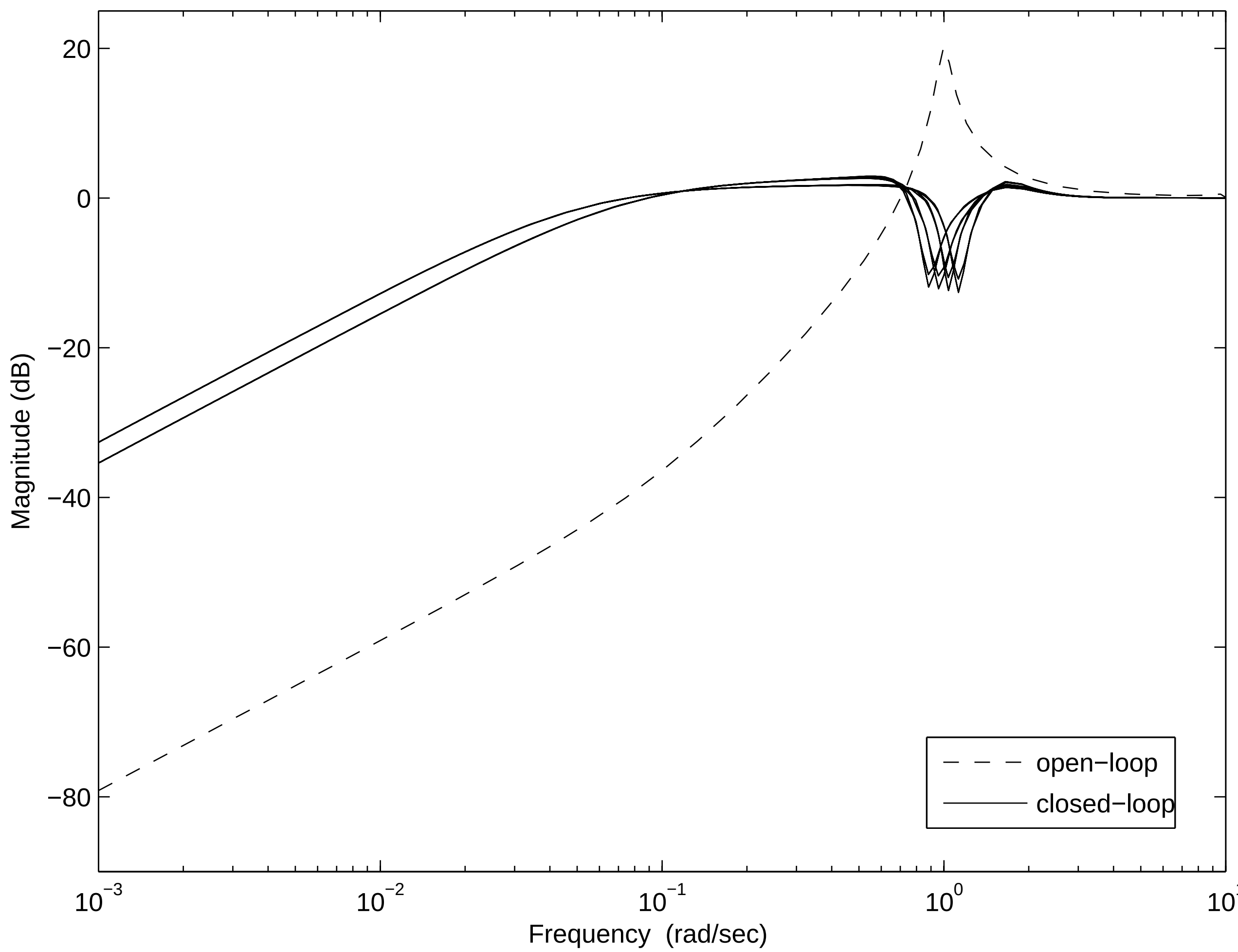

Fig. 3. Bode magnitude diagrams of the sensitivity functions for the first controller.

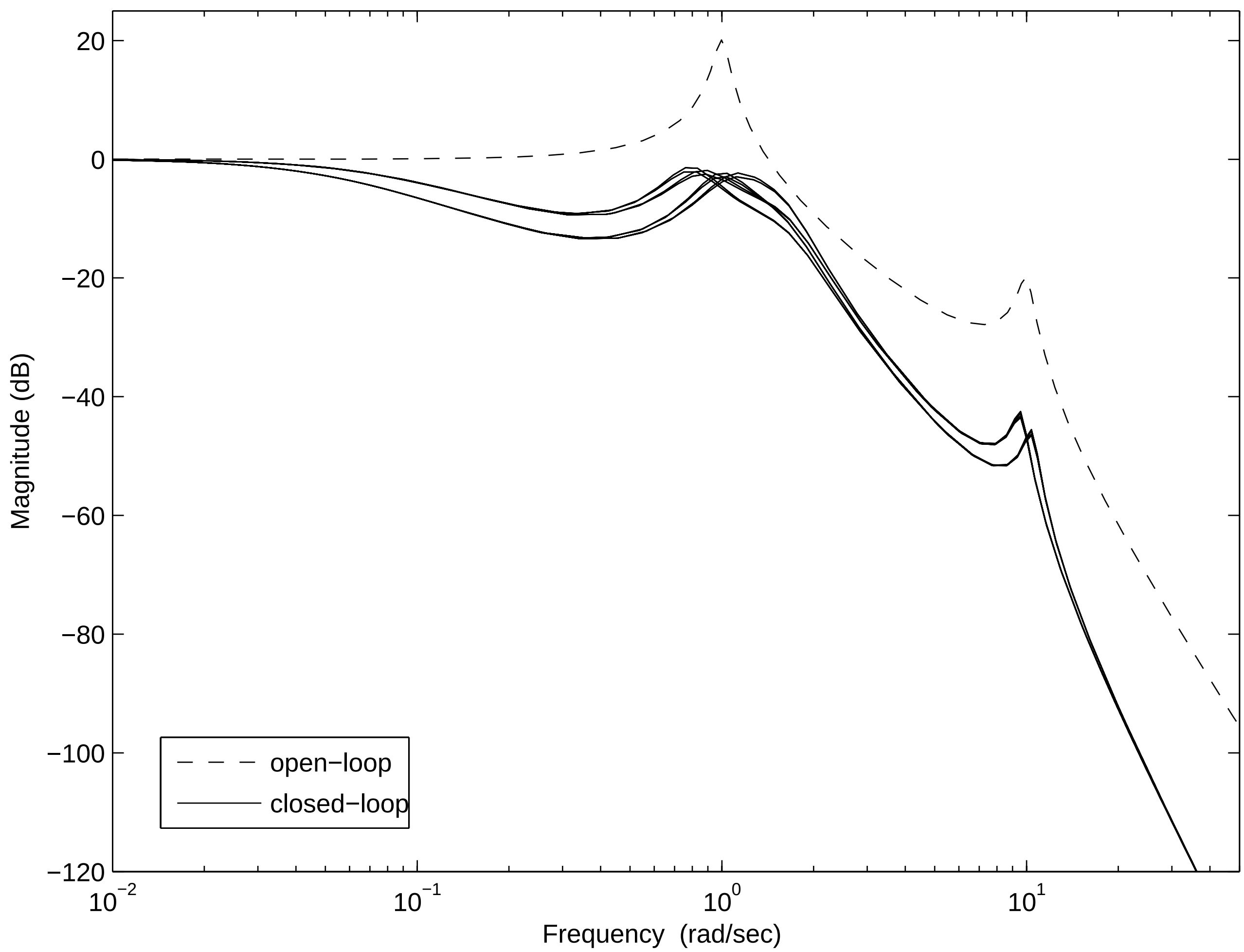

Fig. 4. Bode magnitude diagrams of the complementary sensitivity functions for the first controller.

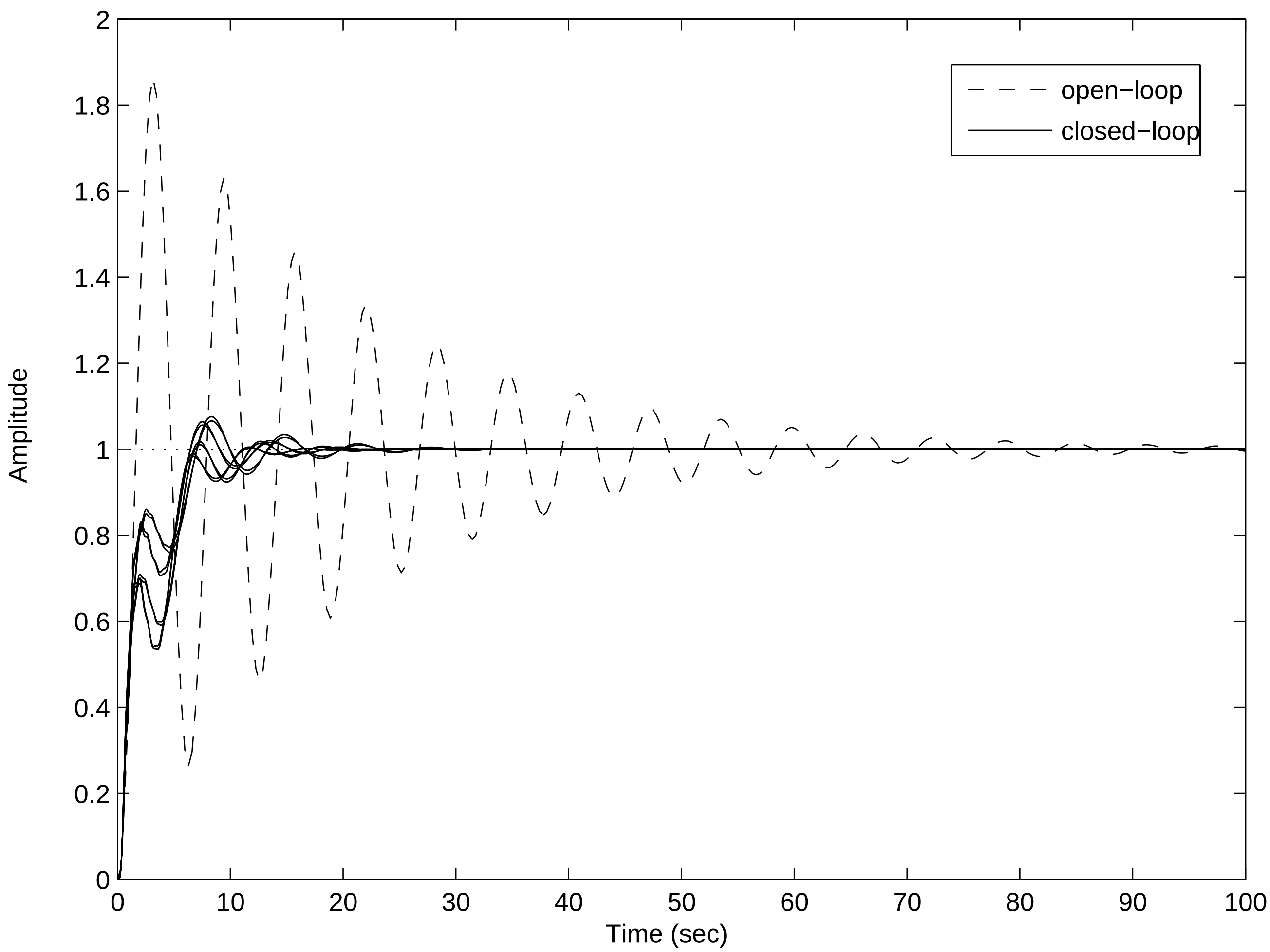

Fig. 5. Unit step responses for the second controller.

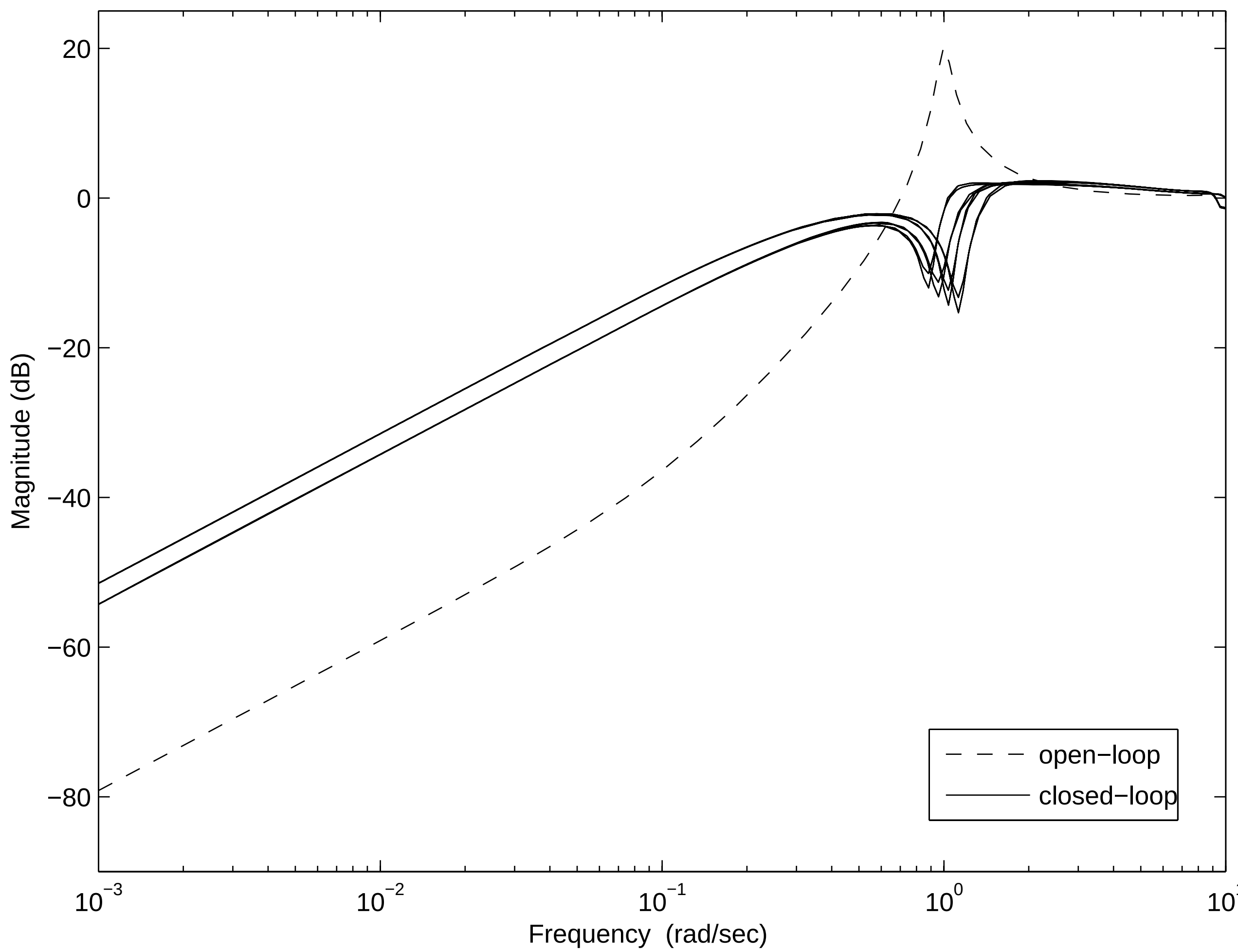

Fig. 6. Bode magnitude diagrams of the sensitivity functions for the second controller.

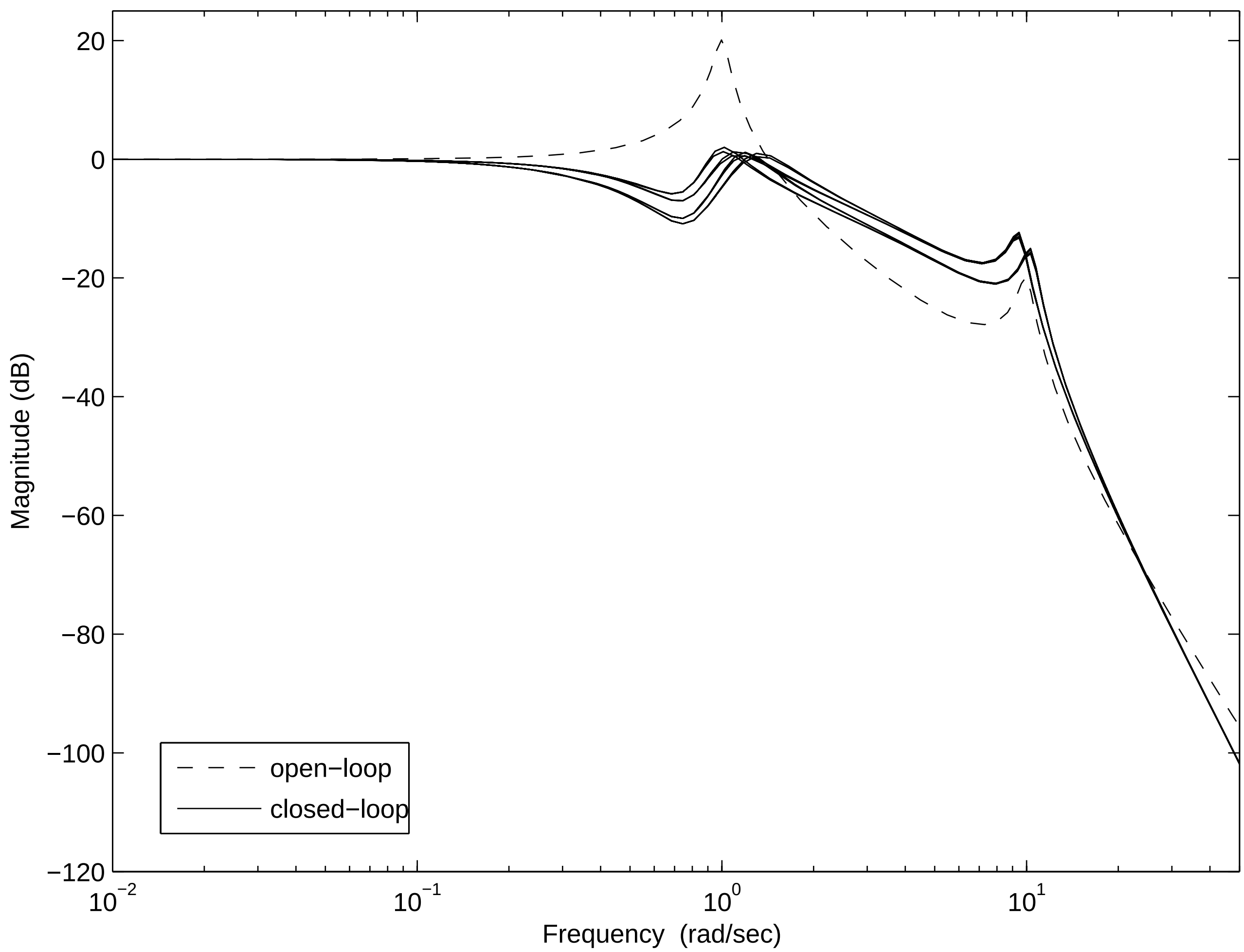

Fig. 7. Bode magnitude diagrams of the complementary sensitivity functions for the second controller.