Real-time H_2 and H_∞ control of a gyroscope using a polynomial approach

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Abstract. In this paper H_2 and H_∞ control techniques are applied to the real-time control of a gyroscope with two degrees of freedom. The controllers are designed based on a polynomial approach and using routines from the Polynomial Toolbox for MATLAB. Real-time results are presented, showing a good performance of the controllers.

Keywords: H_2-Optimization, H_∞-Optimization, Control system design, Polynomial approach, Gyroscope, Polynomial Toolbox.

1 Introduction

Practical applications of modern control techniques to physical systems are fundamental in control engineering. These modern techniques allow to control complex dynamic systems satisfying particular design objectives. The efficiency of the control schemes is tested and verified in the application process.

Computational routines and software, on the other hand, are an important tool for the application of control schemes. The analysis of dynamic systems and the design of controllers, usually involving complex computations, can be easily carried out using computer programs.

The aim of this work is to apply H_2 and H_∞ optimization techniques to the control of a gyroscope with two degrees of freedom. The controllers are designed using routines from the Polynomial Toolbox 2.5 for MATLAB[1], which are based on a polynomial approach. One of the advantages of the polynomial approach

[1]For more information on the Polynomial Toolbox, see www.polyx.com. MATLAB is a trademark of The MathWorks Inc.

is that the controllers for the linear model of the system can be designed directly from a transfer function description, which can be usually obtained from input-output information, avoiding the need for a state-space model of the system.

Roughly speaking, H_2-optimization consists in finding a controller which minimizes the H_2 norm of the closed-loop transfer function and internally stabilizes the system. The closed-loop transfer function to be minimized is located between the external signal and the control error signal, where the external signal comprises external inputs, including perturbations, measuring noise and reference inputs.

The standard H_2 problem was solved by Doyle, Glover, Khargonekar, and Francis in [2], where the authors present a solution to this problem considering a state-space description of a linear multivariable system. In [4], Hunt, Šebek and Kučera present a solution to the standard H_2 problem based on the polynomial solution to the LQG problem from Kučera [5]. The proposed polynomial solution is based on square complements and Diophantine equations. Another polynomial solution to the standard H_2 problem is given by Meinsma [9], which is based on factorizations over polynomials and stable matrices. Later on, Kwakernaak presents another solution based on factorization over polynomial matrices and Diophantine equations in [8]. This solution uses a generalized plant as a starting point, which allows to solve a number of problems, for example the mixed sensitivity problem and Wiener Hopf optimization, in a simple and methodological way.

It should be stated that the previously mentioned solutions to the standard H_2 problem are not entirely equivalent. Because of the different mathematical tools applied, the problems are solved at different levels of generality under different assumptions.

The design of the H_2 controller to be applied in this work to the gyroscope is based on the results of Kwakernaak [8]. The reasons for this choice is due to the simplicity of the polynomial approach and to the fact that there exist computer routines in the software Polynomial Toolbox, which allow to obtain the corresponding controller in a relatively simple way.

Investigation on H_∞-optimization for control systems was initiated in 1979 by Zames [11], who considered the minimization of the maximum magnitude over frequency of the sensitivity function of a scalar linear feedback system. There exist various solutions to the standard H_∞ problem, the most important being the so-called "two Riccati equation" solution [3, 2], which relies on a state space representation of the problem and requires the solution of two indefinite algebraic Riccati equations. A frequency domain solution to the standard H_∞ problem is presented in [6] based on polynomial matrix techniques and spectral factorizations. The corresponding scheme consists on a generalized plant interconnected with the feedback compensator, and the H_∞-optimization problem consists basically in finding a controller K which minimizes the H_∞ norm and stabilizes the closed-loop system. While the H_2 norm of a signal is the mean energy with respect to the frequency, the H_∞ norm is the maximum energy with respect to the frequency. The design of the H_∞ controller to be applied to the gyroscope will be based on the frequency domain solution of [6].

Real-time results of the application of H_2 and H_∞ control strategies to the gyroscope will be presented, showing a good performance of the controllers. Also, the application proposed in this paper can serve very well as a lab for students. Indeed, the polynomial approach is relatively simple to apply, and the analysis and design can be easily carry out using routines from the Polynomial Toolbox.

This work is organized as follows. The system under study, a gyroscope of two axes, is presented in Section 2. Section 3 deals with the application of H_2-optimization to the gyroscope. The corresponding controller is obtained using routines from the Polynomial Toolbox and an adequate choice of the filters. Real-time simulation results of this controller applied to the gyroscope are also presented in this section. In Section 4, application of H_∞-optimization is considered, presenting also the controller design and real-time simulation results. Finally, we end with some conclusions.

2 Description of the system

Gyroscopes are used to measure the angular movement with respect to a fixed structure, and are a key component of plane automatic pilots, rocket guidance systems, spatial vehicle altitude systems, navigation gyrocompasses, etc. [1].

The system considered in this work is a gyroscope with two axes, shown schematically in Fig. 1, which is a lab experiment developed by Quanser Inc. [10]. The gyroscope consists basically of the following components: a support plate holding the gyro module with a rotor which rotates at a constant speed, its movement being produced by a DC motor, sensors for the angles α and ψ, and a data acquisition card connecting the gyroscope to a computer.

Angle α defines the angular position of the structure with the rotor, with respect to the gyro module, while angle ψ is located between the gyro module and the support plate.

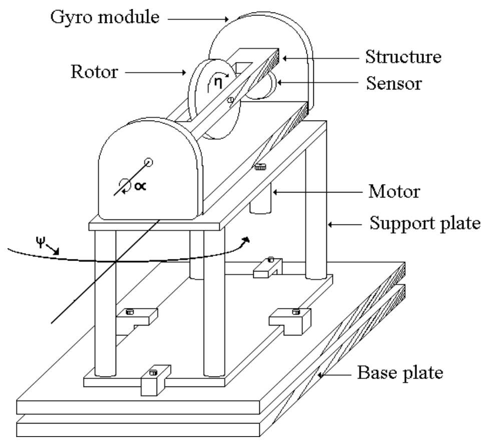

Fig. 1. Scheme of the gyroscope.

The purpose of the controller to be designed for this system is to maintain the direction at which the gyroscope is pointing, while the support plate rotates relative to the base plate. In other words, the gyro module must keep its position relative to the base plate in the presence of perturbations or any movement of the support plate. This mimics the problem of a ship on which a radar is mounted, and it is desired to maintain the direction in which the radar beam points independent of the unknown yawn of the ship due to disturbances and steering. The control input of the system is the voltage δ applied to the DC motor, and the output will be considered to be the angle ψ.

As it was previously mentioned, one of the advantages of the polynomial based techniques to be applied in this work is that the controllers for the linear model of the system can be designed directly from a transfer function description, avoiding the need for a state-space model of the system.

The transfer function of the gyroscope between the input δ and the output ψ, and obtained from the dynamic equations of the system is given by

$$P(s) = \frac{n(s)}{d(s)} = \frac{39.579128s^2 - 179088.874403}{s^4 + 1.728537s^3 + 139071.003587s^2 - 7821.338097s}.$$

The zeros and poles of the system represented by the transfer function $P(s)$ are given by

$$\texttt{zeros} = \begin{array}{r} 67.266865 \\ -67.266865 \end{array}$$

$$\texttt{poles} = \begin{array}{l} \;\;0 \\ \;\;0.056239 \\ -0.892388 + 372.921315i \\ -0.892388 - 372.921315i \end{array}$$

from which it can be seen that the system is non-minimum-phase and unstable.

3 H_2 Control

In this section we present first the design of the H_2 controller and afterwards the real time simulations of the controller applied to the gyroscope. The controller to be designed should be such that the gyro module keeps its position in presence of perturbations or movements of the base plate, while providing stability, performance and robustness to the closed-loop system.

3.1 Controller design

The controller will be designed based on the procedure indicated in [8]. Let us consider the block diagram of Fig. 2, where $P(s)$ is the transfer function of the system, $K(s)$ is the compensator to be designed, v comprises the external inputs, including perturbations, measurement noise and reference inputs, z is the control error signal, y is the measured output, u is the control input, and $V_1(s)$, $V_2(s)$, $W_1(s)$ and $W_2(s)$ are shaping filters. The H_2 mixed sensitivity problem consists in finding a controller $K(s)$ which minimizes the H_2 norm of the closed-loop transfer function and internally stabilizes the system. This generalized problem allows for colored disturbances and measurement noise, whose frequency contents are determined by $V_1(s)$, and $V_2(s)$, and frequency weightings of the controlled output and of the input determined by $W_1(s)$ and $W_2(s)$.

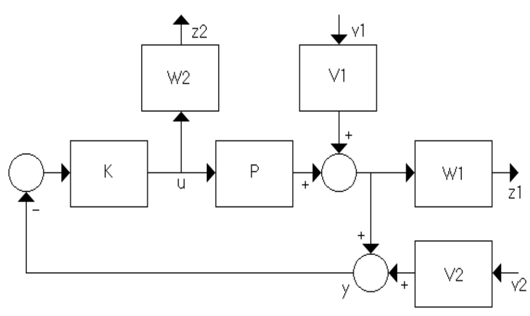

Fig. 2. The H_2 mixed sensitivity problem.

The design objectives of achieving stability, performance and robustness can be accomplished by making the loop gain large at low frequencies, small at high frequencies, and keeping the loop gain away from the critical point -1 at crossover frequencies.

The loop gain has a direct effect on important closed-loop transfer functions which determine the 2-norm, such as the sensitivity $S(s)$ and the complementary sensitivity $T(s)$. For the configuration of Fig. 2, these functions are given by

$$\begin{aligned} S(s) &= [I + P(s)K(s)]^{-1} \\ T(s) &= P(s)K(s)[I + P(s)K(s)]^{-1} \end{aligned} \tag{1}$$

where $P(s)$ is the open-loop transfer function of the system.

The sensitivity function $S(s)$ determines the effect of the disturbance on the output of the control system. The complementary sensitivity $T(s)$ satisfies the identity $S(s)+T(s)=I$, and it is important for the closed-loop response, the effect of measurement noise and the amount of control effort. In terms of these two functions the design objectives can be accomplished by making the sensitivity $S(s)$ small at low frequencies, making the complementary sensitivity $T(s)$ small at high frequencies, and preventing both $S(s)$ and $T(s)$ from peaking at crossover frequencies.

For the design of the controller which solves the H_2 problem we are considering in this work, we will use the instruction `H2` from the Polynomial Toolbox 2.5 for MATLAB

$$[\mathtt{Y},\mathtt{X},\mathtt{clpoles},\mathtt{fixed}]=\mathtt{H2}(\mathtt{N},\mathtt{D},\mathtt{ncon},\mathtt{nmeas})$$

where `N` and `D` are a left coprime matrix fraction description of the generalized plant `G`, `ncon` is the number of driving inputs, and `nmeas` is the number of measured outputs. This instruction returns the controller $K=Y/X$, the nonfixed closed-loop poles, and the fixed poles of the plant.

For simplicity, we consider that $V_2(s)=0$, which is equivalent to supposing that there does not exist measuring noise. In order to use the instruction `H2` from the Polynomial Toolbox, we need the generalized plant given by

$$G(s)=\begin{bmatrix} W_1(s)V_1(s) & W_1(s)P(s) \\ 0 & W_2(s) \\ -V_1(s) & -P(s) \end{bmatrix}. \tag{2}$$

The weighting functions $V_1(s)$, $W_1(s)$ and $W_2(s)$ will be assigned following the procedure indicated in [8] as follows.

Several recommendations are given in [8] for the choice of $V_1(s)$, one of them being the following: if the transfer function of the plant $P(s)$ is strictly proper then $V_1(s)$ is recommended to be of the form $P(s)\frac{s+\alpha_1}{s}$, where the constant α_1 is a design parameter. It can be seen that the presence of a pole at 0 in $V_1(s)$ forces the sensitivity function $S(s)$ to be 0 at $s=0$. If $S(s)$ does not have a zero at 0 then the 2-norm cannot be finite. If the plant $P(s)$ has a pole at 0, then it is not necessary to include a pole at 0 in $V_1(s)$ except if it is desired to design a system with more than one integrator.

In our case, and since the transfer function of the gyroscope has already a pole at 0, instead of the previous choice, function $V_1(s)$ will have the form

$$V_1(s)=\frac{m(s)}{d(s)}$$

where $d(s)$ is the denominator of the plant $P(s)$ and $m(s)$ is a polynomial to be properly assigned. Since the roots of $m(s)$ will actually be poles of the closed-loop system, the selection of $V_1(s)$ is one of the key steps in the controller design. The polynomial $m(s)$ in the case of the gyroscope was assigned as

$$m(s)=(s+12)(s+5)(s+0.001)$$

basically from time simulations, trying different pole locations until a good performance was obtained. Then $V_1(s)$ is strictly proper and has a pole excess (difference between the number of poles and the number of zeros) of one, as recommended in [8].

For sensible control systems the sensitivity function $S(s)$ has the property that $S(\infty)=1$ and therefore it is proper but not strictly proper. For this reason, whichever way the weighting function $V_1(s)$ is chosen, the product $W_1(s)V_1(s)$ needs to be strictly proper to allow the 2-norm to be finite. Since we have that

$V_1(s)$ has a pole at 0, then $W_1(s)$ may be used for fine tuning. A safe initial choice, and the value we will actually use is $W_1(s) = 1$.

Since $V_1(s)$ has a pole excess of 1, the input sensitivity function

$$U(s) = \frac{K(s)}{1 + P(s)K(s)}$$

must be such that $W_2(s)U(s)$ has pole excess 0. Therefore, $W_2(s)$ must be of the form

$$W_2(s) = c(1 + rs) = 0.1(1 + 0.1s)$$

where the values of c and r were obtained experimentally.

Substituting the functions $V_1(s)$, $W_1(s)$ and $W_2(s)$ in the generalized plant $G(s)$ given by (2), we have that the following is a left coprime polynomial matrix fraction description of $G(s)$

$$G(s) = D(s)^{-1}N(s) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & d(s) \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 0.1(1+0.1s) \\ -m(s) & -n(s) \end{bmatrix}.$$

For this generalized plant, the instruction `H2` from the Polynomial Toolbox 2.5 returns the controller $K = Y/X$, where

$$\begin{aligned} \texttt{Y} &= -0.000984 - 1.0285s + 0.007238s^2 + 0.000775s^3 + 0.000076s^4, \\ \texttt{X} &= 1 + 0.34666s + 0.06217s^2 + 0.005256s^3 + 0.000163s^4. \end{aligned}$$

The closed-loop poles of the system are given by

$$\texttt{clpoles} = \begin{array}{l} -0.893871 + 372.946755i \\ -0.893871 - 372.946755i \\ -12.587925 \\ -9.493506 \\ -5.060947 \\ -2.455676 + 2.60193i \\ -2.455676 - 2.60193i \\ -0.001 \end{array}$$

there are no fixed closed-loop poles

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fixed = Empty matrix:  0-by-1,
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and the closed-loop zeros are given by

$$\texttt{clzeros} = \begin{array}{l} 67.266865 \\ -67.266865 \\ -14.949096 + 21.471079i \\ -14.949096 - 21.471079i \\ 19.717299 \\ 0.000956. \end{array}$$

The corresponding sensitivity functions $S(s)$ and $T(s)$ given by (1) are shown in Fig. 3, where it can be seen that they have the specified properties to accomplish the design objectives of achieving stability, performance and robustness of the closed-loop system. Indeed, the sensitivity $S(s)$ is small at low frequencies, the complementary sensitivity $T(s)$ is small at high frequencies, and both $S(s)$ and $T(s)$ do not peak at crossover frequencies.

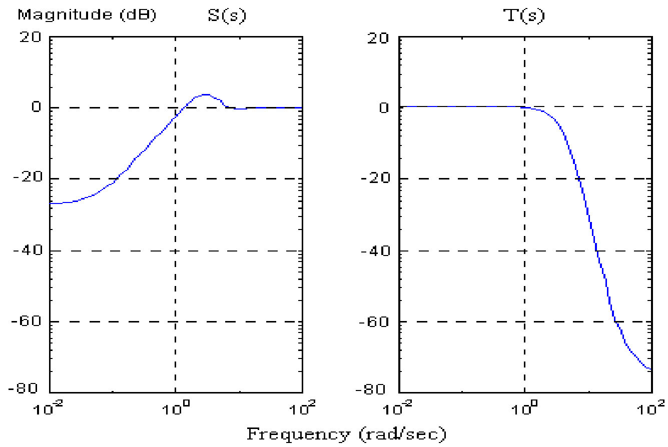

Fig. 3. Sensitivity $S(s)$ and complementary sensitivity $T(s)$ for H_2.

3.2 Real-time results

To test the performance of the designed controller in real time, we introduce a perturbation to the system by moving manually the support plate, producing a sequence of step-like functions of different magnitude. Fig. 4 shows the angular movement $\psi(t)$ between the support plate and the gyro module caused by the input perturbation. Angle $\psi(t)$ stabilizes at the negative value of the input perturbation for every step-like function.

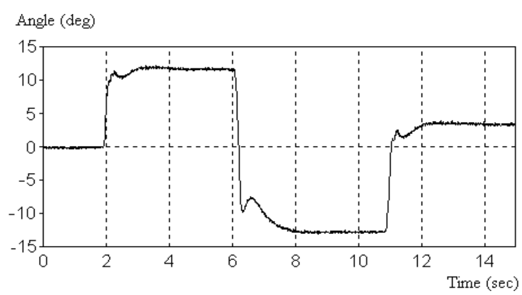

Fig. 4. Angle $\psi(t)$ in real time.

Fig. 5 shows angle $\alpha(t)$, where it can be seen that in the presence of a perturbation, angle $\alpha(t)$ opposes to the movement of the gyro module until the perturbation is rejected.

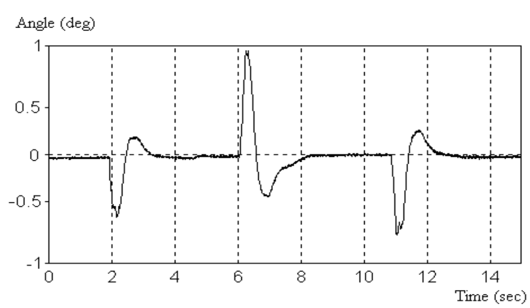

Fig. 5. Angle $\alpha(t)$ in real time.

Fig. 6 shows the control signal $\delta(t)$ applied to the system. The gyro module remains practically without any movement at all while the support plate rotates due to the perturbation introduced, showing a very good performance of the designed H_2 controller.

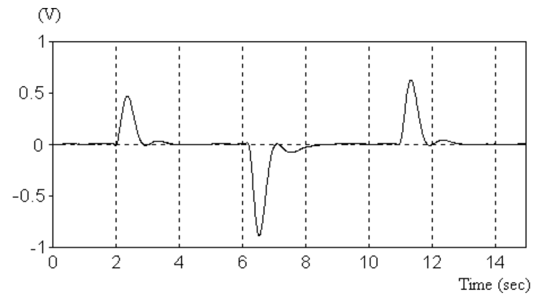

Fig. 6. Control signal $\delta(t)$ in real time.

As far as the design and operation of the control system are concerned, we noticed that if the sensitivity function $S(s)$ is bigger, the gyroscope is more influenced by the perturbations, and the actuators are saturating due to the appearing peaks. This problem appears in real time, but in simulation the system performed better the bigger the sensitivity function was, and the reason for this is because no noise is considered in the simulations. Observe also that one of the closed-loop poles of the system is very close to the imaginary axis. The system performed better in simulation the more this pole was far away from the imaginary axis, but in real time the system did not work, again because of saturation of the actuators. Even though we tried to find a theoretical explanation why this pole must be close to the imaginary axis to avoid saturation, it must be said that still we do not understand this phenomenon properly.

Notice also that the closed-loop system has highly oscillatory poles, which may be a threat to the system stability. These poles are also present in the open-loop and it could be expected that the controller cannot compensate for them. However, no important effect produced by these highly oscillatory poles was observed in the real-time simulations.

4 H_∞ control

4.1 Controller design

In order to solve the H_∞ mixed sensitivity problem, we need again the generalized plant in the form given by (2). For this, the weighting functions $V_1(s)$, $W_1(s)$ and $W_2(s)$ have to be properly chosen, and the procedure to choose them is very similar to the case of the H_2 mixed sensitivity problem [6] (see also [7]); one minor difference is that the function $V_1(s)$ has to be proper for the H_∞ control, and not strictly proper as in the case of the H_2 control.

The function $V_1(s)$ is then a proper rational function of the form

$$V_1(s) = \frac{m(s)}{d(s)}$$

which must satisfy that $W_1(s)V_1(s)$ is proper, and where the roots of $m(s)$ become closed-loop poles of the system. The polynomial $m(s)$ was obtained as

$$m(s) = (s + 42)(s + 15)(s + 5)(s + 0.001)$$

basically from time simulations, trying different pole locations until a good performance was obtained.

As in the case of the H_2 control, we have that $W_1(s) = 1$. For the function $W_2(s)$, and in order that $W_2(s)U(s)$ has pole excess 0, we have that

$$W_2(s) = c(1 + rs) = 1(1 + 0s) = 1$$

where the values of r and s were obtained experimentally.

Substituting the functions $V_1(s)$, $W_1(s)$ and $W_2(s)$ in the generalized plant $G(s)$ given by (2), we have that the following is a left coprime polynomial matrix fraction description of $G(s)$

$$G(s) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & d(s) \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ -m(s) & -n(s) \end{bmatrix}.$$

The H_∞ controller will be obtained using the instruction `mixeds` from the Polynomial Toolbox. Besides the left polynomial factorization of the generalized plant, this instruction requires the parameters `gmin, gmax, accuracy` and `tol` to be specified. The parameters `gmin` and `gmax` are respectively lower and upper bounds for the minimal H_∞ norm, `accuracy` specifies how closely the minimal norm is to be approached, and `tol` defines four tolerances, namely those used in canceling identical pole-zero pairs in the transfer function of the optimal compensator, in various stability tests, in the spectral factorization, and in the left-to-right and right-to-left conversions. For the case of the gyroscope, these values were obtained experimentally from various tests as

$$\begin{aligned} &\texttt{gmin} = 134.5 \\ &\texttt{gmax} = 139 \\ &\texttt{accuracy} = 1 \times 10^{-4} \\ &\texttt{tol} = [1 \times 10^{-4}, 1 \times 10^{-9}, 1 \times 10^{-15}, 1 \times 10^{-8}]. \end{aligned}$$

With these data and parameters, the instruction `mixeds` produces the compensator $K = Y/X$, where

$$\texttt{Y} = 0.000414 + 0.4373s + 0.2765s^2 - 0.024807s^3 + 0.000133s^4$$
$$\texttt{X} = -0.51135 - 0.6715s - 0.13752s^2 - 0.008584s^3 - 0.000135s^4.$$

The closed-loop poles and zeros of the system are given by

$$\begin{aligned} \texttt{clpoles} = \; & -0.865165 + 372.921263i \\ & -0.865165 - 372.921263i \\ & -42 \\ & -15 \\ & -5 \\ & -0.792475 + 0.791598i \\ & -0.792475 - 0.791598i \\ & -0.001, \end{aligned}$$

$$\begin{aligned} \texttt{clzeros} = \; & 67.266865 \\ & -67.266865 \\ & 1843.909032 \\ & 12.628115 \\ & 1.403627 \\ & 0.00000094. \end{aligned}$$

The corresponding sensitivity functions $S(s)$ and $T(s)$, shown in Fig. 7, have the specified properties to accomplish the design objectives of achieving stability, performance and robustness of the closed-loop system, namely the sensitivity $S(s)$ is small at low frequencies, the complementary sensitivity $T(s)$ is small at high frequencies, and both $S(s)$ and $T(s)$ do not peak at crossover frequencies.

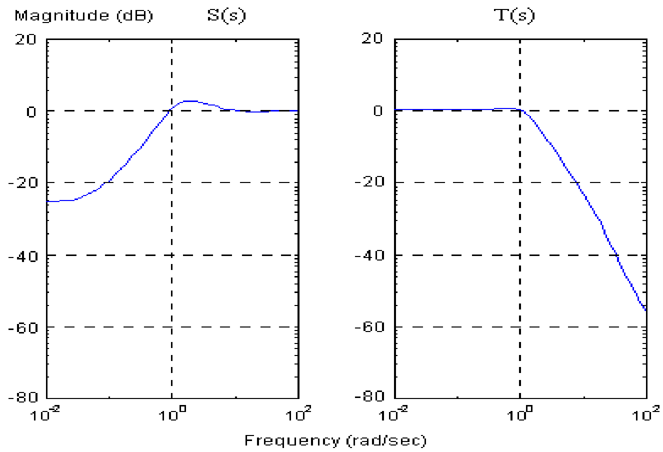

Fig. 7. Sensitivity $S(s)$ and complementary sensitivity $T(s)$ for H_∞.

4.2 Real-time results

For the real-time application, and in order to test the performance of the designed controller, we introduce again a perturbation by moving manually the support plate. The angle $\psi(t)$ caused by this input perturbation is shown in Fig. 8. The corresponding behavior of the angle $\alpha(t)$ is shown in Fig. 9, where it can be seen that $\alpha(t)$ opposes to the movement of the gyro module until the perturbation is rejected. The control signal $\delta(t)$ is shown in Fig. 10.

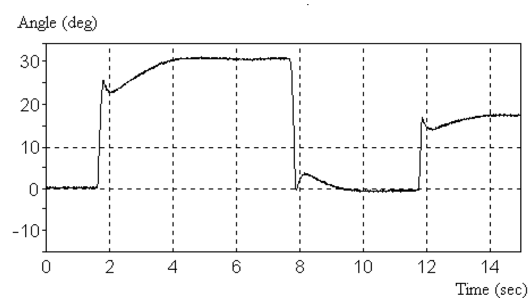

Fig. 8. Angle $\psi(t)$ in real-time

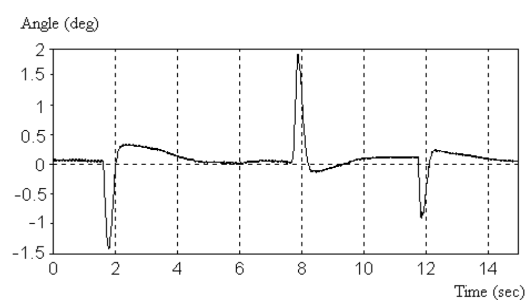

Fig. 9. Angle $\alpha(t)$ in real-time

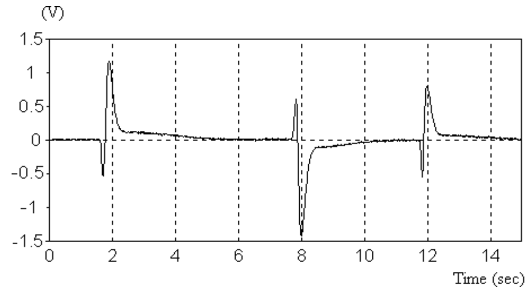

Fig. 10. Control signal $\delta(t)$.

The gyro module remains practically without movement with respect to its initial position in the presence of the introduced perturbation, showing again a good performance of the designed controller.

For the H_∞ controller applied to the gyroscope we noticed that if the sensitivity function $S(s)$ was made too small, theoretically the system would perform better. In the real-time application this option did not work because the control law was out of the physical rank it could assume, producing saturation in the actuators. As in the case of the H_2 control, the closed-loop system has a pole near the imaginary axis, and highly oscillatory poles.

5 Conclusions

In this paper, H_2 and H_∞ control techniques have been applied to the real-time control of a gyroscope of two degrees of freedom. The controllers were designed using routines from the Polynomial Toolbox for MATLAB, based on a polynomial approach. This approach is relatively simple to apply, and the existence of computational routines helps to a great extent in the controllers design.

Real-time results show a good performance of the controllers. The choice of the corresponding filters and parameters for the controllers design was done taking into account the system characteristics and experimental results from simulations. Thus, this selection strongly depends on the particular physical system to be controlled.

Although the H_∞ controller performed generally a little better than the H_2 controller rejecting step-like perturbations, both controllers performed rather similarly in the time domain, both rejected disturbances, and closed-loop poles location does not differ too much.

Future work has to focus on testing the robustness of the controllers, for instance introducing variations on the parameters of the system.

Acknowledgments

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