LMI based design for the Acrobot walking ⋆

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Abstract: This paper aims to further improve previously developed design for Acrobot walking based on partial exact feedback linearization of order 3. Namely, such an exact system transformation leads to an almost linear system where error dynamics along trajectory to be tracked is a 4-dimensional linear time-varying system having 3 time-varying entries only, the remaining entries being either zero or one. In such a way, exponentially stable tracking can be obtained by quadratically stabilizing a linear system with polytopic uncertainty. The current improvement is based on applying LMI methods to solve this problem numerically. This careful analysis significantly improves previously known approaches. Numerical simulations of Acrobot walking based on the above mentioned LMI design are demonstrated as well.

Keywords: Linear matrix inequalities (LMI), underactuated mechanical systems, walking robots.

1. INTRODUCTION

Efficient control of underactuated mechanical systems constitutes one of the most challenging problems of recent decades, see Zikmund and Moog (2005), Fantoni and Lozano (2002) and references therein. Reliable and economic walking is a typical example of studies involving both control and robotic communities. One of the simplest underactuated mechanical systems is the Acrobot. Despite being a seemingly simple system, the Acrobot comprises many important features of underactuated walking robots having degree of underactuation equal to one. As a matter of fact, one can show that any n-link having $n-1$ actuators between its links can be decomposed into a fully actuated system and an acrobot "disturbed" by some influence from that fully actuated (and therefore fully exact feedback linearizable) subsystem, see Spong (1998); Grizzle et al. (2005). As a consequence, effective control of the Acrobot is an important step on the route to underactuated walking. Recently, numerous papers have addressed stabilization of its inverted position, extending its domain of attraction (Bortoff and Spong (1992), Murray (1990), Furuta and Yamakita (1991), Wiklund et al (1993)), or even stable walking-like movement (Čelikovský and Zikmund (2007), Zikmund et al. (2007), Čelikovský et al. (2008)).

This paper aims to further extend the results obtained in Čelikovský et al. (2008). In that paper, asymptotical tracking of a suitable target trajectory generated by an open-loop reference control was obtained. As might have been expected, asymptotical tracking constitutes a principally more complicated problem than stabilization since the corresponding error dynamics has a more complex structure than the Acrobot model itself. In particular, designed tracking feedback could handle limited initial tracking error only and its performance was limited to the case when the Acrobot walking-like movement was very slow. This was caused by a specific and analytic method to stabilize tracking error dynamics. In the present paper, a numerical tuning of such a stabilization using an LMI approach will be used to significantly improve the limited results of Čelikovský et al. (2008).

The rest of the paper is organized as follows. The next section briefly presents the model of the Acrobot together with the main theoretical pre-requisites necessary for further numerical analysis. Section 3 describes the essence of the LMI approach while numerical optimization results and subsequent simulations of Acrobot walking are presented in Section 4. The final section draws briefly some conclusions and discusses some open future research outlooks toward efficient underactuated walking.

2. ACROBOT

The Acrobot depicted on Figure 1 is a special case of an n-link chain with $n-1$ actuators attached by one of its ends to a pivot point through an unactuated rotary joint. Such a system can be modelled by the usual Lagrangian approach, see Greiner (2003). The corresponding Lagrangian is as follows

$$\mathcal{L}(q,\dot{q}) = K - V = \frac{1}{2}\dot{q}^T D(q)\dot{q} - V(q) \qquad (1)$$

where q denotes an n-dimensional vector on the configuration manifold Q and $D(q)$ is the inertia matrix, K is the kinetic energy and V is the potential energy of the system. The resulting Euler-Lagrange equation is

⋆ Supported by the Grant Agency of the Czech Republic through the grant no. 102/08/0186 and by Ministry of Education and Sports of the CR through the grant no. LA09026.

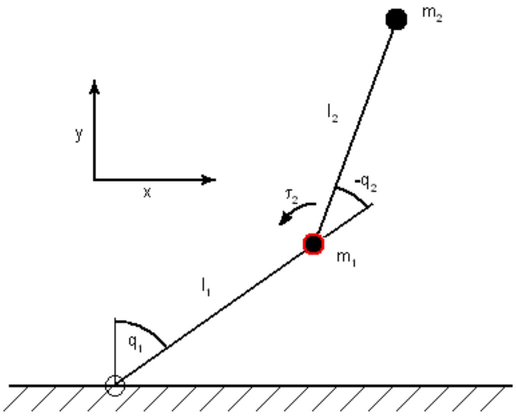

Fig. 1. Acrobot.

$$\begin{bmatrix} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} - \frac{\partial \mathcal{L}}{\partial q_1} \\ \vdots \\ \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_n} - \frac{\partial \mathcal{L}}{\partial q_n} \end{bmatrix} = u = \begin{bmatrix} 0 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}, \tag{2}$$

where u stands for the vector of external controlled forces. System (2) is a so-called **underactuated** mechanical system having degree of underactuation equal to one, see Spong (1998). Moreover, the underactuated angle is at the pivot point. Equation (2) leads to a dynamic equation of the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u \tag{3}$$

where $D(q)$ is the inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ contains gravity terms and u stands for the vector of external forces.

For the Acrobot, these computations lead to a second-order nonholonomic constraint and a kinetic symmetry, i.e. the inertia matrix depends only on the second variable q_2

$$D(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_2 + \theta_3 \cos q_2 & \theta_2 \end{bmatrix}, \tag{4}$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3 \sin q_2 \dot{q}_2 & -(\dot{q}_2 + \dot{q}_1)\theta_3 \sin q_2 \\ \theta_3 \sin q_2 \dot{q}_1 & 0 \end{bmatrix}, \tag{5}$$

$$G(q) = \begin{bmatrix} -\theta_4 g \sin q_1 - \theta_5 g \sin(q_1 + q_2) \\ -\theta_5 g \sin(q_1 + q_2) \end{bmatrix}, \tag{6}$$

where the 2-dimensional configuration vector (q_1, q_2) consists of angles defined on Figure 1 and

$$\begin{gathered} \theta_1 = (m_1 + m_2)l_1^2 + I_1,\ \theta_2 = m_2 l_2^2 + I_2, \\ \theta_3 = m_2 l_1 l_2,\ \theta_4 = (m_1 + m_2)l_1,\ \theta_5 = m_2 l_2. \end{gathered} \tag{7}$$

The **partial exact feedback linearization** method is based on a system transformation into a new system of coordinates that display linear dependence between an output and a new input, see Isidori (1996). From a theoretical point of view, the mechanical system dynamics is described by an n-dimensional state-space equation. Static state-feedback linearization of a suitable output function of relative degree r yields a linear subsystem of dimension r. In other words, the maximal feedback linearization problem consists in linearizing a function with maximal relative degree. In Grizzle et al. (2005) it was shown that if the generalized momentum conjugate to the cyclic variable is not conserved (as it is the case of the Acrobot) then there exists a set of outputs that defines one-dimensional exponentially stable zero dynamics. This means that it is possible to find a function $\overline{y}(q,\dot{q})$ with relative degree 3 that transforms the original system (3) by a local coordinate transformation

$$z = T(q,\dot{q}), \quad z_1 = \overline{y},\ z_2 = \dot{\overline{y}},\ z_3 = \ddot{\overline{y}},\ z_4 = f(q,\dot{q}), \tag{8}$$

into a new input/output linear system with unobservable nonlinear dynamics of dimension 1

$$\begin{gathered} \dot{z}_1 = z_2,\ \dot{z}_2 = z_3,\ \dot{z}_3 = \alpha(q,\dot{q})v + \beta(q,\dot{q}) = w, \\ \dot{z}_4 = \psi_1(q,\dot{q}) + \psi_2(q,\dot{q})\tau_2. \end{gathered} \tag{9}$$

In the case of the Acrobot there are two independent functions with relative degree 3 transforming the system into the desired form [1] (9), namely

$$\sigma = \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)\dot{q}_1 + (\theta_2 + \theta_3 \cos q_2)\dot{q}_2, \tag{10}$$

$$p = q_1 + \frac{q_2}{2} + \frac{2\theta_2 - \theta_1 - \theta_2}{\sqrt{(\theta_1 + \theta_2)^2 - 4\theta_3^2}} \arctan \left(\sqrt{\frac{\theta_1 + \theta_2 - 2\theta_3}{\theta_1 + \theta_2 + 2\theta_3}} \tan \frac{q_2}{2} \right). \tag{11}$$

The zero dynamics is used to investigate internal stability when the corresponding output is forced to zero. For the most simple cases $\overline{y} = Cp$ or $\overline{y} = C\sigma$ the resulting zero dynamics is only critically stable. However, considering the output function $\overline{y} = C_1 p(q) + C_2 \sigma(q,\dot{q})$ one gets the following zero dynamics $\dot{p} + C_1[C_2 d_{11}(q_2)]^{-1} p = 0$ which is asymptotically stable whenever C_1/C_2 is positive, $d_{11}(q_2)$ being the corresponding part of the inertia matrix D in (3). Unfortunately, the corresponding transformations have a complex set of singularities, unless C_1 is very small, which is not suitable for practical purposes.

In Čelikovský et al. (2008), it was shown that using the set of functions with maximal relative degree, the following transformation

$$\xi_1 = p, \xi_2 = \sigma, \xi_3 = \dot{\sigma}, \xi_4 = \ddot{\sigma} \tag{12}$$

can be defined. Notice, that by (10)-(11) and some straightforward but laborious computations the following relation holds

$$\dot{p} = d_{11}(q_2)^{-1}\sigma, \tag{13}$$

where $d_{11}(q_2) = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)$ is the corresponding element of the inertia matrix D in (3). Applying (12), (13) to (3) we obtain Acrobot dynamics in partial exact linearized form

$$\begin{gathered} \dot{\xi}_1 = d_{11}(q_2)^{-1}\xi_2,\ \dot{\xi}_2 = \xi_3,\ \dot{\xi}_3 = \xi_4, \\ \dot{\xi}_4 = \alpha(q,\dot{q})\tau_2 + \beta(q,\dot{q}) = w \end{gathered} \tag{14}$$

with new coordinates ξ and input w being well defined whenever $\alpha(q,\dot{q})^{-1} \neq 0$.

[1] Actually, by (2), $\dot{\sigma} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = \frac{\partial \mathcal{L}}{\partial q_1}$ and therefore by (1), $\dot{\sigma} = -\frac{\partial V(q)}{\partial q_1}$ as $D(q) \equiv D(q_2)$ by (4). In other words, $\dot{\sigma}$ has relative degree 2, i.e. σ has relative degree 3. Moreover, by straightforward differentiation it holds $\dot{p} = d_{11}(q_2)^{-1}\sigma$, i.e. $\dot{p}$ has relative degree 2, i.e. p should have relative degree 3 as well.

System (14) is almost linear, but there is a nonlinearity $d_{11}(q_2)^{-1}$ in the first row that depends on q_2 only. Instead of expressing this nonlinearity in coordinates ξ and trying to study its exact influence, one can use some favorable qualitative properties. Namely, one can easily see that

$$a_{min} \leq d_{11}(q_2)^{-1} \leq a_{max} \tag{15}$$

$$a_{min} = \frac{1}{m_2(l_1+l_2)^2 + m_1 l_1^2 + I_1 + I_2} \tag{16}$$

$$a_{max} = \frac{1}{m_2(l_1-l_2)^2 + m_1 l_1^2 + I_1 + I_2}. \tag{17}$$

Notice, that the quantity

$$a_{max} - a_{min} = \frac{4l_1 l_2 m_2 (m_2(l_1+l_2)^2 + m_1 l_1^2 + I_1 + I_2)^{-1}}{(m_2(l_1-l_2)^2 + m_1 l_1^2 + I_1 + I_2)} \tag{18}$$

is quite small and therefore the nonlinearity $d_{11}(q_2)^{-1}$ is actually varying in a quite narrow range. Therefore, its derivative also evolves in a favorable way, namely

$$\frac{\partial[d_{11}(q_2)^{-1}]}{\partial q_2} = (2\theta_3 \sin q_2) d_{11}(q_2)^{-2}, \tag{19}$$

$$\left|\frac{\partial[d_{11}^{-1}]}{\partial q_2}\right| \leq 2\theta_3 a_{max}^2. \tag{20}$$

The above favorable properties of Acrobot partial linearization will be used in the sequel for a feedback design ensuring exponentially tracking of a given walking-like trajectory. We assume that an open-loop control generating a suitable reference trajectory is given in partial exact linearized coordinates (14), and our task is to track the following reference system

$$\dot\xi_1^{ref} = d_{11}^{-1}(q_2^{ref})\xi_2^{ref},\ \dot\xi_2^{ref} = \xi_3^{ref},$$
$$\dot\xi_3^{ref} = \xi_4^{ref},\ \dot\xi_4^{ref} = w^{ref}. \tag{21}$$

The following theorem gives a constructive and analytic way to asymptotically track reference system (21).

Theorem 1. Consider system (14) with the following feedback

$$\begin{aligned} &w = w^{ref} + \\ &\Theta^3 K_1 e_1 + \Theta^3 K_2 e_2 + \Theta^2 K_3 e_3 + \Theta K_4 e_4, \\ &e =: \xi - \xi^{ref}. \end{aligned} \tag{22}$$

Further, let $K_1 < 0$ and $K_{2,3,4}$ be such that the polynomial $\lambda^3 + K_4\lambda^2 + K_3\lambda + K_2$ is Hurwitz. Then there exist $\Theta > 0, \mathcal{R} > 0, \mathcal{B} > 0$ such that for all reference trajectories given by (21) and satisfying

$$\forall t \geq 0\ \ |s(\phi_2(\xi^{ref})(t))| \geq \mathcal{B} > 0, \tag{23}$$

$$|\xi_2^{ref}(t)| \leq \mathcal{R},\ \forall t \geq 0, \tag{24}$$

where ϕ_2 and $s(q)$ are certain functions given in Čelikovský et al. (2008), it follows that $e(t) \to 0,\ t \to \infty$. locally exponentially for e given by (22).

The above theorem is based on a certain specific adaptation of high-gain technique, enabling to produce an exact mathematical proof of stability. The drawback is rather high stabilizing gains leading to an unreasonable high torque at the actuated Acrobot joint. Moreover the convergence is slow and proved only for slow walking speed. As a matter of fact, simulations show that the stabilizer works even for walking speeds significantly higher than those necessary for the theoretical proof.

Therefore, a natural idea is to try to stabilize the error dynamics using more sophisticated numerical methods, like linear matrix inequalities (LMI). To be more specific, let us repeat that in Čelikovský et al. (2008) it was shown that subtracting (21) from (14) with (22) one obtains

$$\dot e_1 = d_{11}^{-1}(\phi_2(\xi_1,\xi_3))\xi_2 - d_{11}^{-1}(\phi_2(\xi_1^{ref},\xi_3^{ref}))\xi_2^{ref},$$
$$\dot e_2 = e_3, \dot e_3 = e_4, \dot e_4 = \Theta^3 K_1 e_1 + \Theta^3 K_2 e_2 + \Theta^2 K_3 e_3 + \Theta K_4 e_4.$$

Straightforward computations based on Taylor expansions give

$$\dot e_1 = \mu_2(t)e_2 + \mu_1(t)e_1 + \mu_3(t)e_3 + o(e) \tag{25}$$

$$\dot e_2 = e_3,\ \ \dot e_3 = e_4, \tag{26}$$

$$\dot e_4 = \Theta^3 K_1 e_1 + \Theta^3 K_2 e_2 + \Theta^2 K_3 e_3 + \Theta K_4, \tag{27}$$

$$\mu_1(t) = \xi_2^{ref}(t)\frac{\partial[d_{11}^{-1}]}{\partial q_2}\frac{\partial \phi_2}{\partial \xi_1}(q_2^{ref}(t)), \tag{28}$$

$$\mu_2(t) = d_{11}^{-1}(q_2^{ref}(t)), \tag{29}$$

$$\mu_3(t) = \xi_2^{ref}(t)\frac{\partial[d_{11}^{-1}]}{\partial q_2}\frac{\partial \phi_2}{\partial \xi_3}(q_2^{ref}(t)), \tag{30}$$

$$q_2^{ref}(t) = \phi_2(\xi_1^{ref}(t), \xi_3^{ref}(t)),\ \ q_2 \in [0, 2\pi). \tag{31}$$

In Čelikovský et al. (2008) it was shown that

$$|\mu_1(t)| \leq 2\theta_3 a_{max}^2 (\theta_4 + \theta_5)\frac{\mathcal{R}}{\mathcal{B}} \tag{32}$$

$$|\mu_3(t)| \leq 2\theta_3 a_{max}^2 \frac{\mathcal{R}}{\mathcal{B}},\ \ 0 < a_{min} \leq \mu_2(t) \leq a_{max}. \tag{33}$$

It turns out that the above bounds can be quite easily evaluated numerically.

3. LMI BASED STABILIZATION OF THE ERROR DYNAMICS

It was shown at the end of the previous section that for reference trajectory tracking one has to solve the following stabilization problem. Consider the open-loop continuous time-varying linear system

$$\dot e = A(t)e + Bu, \tag{34}$$

where

$$A(t) = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

The tracking problem consists in finding the state-feedback controller

$$u = Ke,\ \ K = (\,K_1\ K_2\ K_3\ K_4\,), \tag{35}$$

producing the following closed-loop system

$$\dot e = (A + BK)\,e = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K_1 & K_2 & K_3 & K_4 \end{pmatrix} e, \tag{36}$$

where bounds for $\mu(t) = (\mu_1(t), \mu_2(t), \mu_3(t))$ are given by (32)-(33).

Despite entries of $\mu(t)$ are **known** functions, the appealing idea is to treat them as **unknown disturbances** satisfying the above mentioned given constraints. If constraints are tight enough, one can think about solving quadratic stability conditions and design a unique feedback stabilizing such an “uncertain” system. Obviously, such a feedback would be at the same time solving our tracking problem.

To pursue such an idea, let us obtain LMI conditions for quadratic stability. Let us recall here that quadratic stability is a particular case of robust stability, valid for arbitrarily fast time-variation of the uncertain parameters, and certified by a unique quadratic-in-the-state parameter-independent Lyapunov function. Consider the well-known Lyapunov inequality to be solved for all values of $\mu(t)$ by finding a suitable symmetric positive definite matrix S and a vector K:

$$(A(\mu) + BK)^{\mathrm{T}} S + S(A(\mu) + BK) \preceq 0, \tag{37}$$

$$S = S^{\mathrm{T}} \succ 0. \tag{38}$$

Such a problem is in fact bilinear with respect to the unknowns. Denoting

$$Q = S^{-1}, Y = KS^{-1} \tag{39}$$

we derive the following LMI condition for quadratically stabilizing feedback design:

$$QA^{\mathrm{T}}(\mu) + A(\mu)Q + Y^{\mathrm{T}}B^{\mathrm{T}} + BY \preceq 0. \tag{40}$$

Notice that the pair $(A(\mu), B)$ is controllable if and only if

$$\mu_1\mu_3 + \mu_2 \neq 0. \tag{41}$$

Obviously, if the set of possible values of μ contains, or stays close to, the singular set given by (41), LMI (40) becomes infeasible, or almost infeasible.

4. NUMERICAL ANALYSIS AND SIMULATIONS

As already indicated, bounds on $\mu(t)$ during a single step of the so-called passive walking, cf. Čelikovský et al. (2008), can be obtained numerically, see Figs. 2 and 6. Two cases of LMI solving will be considered: when the $\mu(t)$ trajectory is estimated by box-like (rectangular) set and secondly by a prism-like (non-rectangular) set.

4.1 Convex rectangular parameter set

In the first case the convex set is defined as a rectangular box, see Fig. 2. Each vertex of the box is defined by a combination of upper- and lower-bounds on entries of μ. Summarizing, we have 8 constraints

$$\begin{gathered} QA_i^{\mathrm{T}} + A_iQ + Y^{\mathrm{T}}B^{\mathrm{T}} + BY \preceq 0, i = 1, \ldots, 8, \\ A_1 = A^{\mathrm{T}}\left(\overline{\mu_1}, \overline{\mu_2}, \overline{\mu_3}\right), \; A_2 = A^{\mathrm{T}}\left(\underline{\mu_1}, \overline{\mu_2}, \overline{\mu_3}\right), \; \ldots, \\ A_7 = \left(\underline{\mu_1}, \underline{\mu_2}, \overline{\mu_3}\right), \; A_8 = \left(\underline{\mu_1}, \underline{\mu_2}, \underline{\mu_3}\right). \end{gathered} \tag{42}$$

These LMIs are solved using the YALMIP parser and the SeDuMi solver with Matlab, giving the state-feedback matrix

$$K = 10^5 \cdot (-3.5810 \;\; -1.8147 \;\; -0.1854 \;\; -0.0037).$$

One can see that these gains are quite large. Moreover, the resulting torques are unrealistic during a short time interval at the beginning of the step. In the step trajectory simulations, the initial positions errors are zero but velocities errors are about 20%. Because the initial torque is unrealistic for the actual model of Acrobot, we set the saturation limit to the range $\pm 25\,\mathrm{Nm}$, see Fig. 5. The effect of the saturation limit is clearly visible on Fig. 3 and Fig. 4. Experimentally, the saturation limit could not be further lowered, yet it is still almost unrealistic.

Summarizing, using the rectangular box to estimate the values of $\mu(t)$ produces highly conservative and practically unacceptable design. Fortunately, tighter bounding sets can be used to estimate the values of $\mu(t)$, as shown in the next subsection.

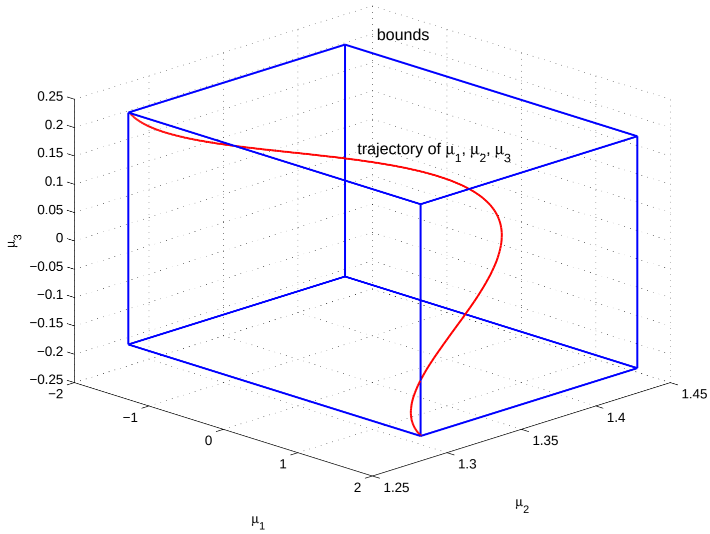

Fig. 2. Trajectory $\mu(t)$ and rectangular bounds.

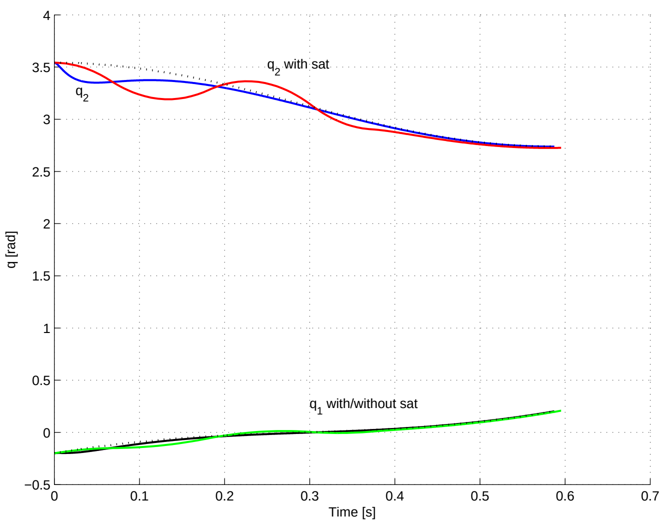

Fig. 3. Angular positions q_1, q_2 with and without saturation and references (dotted line) for rectangular bounds on μ.

4.2 Convex prismatic parameter set

In the second case we reduce the parameter set into a convex set much closer to the actual trajectory $\mu(t)$. The number of LMI constraints is thereby reduced to 6: two constraints are the same as previously, the remaining 4 constraints are defined via vertices relatively close to each other and centered around parameters value at the middle of the step. It is nicely seen from Figure 6 that this set is reasonably small and close to a triangle.

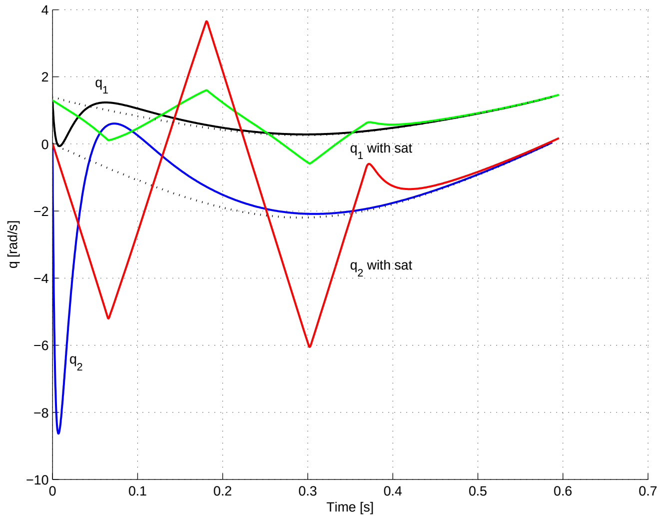

Fig. 4. Angular velocities q_1, q_2 with and without saturation and references (dotted line) for rectangular bounds on μ.

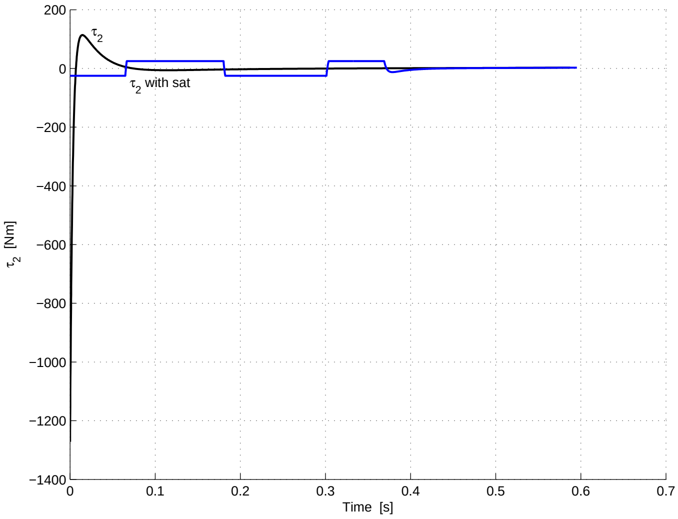

Fig. 5. Torque τ_2 with and without saturation for rectangular bounds on μ.

Solving the resulting LMI yields the state-feedback matrix

$$K = 10^4 \cdot (-1.9087 \quad -1.2097 \quad -0.1781 \quad -0.0090).$$

The gains are significantly smaller than previously.

The initial positions errors are zero while velocities errors are about 20%. For the sake of comparison, they are the same as for the rectangular parameter set. The initial torque is much smaller now, yet still quite unrealistic for the actual model of Acrobot. Therefore, a saturation limit in the range $\pm 10\,\mathrm{Nm}$ was used, see Fig. 9. In Fig. 7 and in Fig. 8 one can see the effect of saturation limit. Convergence is very good now and saturation limits now ensure a realistic implementation.

Finally, Figure 10 shows the animation of the Acrobot walking step with the prismatic parameter set based controller and torque saturation of $\pm 10\mathrm{Nm}$.

5. CONCLUSIONS AND OUTLOOKS

An LMI-based design for the stabilization of error dynamics resulting from tracking a walking-like trajectory of the Acrobot

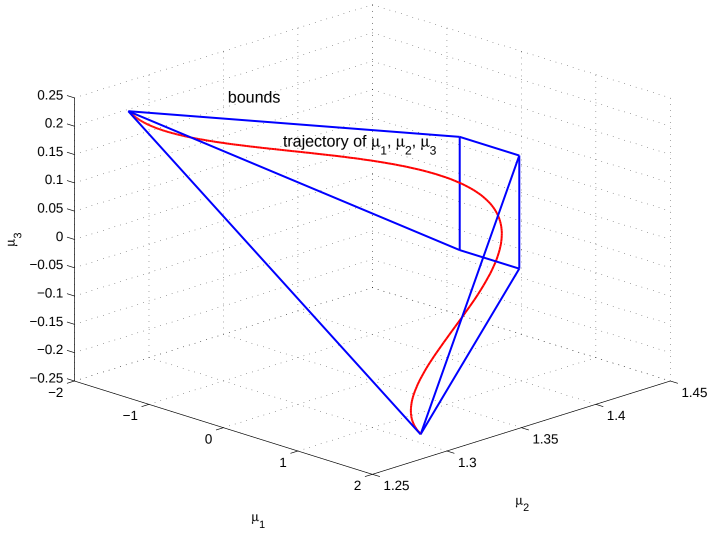

Fig. 6. Trajectory $\mu(t)$ and prismatic bounds.

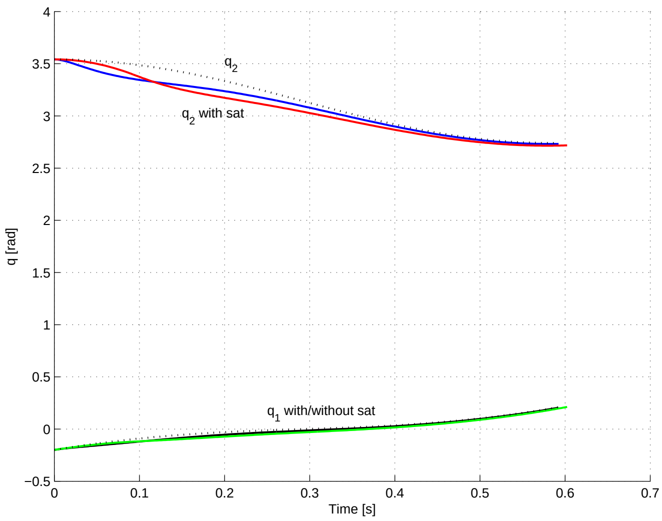

Fig. 7. Angular positions q_1, q_2 with and without saturation and references (dotted line) for prismatic bounds on μ.

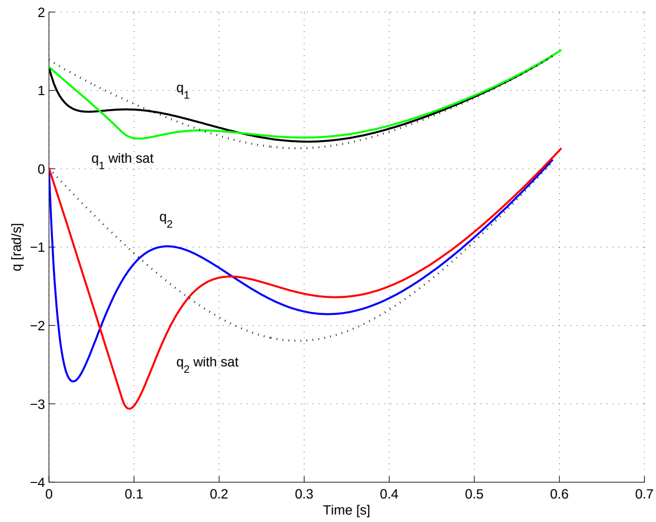

Fig. 8. Angular velocities q_1, q_2 with and without saturation and references (dotted line) for prismatic bounds on μ.

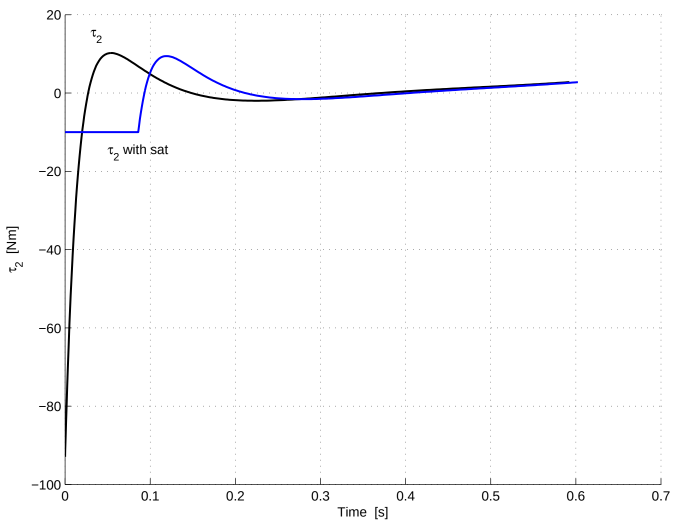

Fig. 9. Torque τ_2 with and without saturation for prismatic bounds on μ.

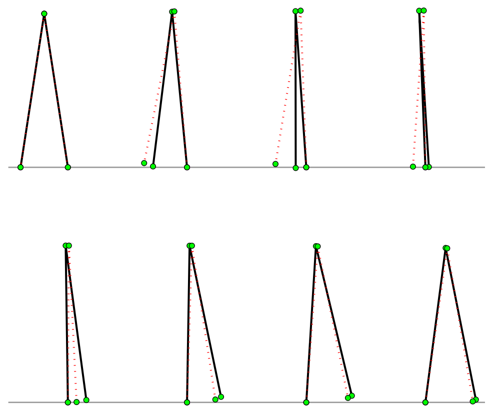

Fig. 10. Animation of a single step with sampling time $0.08\,\mathrm{s}$. The dotted line is the reference, the full line represents the actual Acrobot.

has been suggested. Compared to earlier analytic results in Čelikovský et al. (2008), it gives now quite realistic torque at the Acrobot actuator.

Yet, further torque optimization is possible via a further restriction of the set estimating parameter values. Namely, so far we have modeled the parameter trajectory as a polytope in the parameter space, and this allowed for the application of simple vertex LMI conditions corresponding to the search of a quadratic Lyapunov function. More sophisticated LMI conditions, based on representations of positive polynomials, can be derived for parameters varying along a curve, or within a general basic semialgebraic set (conjunction of multivariate polynomial inequalities). In the same vein, we could also derive LMI conditions to search for parameter-dependent polynomial-in-the-state Lyapunov functions, so as to reduce conservatism, if necessary.

Nevertheless, the issue of defining criterion to minimize the input torque action remains open. First problem is that criterion should be linear in LMI variables Y, Q while gains K are nonlinear function of them, i.e. they can not be directly taken as the linear cost function. Secondly, gains K affect real torques indirectly because there is nonlinear change of coordinates and feedback transformation between real torque τ_2 and virtual input w, resulting from partial feedback linearization.

Regarding saturations of the control signal, we could also model them as sector-bounded nonlinearities and, as a post-processing phase, assess stability of the resulting closed-loop system in the presence of saturations via appropriate Lyapunov-based LMI conditions.

These ideas are currently subject of ongoing research.

ACKNOWLEDGEMENTS

We would like to thank Fabrizio Dabbene from Politecnico di Torino for fruitful discussion and useful tips.

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