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A Biped Walking Pattern Generator based on "Half-Steps" for Dimensionality Reduction

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Abstract— **We present a new biped walking pattern generator based on "half-steps". Its key features are a) a 3-dimensional parametrization of the input space, and b) a simple homotopy that efficiently smooths the walking trajectory corresponding to a fixed sequence of steps. We show how these features can be ideally combined in the framework of sampling-based footstep planning. We apply our approach to the robot HRP-2 and are able to quickly produce smooth and dynamically stable trajectories that are solutions to a difficult problem of footstep planning.**

I. INTRODUCTION, RELATED WORK AND OVERVIEW

A. Introduction and related work

In this article, we consider the following framework for fast footstep planning for humanoid robots:

1) Use a low dimensional space of steps to plan a first "raw" feasible walking sequence towards a goal location, using a sampling-based planning algorithm.
2) Smooth the gait obtained, and play the result on the robot.

A* or similar search algorithms have been extensively used in the context of footstep planning for humanoid robots (see for example [11], [2]). One of the problems often encountered is the gap between the trajectories used for the search of footsteps, and the trajectories used for the final whole-body motion generation. For instance, the sequence of footsteps planned might lead to self-collisions when used as an input for the whole-body motion generation. In this article, we seek coherence at this level and introduce a new walking pattern generator aimed at being used in both steps of the above framework. The fact that it is based on half-steps (we define what is a half-step in section II-A) gives it a low-dimensonal input space: compared to pattern generators based on full steps, the dimensionality is divided by 2. Our approach shares several similarities with the article [11], where Kuffner et al. present an algorithm for planning safe navigation strategies for biped robots moving in obstacle-cluttered environments. In [11], in order to reduce the number of transition trajectories between two consecutive footstep placements, the authors introduce two intermediate postures Q_{right} and Q_{left} that serve as via points for all footstep transitions. Q_{right} and Q_{left} correspond to default postures in which the robot is balanced entirely on either the right or left foot respectively, with the other foot suspended high above the walking surface. We use the same intermediate postures as extremities for our half-steps. The main difference between our approach and the one of [11] is that at the planning phase, instead of looking for statically stable motions like in [11], we directly use the low-dimensional space of dynamic motions (the half-steps) offered by our walking pattern generator. Thanks to its low dimensionality, we can replace tests on this space by approximation functions that have been learned offline (we use the same approximation algorithm as in [12], but avoid the problems of dimensionality it encountered). Those approximations can be evaluated in a few microseconds which makes them suitable for sampling-based planning techniques. Besides, since the domains of feasibility approximated are continuous, we have more flexibility than the approaches based on the use of the A* search algorithm with a finite set of actions (see [3], [2]), and even more than the approaches where footsteps of a finite action set can be locally adjusted ([5]). The walking sequences we obtain after the planning phase are sequences of half-steps that all start and finish with zero speed. Even if the half-steps are dynamic, they are still statically stable at their extremities, and as a result the gaits obtained contain strong speed variations (they frequently reach zero speed) and unnecessary sway motions of the CoM. For this reason even if those raw sequences are better than statically stable motions, they are still very poor compared to trajectories generated by state-of-the-art walking pattern generators. We show that we can cope with this issue by using a very simple homotopy which continuously deforms a raw sequence into a smoother and more dynamic sequence where the zero speed configurations have totally disappeared.

The following overview states the different components of our contribution.

B. Overview

The overall algorithm that we use for fast footstep planning is described by the following steps:

1) Offline approximation using the new pattern generator proposed in this paper in order to learn domains of feasible half-steps. *Contribution 1*: thanks to the low dimensionality of the space of half-steps, accurate approximations can be obtained in a reasonable time (1 hour).
2) Using the RRT* algorithm recently introduced in [10], we quickly obtain a feasible (no self-collision, 2D obstacle avoidance) raw sequence of half-steps (it took

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about 14 seconds for the generation of a sequence of 28 half-steps). The RRT* algorithm works by growing a random tree in the configuration space, and the functions approximated help us to validate extremely quickly the edges (i.e. the half-steps) between two configurations. *Contribution 2*: the low dimensionality and the coherence of our approach enabled us to approximate the correct feasibility tests without any additional restriction, and thus we obtain results that are more sound than in [12], and more expressive than if the domain of feasibility was defined by an expert user, as in [4] for example.

3) *Contribution 3*: once a raw sequence is obtained, a simple homotopy is used to smooth it into a fluid gait. It is still compulsory to check through simulation that the smoothed sequences stay feasible, but since we use a dichotomy to set the parameters that govern our homotopy, the number of simulations needed always stays reasonable (the smoothing of 28 half-steps is done in 12 seconds).

In section II, we present the principles of our walking pattern generator based on half-steps, and introduce the operators that we use to perform the homotopy on raw sequences of half-steps. In section III, we briefly show how we practically approximated continuous domains of feasible half-steps (feasible for the robot HRP-2). Finally, in section IV, we show how we applied the recent algorithm RRT* [10] which, thanks to our approximation functions, could consider several hundreds of thousands of half-steps in only a couple of seconds, and thus rapidly obtained a feasible raw solution. Section IV also shows that the overall algorithm performed well when applied to the robot HRP-2 on a footstep planning problem that classical approaches don't solve well.

II. A WALKING PATTERN GENERATOR BASED ON HALF-STEPS

We use a classical simplified model of the robot dynamics: the Linear Inverted Pendulum Mode (see [7]). In this model the mass of the robot is assumed to be concentrated in its CoM which is supposed to be rigidly linked to and above the robot waist at all time. Besides, the robot is supposed to have only point contacts with the walking surface. Thus it behaves like an inverted pendulum, and an analysis of the subsequent equations leads to a further approximation which enables the decoupling of the dynamic differential equations for the x-axis and y-axis. They can be written as follows:

$$p_x = x - \frac{z_c}{g}\ddot{x} \quad (1)$$

$$p_y = y - \frac{z_c}{g}\ddot{y} \quad (2)$$

where (x, y) are the (x-axis,y-axis) coordinates of the CoM of the robot, and z_c the height of the robot center of mass which is supposed constant. (p_x, p_y) are the (x-axis,y-axis) coordinates of the virtual Zero Moment Point (ZMP), which is a very important point in humanoid robotics: a classical stability criterion for biped walking is that the ZMP should stay at all time inside the polygon of support (defined as the convex hull of the set of points of the robot in contact with the walking surface; see [14]).

In the article [6], Harada et al. show how analytical trajectories for both the CoM and the ZMP can be derived from these equations. The ZMP trajectory is a polynomial of the time variable t, and the CoM trajectory $\binom{x(t)}{y(t)}$ has the general following form:

$$cosh(\sqrt{\frac{g}{z_c}} \cdot t)\begin{pmatrix}V_x\\V_y\end{pmatrix} + sinh(\sqrt{\frac{g}{z_c}} \cdot t)\begin{pmatrix}W_x\\W_y\end{pmatrix} + \begin{pmatrix}r_x(t)\\r_y(t)\end{pmatrix} \quad (3)$$

where $r_x(t)$ and $r_y(t)$ are polynomials entirely determined by $p_x(t)$ and $p_y(t)$ (which are also polynomials).

From this equation we see that for a given ZMP profile, there are just enough free parameters (V_x, V_y, W_x, W_y) to set the initial horizontal position and speed of the CoM:

$$\begin{pmatrix}x(0)\\y(0)\end{pmatrix} = \begin{pmatrix}V_x + r_x(0)\\V_y + r_y(0)\end{pmatrix} \quad (4)$$

$$\begin{pmatrix}\dot{x}(0)\\\dot{y}(0)\end{pmatrix} = \begin{pmatrix}\sqrt{\frac{g}{z_c}} \cdot W_x + \dot{r_x}(0)\\\sqrt{\frac{g}{z_c}} \cdot W_y + \dot{r_y}(0)\end{pmatrix} \quad (5)$$

Using these equations, in the next section we show how to produce the lower body C-space (configuration space) trajectory corresponding to an isolated half-step. Thanks to a few assumptions on the inverse geometry of the legs, this problem can be reduced to the production of trajectories for the waist and the feet. With a few additional assumptions we can show that this C-space trajectory of the lower body is, for any half-step, entirely defined by the 7 following functions of the time:

- the CoM horizontal position: $x(t)$, $y(t)$ (equal to the waist horizontal position)
- the waist horizontal orientation (the yaw): $\theta(t)$
- the swing foot position: $SF_x(t)$, $SF_y(t)$, $SF_z(t)$
- the swing foot horizontal orientation $SF_\theta(t)$

A. Producing isolated half-steps

There are two types of half-steps: upward and downward. Any full step can be divided into two parts: the first one is the upward half-step where the swing foot ends up at its highest position, and the second is the downward half-step where the swing foot starts at its highest position to finish on the ground, reaching the next footprint. In this section we only consider upward half-steps, but the method for the generation of downward half-steps trajectories is similar.

Now, let us consider an upward half-step. In order to reduce the dimensionality of the parameter space, we make several assumptions. First, we fix and denote by T the duration of any half-step. Then, we assume that the initial and final speed of the ZMP and swing foot are 0, but we *don't* assume that the CoM initial and final speed are zero. Furthermore, the initial vertical projection on the ground of the CoM is equal to the ZMP initial position, i.e. the barycenter of the feet centers. Taking the center of the support foot as the origin of the Euclidean space, it gives

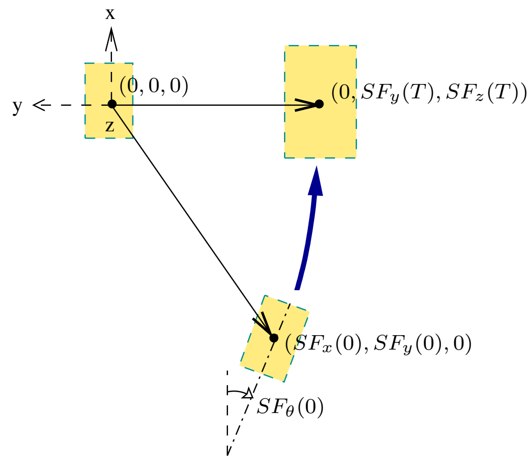


Fig. 1. Here we show an upward half-step from above. It is fully determined by the 5 parameters $SF_x(0)$, $SF_y(0)$, $SF_\theta(0)$, $SF_y(T)$ and $SF_z(T)$. A downward half-step is also fully determined by 5 parameters.

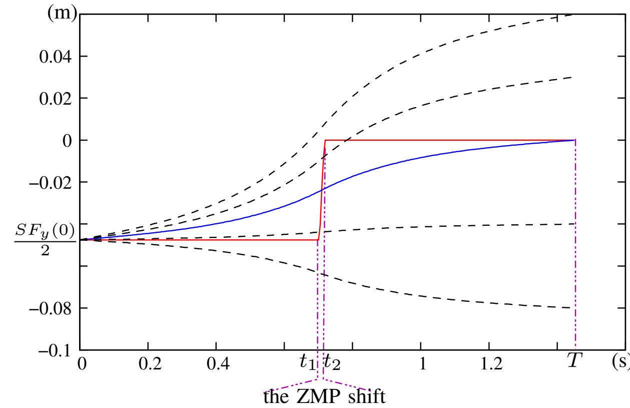


Fig. 2. We consider the upward half-step of Fig. 1, and show in red the corresponding $p_y(t)$. To this trajectory correspond an infinity of C^2 solutions for $y(t)$ which all verify $y(0) = p_y(0) = \frac{SF_y(0)}{2}$, each of them being fully defined by $\dot{y}(0)$. We show several such C^2 solutions; the curve in blue is the solution retained: it is the unique one verifying $y(T) = 0$.

us: $x(0) = p_x(0) = \frac{SF_x(0)}{2}$, and $y(0) = p_y(0) = \frac{SF_y(0)}{2}$. We also assume that the final horizontal positions of the CoM and ZMP coincide at the center of the support foot ($x(T) = p_x(T) = y(T) = p_y(T) = 0$), and that the final orientations of the swing foot and waist are equal to the orientation of the support foot ($\theta(T) = SF_\theta(T) = 0$). Besides, the line passing through the centers of the feet final positions is orthogonal to the support foot orientation: $SF_x(T) = 0$. As a consequence of those restrictions, the final and initial configurations are entirely determined by 5 parameters (as shown on Fig. 1):

$$SF_x(0),\ SF_y(0),\ SF_\theta(0),\ SF_y(T) \text{ and } SF_z(T).$$

Besides, concerning the derivatives at the boundaries, the only free parameters are $\dot{x}(0)$, $\dot{x}(T)$, $\dot{y}(0)$, and $\dot{y}(T)$. This adds up to a total of 9 free parameters.

Now, we show how the ZMP trajectory is defined. An upward half-step is divided into 3 phases: during the first one, of duration t_1, the ZMP stays at the barycenter of the feet (and the feet keep their positions as well), so we have $p_x(t) = \frac{SF_x(0)}{2}$, $p_y(t) = \frac{SF_y(0)}{2}$, and $\dot{p_x}(t) = \dot{p_y}(t) = 0$. Then there is the "shift" phase, during which the ZMP quickly shifts from its initial position to its final position, reached at time t_2. Then, from t_2 to T, the ZMP stays at its final position, so we have $p_x(t) = p_y(t) = \dot{p_x}(t) = \dot{p_y}(t) = 0$. During the "shift" phase we set p_x and p_y as third-degree polynomials determined by the respective boundary conditions $p_x(t_1) = \frac{SF_x(0)}{2}$, $p_x(t_2) = \dot{p_x}(t_1) = \dot{p_x}(t_2) = 0$, and $p_y(t_1) = \frac{SF_y(0)}{2}$, $p_y(t_2) = \dot{p_y}(t_1) = \dot{p_y}(t_2) = 0$. For the downward half-step, even if the phase of double support and single support are inverted, we keep the same durations: the ZMP shift occurs between time t_1 and t_2. In practice, we set $t_1 = T - t_2$.

With the ZMP profile and the $SF_x(0)$ and $SF_y(0)$ parameters set, if we fix $\dot{x}(0)$, and $\dot{y}(0)$, we have a unique C^2 solution for $x(t)$ and $y(t)$ over $[0, T]$. The boundary conditions fix the parameters V_x, V_y, W_x, W_y (eq. (4) and eq. (5)), and eq. (3) gives us the analytical expression of the solution. It is remarkable that $\dot{x}(0)$ and $\dot{y}(0)$ have a linear influence over respectively $x(T)$ and $y(T)$, and that in the conditions of our study, x and y are necessarily monotone. We use a simple method based on these properties to ensure numerical stability and compute the unique values $\dot{x}(0)$ and $\dot{y}(0)$ that lead to $x(T) = 0$ and $y(T) = 0$ (see Fig. 2).

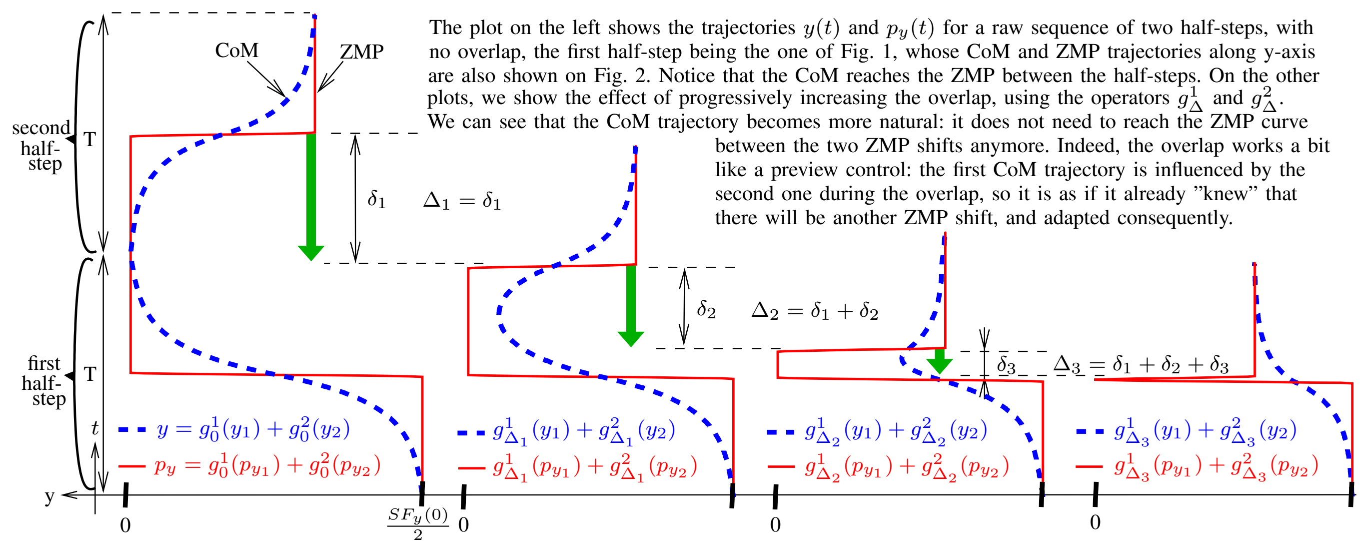
If the durations t_1 and $T - t_2$ are long enough, the practical values obtained for $\dot{x}(0)$ and $\dot{y}(0)$ can be neglected, and thus the CoM trajectories obtained are supposed to be C^2 continuous over $(-\infty, \infty)$. When we tested our approach on the robot HRP-2, which executes trajectories with an additional closed-loop control system aiming at preserving the balance (see [8]), the very small discontinuities of the derivatives were cancelled by this controller. Actually, they might even be completely erased by the time discretization since they were not noticeable in the torque profiles.

For the trajectories other than $x(t)$ and $y(t)$ ($\theta(t)$, $SF_x(t)$, $SF_y(t)$, $SF_z(t)$, $SF_\theta(t)$)), we simply use third-order polynomials that ensure C^2 continuity and satisfying profiles, with a few specific constraints (e.g. in our implementation the swing foot always leave and reach the ground vertically). So, we can completely define a half-step with 5 parameters (whether it is an upward half-step or a downward half-step). In our application, we decided to fix the maximum height of the swing foot ($SF_z(T)$), and the horizontal distance between the feet when the maximum height is reached (which fixes $SF_y(T)$). This puts us in the conditions of [11] where two "via point configurations" Q_{right} and Q_{left} are fixed. With these constraints only 3 parameters are needed to completely define a half-step. Once these parameters are set, we are capable of generating *unique* analytical solutions for the 7 functions of the time that are required to produce the lower body trajectory in the C-space.

B. Smoothing a sequence of half-steps

Using the results of the previous section, we can generate C-space trajectories for isolated half-steps. Since they start

Fig. 3. Progressively increasing the overlap between two half-steps



and finish with zero speed, we can simply join them to produce sequences of half-steps. Alternating upward and downward half-steps will produce a walking motion. During each half-step, the motion is dynamically stable (not statically stable), but at the extremities of each half-step, the configuration is statically stable. This is not a satisfying result because the corresponding walk is really unsteady, unnatural, and contains unnecessary sway motions. In this section, we show how we can start from a simple concatenation of half-steps (that is to say an awkward walk), and then continuously modify it towards a much smoother and quicker sequence that will realize the same steps. We first show how to do so for a sequence of two half-steps, and start with the case of an upward half-step followed by a downward half-step.

1) Upward then downward: We consider an upward half-step followed by a downward half-step. Together the two half-steps make a classical full step: double support phase, then single support phase, and then double support phase again.

We recall that the whole C-space trajectory of the lower body during one half-step is generated by inverse geometry from 7 functions of the time. Since here we are dealing with two consecutive half-steps (with the same support foot), we have to consider 14 functions. Let us first consider for example the position of the waist along the y-axis, respectively for the upward half-step: $y_1(t)$, and the downward half-step: $y_2(t)$. We have $y_1(T) = y_2(0) = 0$. Let us define two operators g_Δ^1 and g_Δ^2 such that:

$$g_\Delta^1(f)(t) = \begin{cases} f(t) \text{ for } t \in [0, T] \\ f(T) \text{ for } t \in [T, 2T - \Delta] \end{cases} \quad (6)$$

$$g_\Delta^2(f)(t) = \begin{cases} 0 \text{ for } t \in [0, T - \Delta] \\ f(t - T + \Delta) - f(0) \text{ for } t \in [T - \Delta, 2T - \Delta] \end{cases} \quad (7)$$

The two functions $g_\Delta^1(y_1)$ and $g_\Delta^2(y_2)$ are C^2 continuous for any $0 \leq \Delta \leq T$, and $g_0^1(y_1) + g_0^2(y_2)$ corresponds to the simple concatenation of y_1 and y_2 without overlap. If the ZMP profiles corresponding to the CoM trajectories y_1 and y_2 are respectively p_{y_1} and p_{y_2}, then starting from the equation (1) it is easy to verify that for any $0 \leq \Delta \leq T$, $y = g_\Delta^1(y_1) + g_\Delta^2(y_2)$ is a solution of the differential equation:

$$g_\Delta^1(p_{y_1}) + g_\Delta^2(p_{y_2}) = y - \frac{z_c}{g}\ddot{y} \quad (8)$$

Therefore the operators g_Δ^1 and g_Δ^2 enable us to obtain new combined CoM and ZMP trajectories that still verify the Linear Inverted Pendulum equations (eq. (1) and eq. (2)). Starting with $\Delta = 0$ and progressively increasing the value of Δ continuously modifies the CoM trajectory (starting from the initial trajectory $g_0^1(y_1) + g_0^2(y_2)$) to make the second ZMP shift (the one of y_2) happen earlier, creating an overlap of duration Δ between the two trajectories y_1 and y_2. Fig. 3 illustrates this effect: when we increase the value of Δ the transition between the two half-steps becomes smoother (in particular it can be noted that the CoM does not need to reach the center of the support foot anymore).

We use the same operators, g_Δ^1 and g_Δ^2, to produce an overlap between the functions of the time corresponding to the waist orientation and swing foot position and orientation. Since the inverse geometry for the legs is a continuous function as long as we stay inside the joint limits, these operators used on the bodies trajectories actually implement a simple homotopy that continuously deforms the initial C-space trajectory into a smoother, more dynamic trajectory.

In the case of an upward half-step followed by a downward half-step, increasing Δ reduces the duration of the single support phase, and therefore it increases the speed of the swing foot. To limit this effect we must bound Δ. Since we also use the operators g_Δ^1 and g_Δ^2 for the swing foot trajectory, a natural upper bound appears: $\Delta < min(T - t_2, t_1)$ (with $\Delta > min(T - t_2, t_1)$ the swing foot height would sometimes be negative).

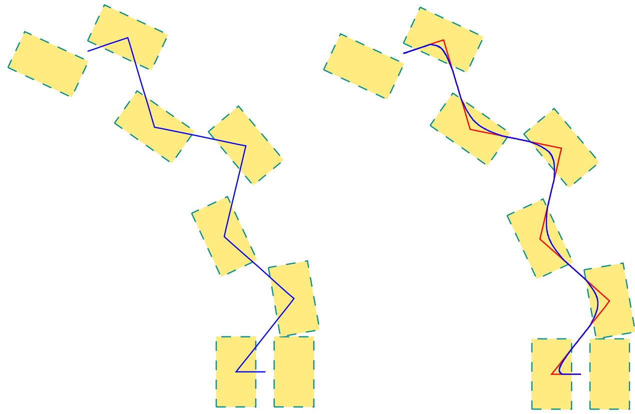


Fig. 4. We illustrate the "smoothing" of a raw sequence of half-steps. On the initial raw sequence (on the left) we can see that the support paths for the ZMP and CoM trajectories are superimposed. Then, after adjusting the overlaps, the ZMP support path stays the same but the CoM support path becomes smoother (on the right, ZMP in red and CoM in blue).

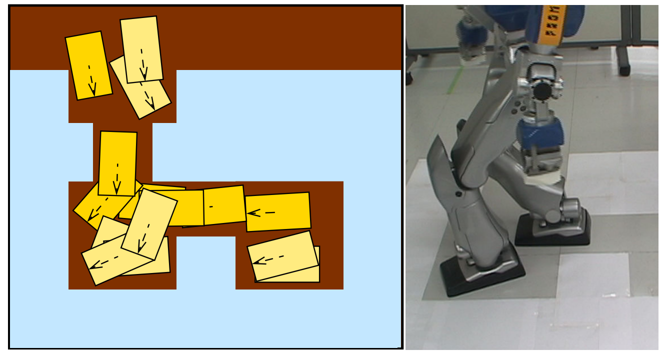


Fig. 5. The goal here was to escape from the small "island" in the bottom-right and join the border. The footprints of the solution found are in yellow (light yellow for the left foot, dark yellow for the right foot), with dashed arrows indicatating the orientation of the feet. On an Intel Xeon 2GHz CPU, the raw sequence of 28 half-steps was found with the RRT* algorithm in about 14s (during which 356944 calls were made to our approximation functions), and then the computation time needed for the sequence smoothing was about 12s. We successfully performed the escape sequence with the robot HRP-2.

After adjusting Δ we obtain a more natural step, where the waist does not have an exaggerated sway motion. On top of that, even if we did fix the configuration of the lower body when the swing foot is at its maximum height, after the "smoothing" this special configuration will not be reached anymore (it is replaced by a flexible mixture between two configurations), so even if only 3 parameters were used to define the half-steps, combining them with overlap considerably widens the range of possible steps.

2) Downward then upward: We can apply the same technique to produce an overlap in the case of a downward half-step followed by an upward half-step. Since the last phase of the downward half-step and the first phase of the upward half-step are double support phases, the constraint on the swing foot motion disappears and the maximum bound on Δ becomes simply T.

3) For longer sequences of half-steps: We simply repeat the procedure to smooth the whole trajectory. If we smooth a collision-free sequence of steps, we must verify that the result remains collision-free. The simple strategy we use is to set one overlap at a time with a dichotomy in order to quickly find an acceptable value as large as possible.

Fig. 4 shows the results obtained with an example of raw sequence. After the smoothing, the CoM trajectory is more fluid. The new gait obtained is much faster (about 3 times faster) and more natural, because the unnecessary sway motions are reduced and the repeated speed changes are avoided.

III. APPROXIMATING FEASIBLITY REGIONS

Since any raw half-step is, as we have seen in section II, completely defined by 3 parameters, it gives us an input space whose dimensionality is small enough to allow efficient approximations of the feasibility region. The feasibility of a half-step can depend on many factors; here, we simply assume that a half-step produced by our pattern generator is feasible if it does not involve self-collisions and does not violate any joint limit. The distances to these two constraints are checked after time discretization (we check about 300 configurations per half-step); for the self-collisions, we use the efficient SPQ algorithm (see [1]).

The algorithm that we used for the approximation is the one introduced in [12]. It is based on stratified recursive sampling: it recursively divides the input space into small boxes, focusing on the frontier region which separates feasible half-steps from unfeasible ones. It uses QP problems to produce local approximations of the frontier. The whole approximation process (i.e. the approximation of the feasibility tests for both upward and downward half-steps) took about 1 hour during which 140,000 samples were collected (this is to be compared with the 11 days that were required for a good approximation in [12]; the dimensionality reduction is the origin of this dramatic saving of time).

IV. FAST FOOTSTEP PLANNING

As we have seen in the previous section, the low dimensionality of our walking pattern generator based on half-steps enabled us to accurately learn feasibility regions. Since the aproximation functions created can be evaluated extremely quickly, they can be used during the planning phase to consider a very large number of raw sequences of half-steps that are all feasible. For example, let us consider a known environment with flat ground and only 2D obstacles (holes in the walking surface). When the robot receives a goal location, it first uses a sampling-based method to look for an acceptable raw sequence of feasible half-steps. Then, once a feasible raw sequence leading to the goal location while avoiding the obstacles has been found, we use the technique presented in section II to transform it into a smoother gait sequence, and execute the result on the robot.

During the planning phase, we use the recent RRT* algorithm (see [10]) to grow a tree of random half-steps exploring the environment and looking for a path leading to the goal location. In our case each node in the tree corresponds to a position (on the walking surface) and an orientation of either the left or the right foot (and the swing foot is at its reference position of maximum height). Thanks to the efficient and modular open-source implementation of the RRT* algorithm by Karaman and Frazzoli, we could easily modify the code to use it with our problem. Yet, footsteps planning has some important specificities, and we believe that a specific RRT-like algorithm would lead to better results, such as for example the one introduced in [15]. We will investigate this in further work.

Fig. 5 shows our experimental setup: the robot must escape from a narrow environment where only a small range of steps are possible. We chose this problem because a lot of known methods would fail to solve it. For example, with a bounding box technique (see [16]), it is impossible to find a solution because the bounding box cannot escape. Also, the footstep planning strategies that consider only a finite set of steps (and use the A* search algorithm) might fail too if the set considered is not expressive enough. In particular when the set of feasible steps (discrete or continuous) is defined by an expert user, it might be too "safe". Furthermore, in such a narrow environment it is very important to have a strong coherence between the footstep planner and the walking pattern generator. For example, if you first obtain a statically stable solution like in [11], and directly use the same sequence of footsteps with a classical walking pattern generator, self-collisions are likely to occur, especially if the footsteps planned are very closed one to another. The method proposed in [9] avoids this problem by integrating geometric constraints into leg motion generation, but it cannot guarantee that a feasible pattern is always generated. An alternative is to deform the statically stable trajectory produced by [11] into a dynamic one while checking that self-collisions don't appear. Methods like [13] can be used, but they might be time-costly whereas our homotopy is very simple: we obtain the fully dynamic trajectory by a sequential optimization (the dichotomy) of just one parameter per half-step.

V. CONCLUSION AND FUTURE WORK

In this article we presented a new walking pattern generator based on half-steps. Since 3 parameters entirely define a half-step, we could easily learn offline (through simulation) the domains of feasibility of the half-steps, and then use the result in an implementation of RRT* which enables us to very quickly obtain a feasible concatenation of half-steps leading to the goal location. We also introduced two concatenation operators which continuously produce an overlap between consecutive half-steps. Starting from a concatenation of half-steps with zero speed at both extremities, by using these operators and checking (through simulation) that no self-collision appears, we obtain a much smoother and more satisfying trajectory. The continuity of the process brings coherence to our approach: there is no gap between the trajectories used during planning and the fully dynamic ones that are finally played on the robot. This coherence and the domains of feasibility learned offline enabled us to quickly find solutions to the problem of planning dynamic walking sequences in an environment where the robot feet can only move around in a very narrow space (Fig. 5), a problem which is difficult to solve with classical methods. Our main priority for further work is to use the same approach for fast footstep planning in environments with 3D obstacles. More precisely, we will try to approximate the volumes swept by the isolated half-steps, and then use the approximations to test extremely quickly whether a given sequence would collide with the environment or not.

VI. ACKNOWLEDGMENTS

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