A practical approach to feedback control for a mobile robot with trailer

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Abstract: This paper presents a robust method to control a mobile robot towing a trailer. Both problems of trajectory tracking and steering to a given configuration are addressed. This second issue is solved by an iterative trajectory tracking. Perturbations are taken into account along the motions. Experimental results on the mobile robot Hilare illustrate the validity of our approach.

1 Introduction

Motion control for nonholonomic systems have given rise to a lot of work for the past 8 years. Brockett's condition [2] made stabilization about a given configuration a challenging task for such systems, proving that it could not be performed by a simple continuous state feedback. Alternative solutions as time-varying feedback [10, 4, 11, 13, 14, 15, 18] or discontinuous feedback [3] have been then proposed. See [5] for a survey in robot motion control. On the other hand, tracking a trajectory for a nonholonomic system does not meet Brockett's condition and thus it is an easier task. A lot of work have also addressed this problem [6, 7, 8, 12, 16] for the particular case of mobile robots.

All these control laws work under the same assumption: the evolution of the system is exactly known and no perturbation makes the system deviate from its trajectory. Few papers dealing with mobile robots control take into account perturbations in the kinematics equations. [1] however proposed a method to stabilize a car about a configuration, robust to control vector fields perturbations, and based on iterative trajectory tracking.

The presence of obstacle makes the task of reaching a configuration even more difficult and require a path planning task before executing any motion.

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In this paper, we propose a robust scheme based on iterative trajectory tracking, to lead a robot towing a trailer to a configuration. The trajectories are computed by a motion planner described in [17] and thus avoid obstacles that are given in input. In the following, we won't give any development about this planner, we refer to this reference for details. Moreover, we assume that the execution of a given trajectory is submitted to perturbations. The model we chose for these perturbations is very simple and very general. Our method presents thus some common points with [1].

The paper is organized as follows. Section 2 describes our experimental system Hilare and its trailer: two hooking systems will be considered (Figure 1). Section 3 deals with the control scheme and the analysis of stability and robustness. In Section 4, we present experimental results.

2 Description of the system

Hilare is a two driving wheel mobile robot. A trailer is hitched on this robot, defining two different systems depending on the hooking device: on system A, the trailer is hitched above the wheel axis of the robot (Figure 1, top), whereas on system B it is hitched behind this axis (Figure 1, bottom). A is the particular case of B, for which $\ell_r = 0$. This system is however singular from a control point of view and requires more complex computations. For this reason, we deal separately with both hooking systems. Two motors enable to control the linear and angular velocities (v_r, ω_r) of the robot. These velocities are moreover measured by odometric sensors, whereas the angle φ between the robot and the trailer is given by an optical encoder. The position and orientation (x_r, y_r, θ_r) of the robot are computed by integrating the former velocities. With these notations, the control system of B

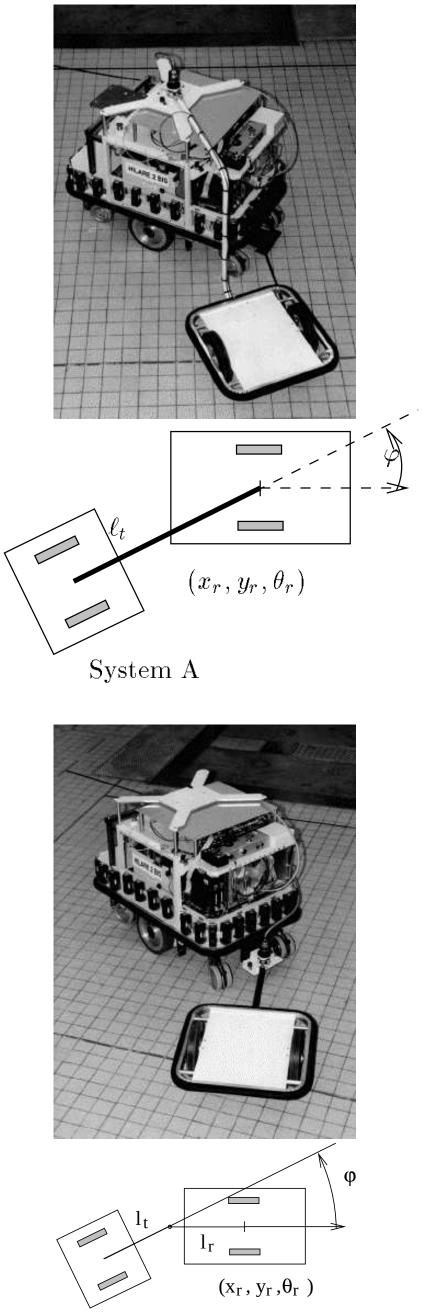

Figure 1: Hilare with its trailer

is:

$$\begin{cases} \dot{x_r} &= v_r \cos\theta_r \\ \dot{y_r} &= v_r \sin\theta_r \\ \dot{\theta_r} &= \omega_r \\ \dot{\varphi} &= -\frac{v_r}{l_t}\sin(\varphi) - \frac{l_r \omega_r}{l_t}\cos(\varphi) - \omega_r \end{cases} \tag{1}$$

3 Global control scheme

3.1 Motivation

When considering real systems, one has to take into account perturbations during motion execution. These may have many origins as imperfection of the motors, slippage of the wheels, inertia effects... These perturbations can be modeled by adding a term in the control system (1), leading to a new system of the form

$$\dot{x} = f(x, u) + \epsilon$$

where ϵ may be either deterministic or a random variable. In the first case, the perturbation is only due to a bad knowledge of the system evolution, whereas in the second case, it comes from a random behavior of the system. We will see later that this second model is a better fit for our experimental system.

To steer a robot from a start configuration to a goal, many works consider that the perturbation is only the initial distance between the robot and the goal, but that the evolution of the system is perfectly known. To solve the problem, they design an input as a function of the state and time that makes the goal an asymptotically stable equilibrium of the closed loop system. Now, if we introduce the previously defined term ϵ in this closed loop system, we don't know what will happen. We can however conjecture that if the perturbation ϵ is small and deterministic, the equilibrium point (if there is still one) will be close to the goal, and if the perturbation is a random variable, the equilibrium point will become an equilibrium subset. But we don't know anything about the position of these new equilibrium point or subset.

Moreover, time varying methods are not convenient when dealing with obstacles. They can only be used in the neighborhood of the goal and this neighborhood has to be properly defined to ensure collision-free trajectories of the closed loop system. Let us notice that discontinuous state feedback cannot be applied in the case of real robots, because discontinuity in the velocity leads to infinite accelerations.

The method we propose to reach a given configuration *in the presence of obstacles* is the following. We first build a collision free path between the current configuration and the goal using a collision-free motion planner described in [17], then we execute the trajectory with a simple tracking control law. At the end of the motion, the robot does never reach exactly the goal because of the various perturbations, but a neighborhood of this goal. If the reached configuration is too far from the goal, we compute another trajec-

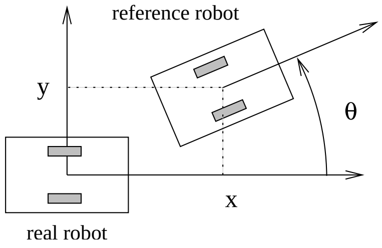

Figure 2: Tracking control law for a single robot

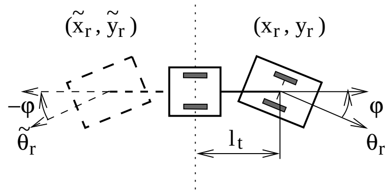

Figure 3: virtual robot

tory that we execute as we have done for the former one.

We will now describe our trajectory tracking control law and then give robustness issues about our global iterative scheme.

3.2 The trajectory tracking control law

In this section, we deal only with system A. We will see that equivalent computations are easier for system B.

A lot of tracking control laws have been proposed for wheeled mobile robots without trailer. One of them [16], although very simple, give excellent results. If (x, y, θ) are the coordinates of the reference robot in the frame of the real robot (Figure 3.2), and if (v_r^0, ω_r^0) are the inputs of the reference trajectory, this control law has the following expression:

$$\left\{\begin{array}{lcl} v_r & = & v_r^0 \cos\theta + k_1 x \\ \omega_r & = & \omega_r^0 + k_3\theta + k_2 \frac{\sin\theta}{\theta} y \end{array}\right. \qquad (2)$$

The key idea of our control law is the following: when the robot goes forward, the trailer need not be stabilized (see below). So we apply (2) to the robot. When it goes backward, we define a virtual robot $(\tilde{x}_r, \tilde{y}_r, \tilde{\theta}_r)$ (Figure 3.2) which is symmetrical to the real one with respect to the wheel axis of the trailer:

$$\left\{\begin{array}{lcl} \tilde{x}_r & = & x_r - 2\ell_t \cos\theta_t \\ \tilde{y}_r & = & y_r - 2\ell_t \sin\theta_t \\ \tilde{\theta}_r & = & \pi + 2\theta_t - \theta_r \end{array}\right.$$

Then, when the real robot goes backward, the virtual robot goes forward and the virtual system $(\tilde{x}_r, \tilde{y}_r, \tilde{\theta}_r, -\varphi)$ is kinematically equivalent to the real one. Thus we apply the tracking control law (2) to the virtual robot.

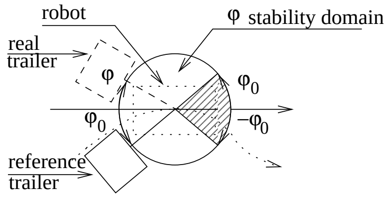

Figure 4: Stability domain for $\hat{\varphi}$

A question arises now: is the trailer really always stable when the robot goes forward ? The following section will answer this question.

3.3 Stability analysis of the trailer

We consider here the case of a forward motion $(v_r > 0)$, the backward motion being equivalent by the virtual robot transformation. Let us denote by $(x_r^0, y_r^0, \theta_r^0, \varphi^0, v_r^0, \omega_r^0)$ a reference trajectory and by $(x_r, y_r, \theta_r, \varphi, v_r, \omega_r)$ the real motion of the system. We assume that the robot follows exactly its reference trajectory: $(x_r, y_r, \theta_r, v_r, \omega_r) = (x_r^0, y_r^0, \theta_r^0, v_r^0, \omega_r^0)$ and we focus our attention on the trailer deviation $\hat{\varphi} = \varphi - \varphi^0$. The evolution of this deviation is easily deduced from system (1):

$$\begin{array}{lcl} \dot{\hat{\varphi}} & = & -\dfrac{v_r}{l_t}(\sin\varphi - \sin\varphi^0) \\ & = & -\dfrac{2v_r}{\ell_t}\cos(\dfrac{\varphi + \varphi^0}{2})\sin(\dfrac{\hat{\varphi}}{2}) \end{array}$$

$|\hat{\varphi}|$ is thus decreasing iff

$$-\frac{\pi}{2} < \varphi^0 + \frac{\hat{\varphi}}{2} < \frac{\pi}{2} \qquad [2\pi] \qquad (3)$$

Our system is moreover constrained by the inequalities

$$-\pi/2 < \varphi, \varphi^0 < \pi/2 \qquad (4)$$

so that $-\pi < \hat{\varphi} < \pi$ and (3) is equivalent to

$$\left\{\begin{array}{ccc} 0 < \varphi^0 < \frac{\pi}{2} & \text{and} & -\pi < \hat{\varphi} < \pi - 2\varphi^0 \\ & \text{or} & \\ -\frac{\pi}{2} < \varphi^0 < 0 & \text{and} & -\pi - 2\varphi^0 < \hat{\varphi} < \pi \end{array}\right. \qquad (5)$$

Figure 4 shows the domain on which $|\hat{\varphi}|$ is decreasing for a given value of φ^0. We can see that this domain contains all positions of the trailer defined by the bounds (4). Moreover, the previous computations permit easily to show that 0 is an asymptotically stable value for the variable $\hat{\varphi}$.

Thus if the real or virtual robot follows its reference forward trajectory, the trailer is stable and will converge toward its own reference trajectory.

3.4 Virtual robot for system B

When the trailer is hitched behind the robot, the former construction is even more simple: we can replace the virtual robot by the trailer. In this case indeed, the velocities of the robot (v_r, ω_r) and of the trailer (v_t, ω_t) are connected by a one-to-one mapping. The configuration of the virtual robot is then given by the following system:

$$\left\{ \begin{array}{rcl} \tilde{x}_r &=& x_r - l_r \cos\theta_r - l_t \cos(\theta_r + \varphi) \\ \tilde{y}_r &=& y_r - l_r \sin\theta_r - l_t \sin(\theta_r + \varphi) \\ \tilde{\theta}_r &=& \theta_r + \varphi + \pi \end{array} \right.$$

and the previous stability analysis can be applied as well, by considering the motion of the hitching point.

The following section addresses the robustness of our iterative scheme.

3.5 Robustness of the iterative scheme

We are now going to show the robustness of the iterative scheme we have described above. For this, we need to have a model of the perturbations arising when the robot moves. [1] model the perturbations by a bad knowledge of constants of the system, leading to deterministic variations on the vector fields. In our experiment we observed random perturbations due for instance to some play in the hitching system. These perturbations are very difficult to model. For this reason, we make only two simple hypotheses about them:

$$\begin{array}{rcl} d_{\mathcal{CS}}(q(s), q^0(s)) &\leq& \delta s \\ d_{\mathcal{CS}}(q(s), q^0(s)) &\leq& \Delta \end{array}$$

where s is the curvilinear abscissa along the planned path, q and q^0 are respectively the real and reference configurations, $d_{\mathcal{CS}}$ is a distance over the configuration space of the system and δ, Δ are positive constants. The first inequality means that the distance between the real and the reference configurations is proportional to the distance covered on the planned path. The second inequality is ensured by the trajectory tracking control law that prevents the system to go too far away from its reference trajectory. Let us point out that these hypotheses are very realistic and fit a lot of perturbation models.

We need now to know the length of the paths generated at each iteration. The steering method we use to compute these paths verifies a topological property accounting for small-time controllability [17]. This means that if the goal is sufficiently close to the starting configuration, the computed trajectory remains in a neighborhood of the starting configuration. In [9] we give an estimate in terms of distance: if q_1 and q_2 are two sufficiently close configurations, the length of the planned path between them verifies

$$\ell(\mathrm{Path}(q_1, q_2)) < \eta d_{\mathcal{CS}}(q_1, q_2)^{\frac{1}{4}}$$

where η is a positive constant.

Thus, if $(q_i)_{i=1,2,\dots}$ is the sequence of configurations reached after i motions, we have the following inequalities:

$$\begin{array}{rcl} d_{\mathcal{CS}}(q_1, q_{goal}) &\leq& \Delta \\ d_{\mathcal{CS}}(q_{i+1}, q_{goal}) &\leq& \eta d_{\mathcal{CS}}(q_i, q_{goal}) \end{array}$$

These inequalities ensure that $distCS(q_i, q_{goal})$ is upper bounded by a sequence $(d_i)_{i=1,2,\dots}$ of positive numbers defined by

$$\begin{array}{rcl} d_1 &=& \Delta \\ d_{i+1} &=& \eta d_i^{\frac{1}{4}} \end{array}$$

and converging toward $\eta^{4/3}$ after enough iterations.

Thus, we do not obtain asymptotical stability of the goal configuration, but this result ensures the existence of a stable domain around this configuration. This result essentially comes from the very general model of perturbations we have chosen. Let us repeat that including such a perturbation model in a time varying control law would undoubtedly make it lose its asymptotical stability.

The experimental results of the following section show however, that the converging domain of our control scheme is very small.

4 Experimental results

We present now experimental results obtained with our robot Hilare towing a trailer, for both systems A and B. Figures 5 and 6 show examples of first paths computed by the motion planner between the initial configurations (in black) and the goal configurations (in grey), including the last computed maneuver in the second case. The lengths of both hooking system is the following: l_r=0, l_t=125cm for A and l_r=60cm, l_t=90cm for B. Tables 1 and 2 give the position of initial and final configurations and the gaps between the goal and the reached configurations after one motion and two motions, for 3 different experiments. In both cases, the first experiment corresponds to the figure. Empty Δq_2 columns mean that the precision reached after the first motion was sufficient and that no more motion was performed.

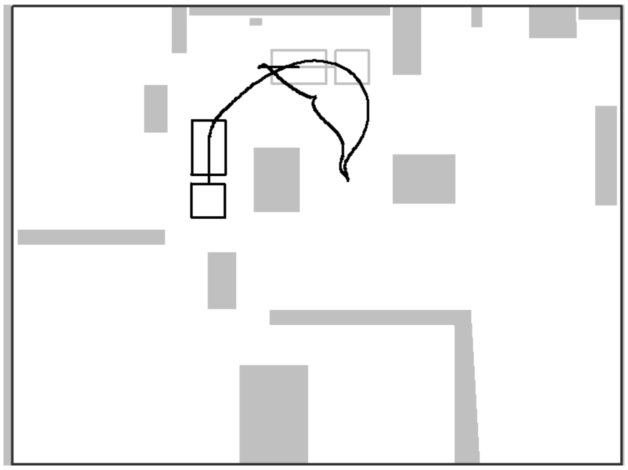

Figure 5: System A: the initial and goal configurations and the first path to be tracked

Comments and Remarks: The results reported in the tables 1 and 2 lead to two main comments. First, the precision reached by the system is very satisfying and secondly the number of iterations is very small (between 1 and 2). In fact, the precision depends a lot on the velocity of the different motions. Here the maximal linear velocity of the robot was 50 cm/s.

5 Conclusion

We have presented in this paper a method to steer a robot with one trailer from its initial configuration to a goal given in input of the problem. This method is based on an iterative approach combining open loop and close loop controls. It has been shown robust with respect to a large range of perturbation models. This robustness mainly comes from the topological property of the steering method introduced in [17]. Even if the method does not make the robot converge exactly to the goal, the precision reached during real experiments is very satisfying.

q_{init}	q_{goal}	Δq_1	Δq_2
$x = 2.10$	$x = 4.26$	$\delta x = 0$	$\delta x = 0.01$
$y = -2.43$	$y = -0.50$	$\Delta y = 0.14$	$\Delta y = -0.02$
$\theta_r = 90$	$\theta_r = 180$	$\Delta\theta_r = 4$	$\Delta\theta_r = 2$
$\varphi = 0$	$\varphi = 0$	$\Delta\varphi = -5$	$\|\Delta\varphi\| < 1$

q_{init}	q_{goal}	Δq_1	Δq_2
$x = 9.42$	$x = 2.04$	$\delta x = 0$	
$y = -2.90$	$y = -2.5$	$\Delta y = -0.02$	
$\theta_r = 91$	$\theta_r = 90$	$\|\Delta\theta_r\| < 1$	
$\varphi = -10$	$\varphi = 0$	$\|\Delta\varphi\| < 1$	

q_{init}	q_{goal}	Δq_1	Δq_2
$x = 4.27$	$x = 10.3$	$\delta x = 0.13$	$\delta x = -0.02$
$y = -0.51$	$y = -7.32$	$\Delta y = 0.01$	$\Delta y = -0.01$
$\theta_r = 179$	$\theta_r = 90$	$\Delta\theta_r = 5$	$\|\Delta\theta_r\| < 1$
$\varphi = 0$	$\varphi = 0$	$\Delta\varphi = -5$	$\|\Delta\varphi\| < 1$

Table 1: System A: initial and final configurations, gaps between the first and second reached configurations and the goal

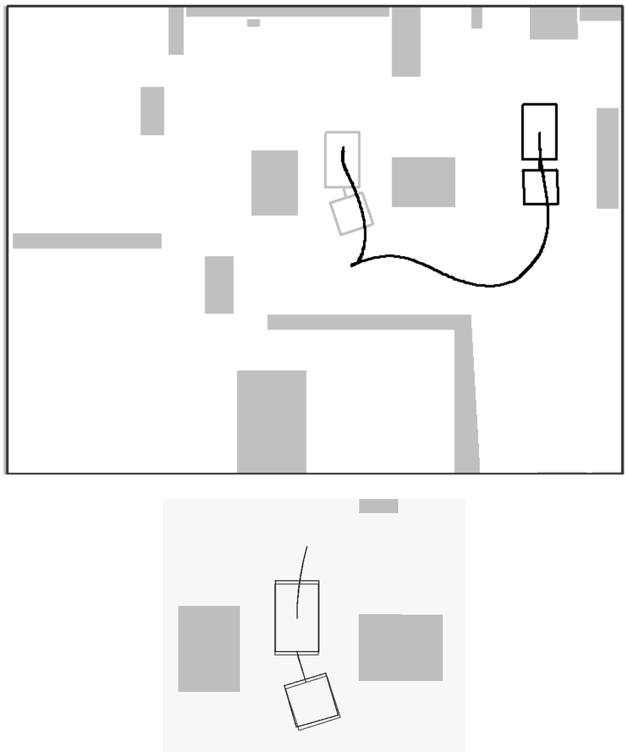

Figure 6: System B: the initial and goal configurations, the first path to be tracked and the final maneuver

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q_{init}	q_{goal}	Δq_1	Δq_2
$x = 9.99$ $y = -2.00$ $\theta_r = 90$ $\varphi = 1$	$x = 5.34$ $y = -2.66$ $\theta_r = 90$ $\varphi = 18$	$\delta x = 0$ $\Delta y = -0.06$ $\Delta\theta_r = 0.54$ $\Delta\varphi = 1$	$\delta x = 0$ $\Delta y = -0.01$ $\Delta\theta_r = -0.75$ $\|\Delta\varphi\| < 1$
q_{init}	q_{goal}	Δq_1	Δq_2
$x = 2.2$ $y = -2.24$ $\theta_r = 90$ $\varphi = 0$	$x = 10.10$ $y = -6.9$ $\theta_r = 90$ $\varphi = 0$	$\delta x = -0.05$ $\Delta y = 0.02$ $\Delta\theta_r = -1$ $\Delta\varphi = 2$	$\delta x = 0$ $\Delta y = -0.01$ $\Delta\theta_r = -2$ $\Delta\varphi = 1$
q_{init}	q_{goal}	Δq_1	Δq_2
$x = 5.31$ $y = -2.61$ $\theta_r = 90$ $\varphi = 19$	$x = 10.08$ $y = -7.7$ $\theta_r = -90$ $\varphi = 28$	$\delta x = 0.02$ $\Delta y = 0.03$ $\|\Delta\theta_r\| < 1$ $\|\Delta\varphi\| < 1$	

Table 2: System B: initial and final configurations, gaps between the first and second reached configurations and the goal

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