LMIs for Robust Stabilization of Systems with Ellipsoidal Uncertainty*

Didier Henrion[1,2,3]
Michael Šebek[4]
Vladimír Kučera[4]

Abstract

A recently developed ellipsoidal inner approximation of the stability domain in the space of polynomial coefficients is used to show that the problem of robust stabilization of a scalar plant affected by ellipsoidal parametric uncertainty boils down to a mere convex LMI problem. An illustrative numerical example is given.

Keywords

Linear Systems, Uncertainty, Robust Design, LMI.

1 Introduction

The problem of robust stabilization of a plant affected by parametric uncertainty is of fundamental importance in control. Even though significant progress has been made recently in the realm of parametric robust control and robust stability analysis, a very few design algorithms are available [3]. One of the main hindrance behind the development of an efficient, systematic robust design tool is the well-known fact that the stability region in the space of polynomial coefficients is non-convex in general [1]. Since most of the design problems can be formulated as optimization problems in the coefficient space, a convex approximation of the non-convex stability region is generally used. Based on this idea, a traditional design methodology consists in assigning characteristic polynomial coefficients

[1]Corresponding author. FAX: +33 5 61 33 69 69. E-mail: henrion@laas.fr

[2]Laboratoire d'Analyse et d'Architecture des Systèmes, Centre National de la Recherche Scientifique, 7 Avenue du Colonel Roche, 31 077 Toulouse, cedex 4, France.

[3]Also with the Institute of Information Theory and Automation, Czech Academy of Sciences, Pod vodárenskou věží 4, 182 08 Praha, Czech Republic.

[4]Center for Applied Cybernetics, Faculty of Electrical Engineering, Czech Technical University in Prague, Technická 2, 166 27 Praha 6, Czech Republic.

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in convex regions instead of assigning polynomial roots directly, see [12] and references therein. The approach may prove conservative, but has the merit of being tractable. This is the approach we pursue in this paper.

The choice of the uncertainty model is crucial in the design procedure. It is now admitted that interval uncertainty is more suitable for robust stability analysis (based on the theorem of Kharitonov and its variations [3]) than for robust design. The assumption that each plant parameter is constrained to an uncertainty range that is independent of all other parameters usually proves overly conservative. For this reason, ellipsoidal parametric uncertainty [4, 10] may be viewed as an interesting alternative uncertainty model. This type of uncertainty arises naturally in the context of parameter estimation for process identification. The identified parameters are known to belong, with a specified level of confidence, to an ellipsoid characterized in terms of a covariance matrix. In contrast to the interval uncertainty, the ellipsoidal uncertainty accounts for dependencies between individual uncertain parameters.

Robust stability analysis conditions for ellipsoidally uncertain systems were proposed in [4, 10] and more recently in [6]. Robust design was studied in [2] where it was shown that the design problem is convex provided the closed-loop polynomial coefficients are placed in a given stability ellipsoid. The stability ellipsoid is obtained heuristically by combining a sensitivity technique with the critical direction method described in [10]. The convex design problem is then tackled with a cutting-plane algorithm requiring analytic calculation of sub-gradients.

Another, totally different approach to robust stabilization was pursued in [14] that applies to systems with ellipsoidal uncertainty. In this reference, stricly positive real functions and the Youla-Kučera parametrization of all stabilizing controllers are used to show that the robust design problem boils down to an infinite-dimensional convex optimization problem over the space of stable rational fractions. There is no efficient method to solve infinite-dimensional problems (in spite of the convexity) so one must resort to a series of finite-dimensional approximations of increasing size when trying to solve the problem practically. Proceeding this way, one has generally no control on the controller order. Controllers of high order may be obtained, for which a low-order robustly stabilizing approximation must be found. Interestingly, the duality theory can be applied to measure the amount of conservatism inherent to the finite-dimensional approximation, as shown recently in [9].

The first contribution of the present paper is in showing that the convex design problem studied in [2] is actually an optimization problem over linear matrix inequalities (LMIs, see [5]) that can be solved readily with off-the-shelf software. The second contribution of our paper is in showing that the ellipsoidal inner approximation of the stability domain in the polynomial coefficient space recently proposed in [11] can be used in robust design. The resulting design algorithm can thus be viewed as an LMI-based counterpart to the linear-programming-based design algorithm proposed in [12], where polytopes were used to model the uncertainty and characteristic polynomial coefficient placement. Contrary to the infinite-dimensional approach of [14] based on rational fractions and the Youla-Kučera parametrization, we fix the order of the controller from the outset and we work in spaces of fixed-degree polynomials. The price one has to pay is in the convex approximation of the non-convex stability region. In [14] the approximation concerns the finite-degree parametrization of the infinite-dimensional space of stable rational fractions.

2 Problem Description

We consider a strictly proper SISO plant

$$p(s) = \frac{p_{N0} + p_{N1}s + \cdots + p_{Nn-1}s^{n-1}}{p_{D0} + p_{D1}s + \cdots + p_{Dn-1}s^{n-1} + s^n} = \frac{p_N(s)}{p_D(s)}$$

of order n and a proper controller

$$c(s) = \frac{\mathbf{c}_{N0} + \mathbf{c}_{N1}s + \cdots + \mathbf{c}_{Nm-1}s^{m-1} + \mathbf{c}_{Nm}s^m}{\mathbf{c}_{D0} + \mathbf{c}_{D1}s + \cdots + \mathbf{c}_{Dm-1}s^{m-1} + s^m} = \frac{c_N(s)}{c_D(s)}$$

of order m, arranged in the standard negative feedback configuration shown in figure 1. Assuming that there is no pole-zero cancellation, the stability of the closed-loop system

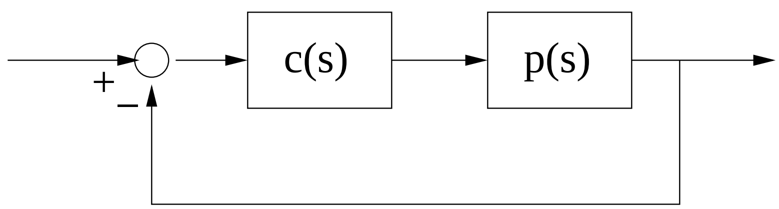

Figure 1: Feedback configuration.

is determined by the location of the roots of the characteristic polynomial

$$q(s) = p_N(s)c_N(s) + p_D(s)c_D(s) = q_0 + q_1 s + \cdots + q_{d-1}s^{d-1} + s^d \tag{1}$$

where $d = n + m$ is the closed-loop system order [13]. In the sequel, we will say that polynomial $q(s)$ is D-stable when the roots of $q(s)$ belong to some given region D of the complex plane. Typical choices for D is the left half-plane (continuous-time systems), the unit disk (discrete-time systems) or a sector thereof.

The coefficients of the plant $p(s)$, the controller $c(s)$ and the characteristic polynomial $q(s)$ are stored in the plant vector p, the control vector $\mathbf{c}$ and the characteristic vector q, respectively

$$\begin{array}{rcl} p &=& \left[\begin{array}{cccccccc} p_{N0} & p_{N1} & \cdots & p_{Nn-1} & p_{D0} & p_{D1} & \cdots & p_{Dn-1} \end{array}\right]' \in \mathbb{R}^{2n} \\ \mathbf{c} &=& \left[\begin{array}{ccccccccc} \mathbf{c}_{N0} & \mathbf{c}_{N1} & \cdots & \mathbf{c}_{Nm-1} & \mathbf{c}_{Nm} & \mathbf{c}_{D0} & \mathbf{c}_{D1} & \cdots & \mathbf{c}_{Dm-1} \end{array}\right]' \in \mathbb{R}^{2m+1} \\ q &=& \left[\begin{array}{cccc} q_0 & q_1 & \cdots & q_{d-1} \end{array}\right]' \in \mathbb{R}^d. \end{array}$$

With these notations, equating coefficients of the powers in s in polynomial equality (1) yields the linear system of equations

$$q = S(\mathbf{c})p + \mathbf{c}_D \tag{2}$$

where

$$
\begin{array}{rcl}
S(\mathbf{c}) & = & \left[\begin{array}{cccccc}
\mathbf{c}_{N0} & & & \mathbf{c}_{D0} & & \\
\mathbf{c}_{N1} & \ddots & & \mathbf{c}_{D1} & \ddots & \\
\vdots & & \mathbf{c}_{N0} & \vdots & & \mathbf{c}_{D0} \\
\mathbf{c}_{Nm-1} & & \mathbf{c}_{N1} & \mathbf{c}_{Dm-1} & & \mathbf{c}_{D1} \\
\mathbf{c}_{Nm} & & \vdots & 1 & & \vdots \\
 & \ddots & \mathbf{c}_{Nm-1} & & \ddots & \mathbf{c}_{Dm-1} \\
 & & \mathbf{c}_{Nm} & & & 1
\end{array}\right] \in \mathbb{R}^{d\times(2m+1)} \\
\mathbf{c}_D & = & \left[\begin{array}{cccccccc} 0 & 0 & \cdots & 0 & \mathbf{c}_{D0} & \mathbf{c}_{D1} & \cdots & \mathbf{c}_{Dm-1} \end{array}\right]' \in \mathbb{R}^d.
\end{array}
$$

It is assumed that the plant parameters are not known exactly: we are given a nominal parameter vector $\bar{p}$ and the actual parameter vector p belongs to the ellipsoid

$$E_p = \{p \ : \ (p-\bar{p})'P(p-\bar{p}) \leq 1\} \tag{3}$$

where $P = P' \succ 0$ is a given positive definite matrix. Plant vector p varies within ellipsoid E_p to account for all admissible parameter variations in plant $p(s)$. Clearly, when $p(s)$ varies, the roots of characteristic polynomial $q(s)$ also vary, see equation (1). The problem we aim to solve can then be formulated as follows.

Robust Control Problem: Find a controller vector $\mathbf{c}$ such that controller $c(s)$ robustly D-stabilizes plant $p(s)$ subject to ellipsoidal uncertainty $p \in E_p$.

3 Ellipsoidal Approximation of the Stability Region

As pointed out in the introduction, the set of coefficients q_i such that polynomial $q(s)$ is D-stable is a non-convex set in general. In view of equation (2), there is a linear dependence between characteristic vector q and controller vector $\mathbf{c}$. Therefore, the set of controller parameters ensuring closed-loop D-stability is also a non-convex set, thus making our robust control problem very intricate in general. This observation led various researchers to develop sufficient D-stability conditions that are convex in the set of polynomial coefficients [1]. In this section, we will quickly recall a sufficient stability condition that we recently proposed in [11] and that approximates the stability region in the set of polynomial coefficients by an ellipsoid.

First we need the following instrumental

Lemma 1 *The roots of polynomial $q(s)$ belong to region D if and only if*

$$H(q) = \sum_{i=0}^{d} \sum_{j=0}^{d} q_i q_j H_{ij} \succ 0 \tag{4}$$

where $H_{ij} = H'_{ij} \in \mathbb{R}^{d\times d}$ are given constant matrices depending on region D only.

In the literature, the above lemma is known as the Hermite stability criterion, see [11] for details.

Inequality (4) is a quadratic matrix inequality describing a region $S = \{q \,:\, H(q) \succ 0\}$ which is non-convex when $d \geq 3$. In the sequel we describe a systematic method to find a vector $\bar{q} \in \mathbb{R}^d$ and a positive definite symmetric matrix $Q \in \mathbb{R}^{d \times d}$ such that the ellipsoid

$$E_q = \{q \,:\, (q - \bar{q})' Q (q - \bar{q}) \leq 1\} \tag{5}$$

is a convex inner approximation of non-convex stability region S, i.e. such that the inclusion $E_q \subset S$ holds. Naturally, we will aim at enlarging the volume of E_q as much as possible.

At this point we need to define the constant matrix $H \in \mathbb{R}^{d(d+1) \times d(d+1)}$ built from the matrices H_{ij} of lemma 1 and the vector $\hat{q} = [q' \; 1]' \in \mathbb{R}^{d+1}$ such that quadratic matrix inequality (4) can be written equivalently as

$$H(q) = (I_d \otimes \hat{q}') H (I_d \otimes \hat{q}) \succ 0$$

where I_d is the identity matrix of dimension d and $\otimes$ is the standard Kronecker product.

Lemma 2 *Solve the convex LMI problem*

$$\begin{array}{ll} \max & \operatorname{trace} Q_{11} \\ \text{s.t.} & \lambda H \succ I_d \otimes \left[\begin{array}{cc} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{array}\right] + \left[\begin{array}{cccc} 0 & S_{21}' & \cdots & S_{d1}' \\ S_{21} & 0 & & S_{d2}' \\ \vdots & & \ddots & \vdots \\ S_{d1} & S_{d2} & \cdots & 0 \end{array}\right] \\ & \lambda > 0, \quad Q_{11} \prec 0, \quad Q_{22} = 1, \quad S_{ij} = -S_{ij}' \end{array} \tag{6}$$

for decision variables $\lambda \in \mathbb{R}$, $Q_{11} \in \mathbb{R}^{d \times d}$, $Q_{12} \in \mathbb{R}^d$, $Q_{22} \in \mathbb{R}$ *and* $S_{ij} \in \mathbb{R}^{(d+1)\times(d+1)}$. *Let*

$$\begin{array}{rcl} \bar{q} & = & -Q_{11}^{-1} Q_{12} \\ Q & = & -Q_{11} / (Q_{22} - Q_{12}' Q_{11}^{-1} Q_{12}). \end{array}$$

Then the ellipsoid E_q *defined in (5) is a convex inner approximation of non-convex stability region* S.

Proof: We give only a sketch of the proof, see [11] for details. For any vector q, LMI (6) implies that

$$\begin{array}{l} \lambda (I_d \otimes \hat{q}') H (I_d \otimes \hat{q}) \\ \quad = \lambda H(q) \succ (I_d \otimes \hat{q}') \left(I_d \otimes \left[\begin{array}{cc} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{array}\right] + \left[\begin{array}{cccc} 0 & S_{21}' & \cdots & S_{d1}' \\ S_{21} & 0 & & S_{d2}' \\ \vdots & & \ddots & \vdots \\ S_{d1} & S_{d2} & \cdots & 0 \end{array}\right] \right) (I_d \otimes \hat{q}) \\ \quad = (Q_{22} - Q_{12}' Q_{11}^{-1} Q_{12}) (I_d \otimes (1 - (q - \bar{q})' Q (q - \bar{q}))) \end{array}$$

Since both λ and $Q_{22} - Q_{12}'Q_{11}^{-1}Q_{12}$ are positive scalars, the above inequality means that $H(q) \succ 0$ as soon as $1 - (q - \bar{q})'Q(q - \bar{q}) \geq 0$, which is precisely the inclusion $E_q \subset S$. □

Matrices S_{ij} in the above LMI problem play the role of additional degrees of freedom. They are similar to the multipliers traditionally used in robust control. Note also that maximizing the trace of Q_{11} in problem (6) amounts to minimizing the trace of Q. Proceeding this way, we indirectly maximize the volume of ellipsoid E_q, while using the degrees of freedom provided by matrices S_{ij}. The interested reader is referred to [11] for more information.

4 Robust Design

Now we show that the problem of finding controller vector $\mathbf{c}$ such that characteristic vector q belongs to stability ellipsoid E_q for all admissible plant uncertainty $p \in E_p$ is a convex LMI feasibility problem.

First, observe that $p \in E_p$ if and only if

$$\begin{bmatrix} p - \bar{p} \\ 1 \end{bmatrix}' \begin{bmatrix} -P & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p - \bar{p} \\ 1 \end{bmatrix} \geq 0$$

or equivalently

$$\begin{bmatrix} p \\ 1 \end{bmatrix}' \begin{bmatrix} I_{2n} & -\bar{p} \\ 0 & 1 \end{bmatrix}' \begin{bmatrix} -P & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{2n} & -\bar{p} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix} \geq 0. \tag{7}$$

Similarly, $q \in E_q$ if and only if

$$\begin{bmatrix} q - \bar{q} \\ 1 \end{bmatrix}' \begin{bmatrix} -Q & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ 1 \end{bmatrix} \geq 0.$$

Recalling equation (2), the above inequality can be written as

$$\begin{bmatrix} p \\ 1 \end{bmatrix}' \begin{bmatrix} S(\mathbf{c}) & \mathbf{c}_D - \bar{q} \\ 0 & 1 \end{bmatrix}' \begin{bmatrix} -Q & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S(\mathbf{c}) & \mathbf{c}_D - \bar{q} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix} \geq 0. \tag{8}$$

The design problem then amounts to finding a condition such that quadratic inequality (8) holds whenever quadratic inequality (7) holds. This problem is frequently encountered in robust control. It is traditionally approached with the so-called S-procedure [5]. The S-procedure states that inequality (8) is satisfied for all vectors p satisfying inequality (7) if and only if there exists a positive scalar t such that

$$\begin{bmatrix} S(\mathbf{c}) & \mathbf{c}_D - \bar{q} \\ 0 & 1 \end{bmatrix}' \begin{bmatrix} -Q & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S(\mathbf{c}) & \mathbf{c}_D - \bar{q} \\ 0 & 1 \end{bmatrix} \succeq t \begin{bmatrix} I_{2n} & -\bar{p} \\ 0 & 1 \end{bmatrix}' \begin{bmatrix} -P & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{2n} & -\bar{p} \\ 0 & 1 \end{bmatrix}.$$

Invoking a standard Schur complement argument [5], the above matrix inequality can be expressed equivalently as an LMI in the controller vector $\mathbf{c}$ and the scalar parameter t. We have shown the main result of this paper.

Theorem 1 *Solve the convex LMI problem*

$$\left[\begin{array}{ccc} Q^{-1} & S(\mathbf{c}) & \mathbf{c}_D - \bar{q} \\ S(\mathbf{c})' & tP & -tP\bar{p} \\ (\mathbf{c}_D - \bar{q})' & -(tP\bar{p})' & 1 + t(\bar{p}'P\bar{p} - 1) \end{array}\right] \succeq 0 \qquad (9)$$

for decision variables $t \in \mathbb{R}$ and $\mathbf{c} \in \mathbb{R}^{2m+1}$. Then the vector $\mathbf{c}$ is such that controller $c(s)$ robustly D-stabilizes plant $p(s)$ subject to ellipsoidal uncertainty $p \in E_p$.

In [2] it was shown that the robust control problem with ellipsoidal uncertainty can be expressed as a convex min-max optimization problem. The design was then carried out with a cutting plane algorithm which requires analytic calculation of sub-gradients. Here we show that this convex problem is actually an LMI problem that can be solved with widely spread semi-definite programming software. Note also that, contrary to the approach developed in [2], we do not require computing a nominally stabilizing controller.

Of course, depending on the problem being considered, LMI problem (9) is not necessarily always solvable. Infeasibility of the LMI problem can either stems from the conservativeness of the ellipsoidal approximation of the stability region proposed in lemma 2, or from the actual inexistence of a robustly stabilizing controller of given order. Using our method, it is not possible to recognize these situations.

Finally, let us point out that, similarly to [2], we can try to robustify the design in the sense that the controller must be stabilizing for all plant parameters p in ellipsoid rE_p, where scaling factor $r \geq 1$ is maximized. Invoking theorem 1, the resulting optimization problem is then a quasi-convex generalized eigenvalue LMI problem [5].

5 Numerical Example

We consider the two mixing tanks arranged in cascade with recycle stream shown in figure 2 and described in [7]. The controller must be designed to maintain the temperature T_b of the second tank at a desired set point by manipulating the power P delivered by the heater located in the first tank. The only available measurement is temperature T_b. The identification of the nominal plant model is carried out using a standard least-squares method [7]. A second-order ($n = 2$) discrete-time model

$$p(z) = \frac{p_{N0} + p_{N1}z}{p_{D0} + p_{D1}z + z^2}$$

is obtained with nominal plant vector

$$\bar{p} = \left[\begin{array}{cccc} 0.0038 & 0.0028 & 0.2087 & -1.1871 \end{array}\right]'.$$

An ellipsoidal uncertainty model is readily available as a by-product of the least-squares identification technique [7]. The positive definite matrix P characterizing the uncertainty

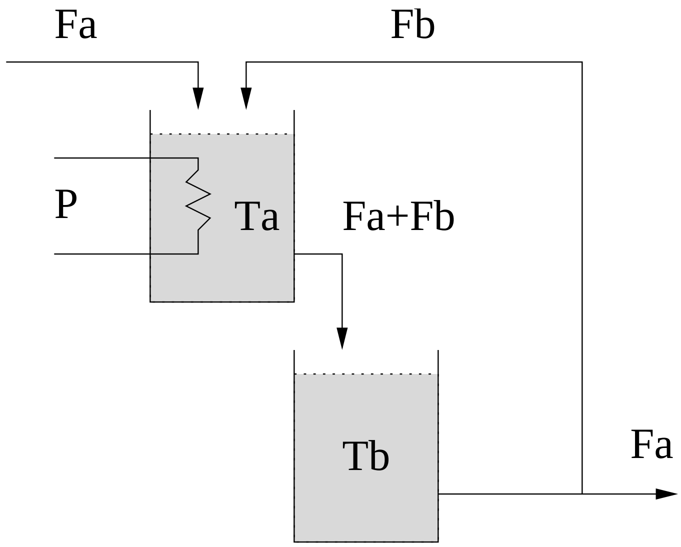

Figure 2: Two-tank system.

ellipsoid E_p defined in (3) is given by

$$P = 10^5 \begin{bmatrix} 2.4179 & 0.0568 & 0.0069 & 0 \\ 0.0568 & 2.4121 & 0.0045 & 0.0062 \\ 0.0069 & 0.0045 & 0.0015 & 0.0014 \\ 0 & 0.0062 & 0.0014 & 0.0015 \end{bmatrix}.$$

Now suppose that we are seeking a first-order ($m = 1$) controller

$$c(z) = \frac{\mathbf{c}_{N0} + \mathbf{c}_{N1} z}{\mathbf{c}_{D0} + z}$$

robustly D-stabilizing plant $p(s)$ for all admissible models $p \in E_p$. Since we are dealing with a discrete-time system, the stability region D is the unit disk here.

Solving the LMI optimization problem of lemma 2 with the user-friendly interface LMITOOL 2.0 for MATLAB [8], we obtain the following inner ellipsoidal approximation of the stability region of a third-order ($d = m + n = 3$) discrete-time polynomial:

$$Q = \begin{bmatrix} 2.3378 & 0 & 0.5397 \\ 0 & 2.1368 & 0 \\ 0.5397 & 0 & 1.7552 \end{bmatrix}, \quad \bar{q} = \begin{bmatrix} 0 \\ 0.1235 \\ 0 \end{bmatrix}.$$

Solving the LMI feasibility problem (9) of theorem 1, we obtain the controller

$$c(z) = \frac{0.3377 + 166.0z}{0.6212 + z}$$

robustly stabilizing plant $p(s)$. The robust root-locus of closed-loop characteristic polynomial $q(s)$ obtained by describing randomly the uncertainty ellipsoid E_p is represented in figure 3. We can check that indeed all characteristic polynomial roots stay in the unit disk D for all admissible uncertainty.

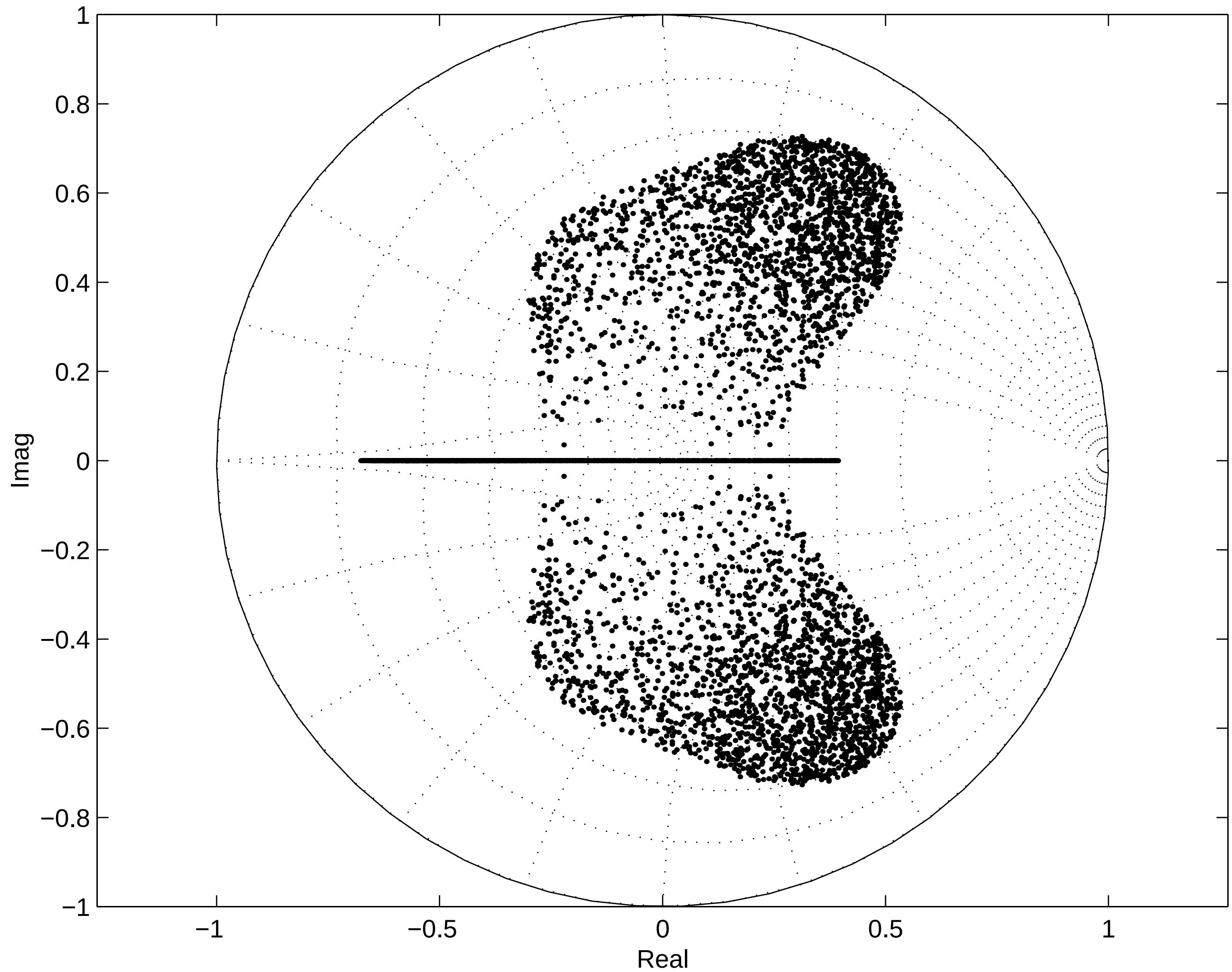

Figure 3: Root-locus of the uncertain closed-loop system.

6 Conclusion

We have shown that the problem of robust characteristic polynomial placement in a given stability ellipsoid for a SISO plant affected by uncertainty located in a given ellipsoid is a convex LMI problem that can be solved easily. Moreover, the stability ellipsoid used for characteristic polynomial placement can be computed via convex LMI optimization, as previously shown in [11].

This work can be extended readily to SIMO or MISO systems using the same formalism as in [4], but its extension to MIMO systems seems to be more difficult. Indeed, up to our knowledge, there exists no matrix version of lemma 1, i.e. a stability criterion for a matrix polynomial that can be expressed as a matrix inequality quadratic in the matrix coefficients. The derivation of such a criterion is one of our current research topics.

One can also think about removing some conservatism in the choice of the ellipsoidal stability region. The stability ellipsoid E_q computed in lemma 2 is given as input data in our main theorem 1 because otherwise the matrix inequality (9) in not simultaneously convex in both controller vector $\mathbf{c}$ and stability ellipsoid parameters Q and $\bar{q}$. Actually, the problem of simultaneously finding a controller $c(s)$ satisfying matrix inequality (9) and a stability ellipsoid E_q satisfying matrix inequality (6) is bilinear in the problem unknowns. A lot of work has been dedicated recently to the study of LMI relaxations of bilinear matrix inequalities. Application of these techniques to our design problem could help to further improve the robustness of the control law.

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