ContainerMinMaxGD: a Toolbox for (Min, +)-Linear Systems

Euriell Le Corronc, Bertrand Cottenceau, and Laurent Hardouin

Laboratoire d’Ingénierie des Systèmes Automatisés, Université d’Angers,
62, Avenue Notre Dame du Lac, 49000 Angers, France,
{euriell.lecorronc,bertrand.cottenceau,laurent.hardouin}@univ-angers.fr,
WWW home page: http://www.istia.univ-angers.fr/LISA/

1 Introduction

According to the theory of Network Calculus based on the (min, +) algebra (see [2] and [5]), analysis and measure of worst-case performance in communication networks can be made easily and several toolboxes such as COINC [1] or DISCO [6] offer to do it. However, the exact computations – sum, inf-convolution, subadditive closure – of such systems are often memory consuming and time costly (see [1] and [4]). That is why we developed a toolbox called ContainerMinMaxGD which handles some “container” of ultimately pseudo-periodic functions and makes approximated computations. The convexity properties of the bounds of a container provide efficient algorithms (linear and quasi-linear complexity) for sum, inf-convolution and subadditive closure.

The ContainerMinMaxGD toolbox\(^1\) is a set of C++ classes which can be found at the following address: http://www.istia.univ-angers.fr/~euriell.lecorronc/Recherche/softwares.php.

2 ContainerMinMaxGD Toolbox

The elementary object handled by the toolbox is called a container and defined as the following intersection illustrated by the grey zone of Fig. 1:

\[
[f_\uparrow, f_\downarrow]_L \triangleq [f_\uparrow, \tilde{f}] \cap [\tilde{f}]_L,
\]

where \([f_\uparrow, f_\downarrow]\) is an interval of functions and \([\tilde{f}]_L\) is the equivalence class of \(\tilde{f}\) modulo the Legendre-Fenchel transform\(^2\) \(L\).

---

\(^1\) It is important to note that this toolbox is an extension of the library MinMaxGD which handles increasing periodic series of the idempotent semiring \(M_{\text{id}}^{\text{inc}}[\gamma, \delta]\) (see [3]).

\(^2\) A non-injective mapping defined by \(L(f)(s) \triangleq \sup_t \{s - f(t)\}\) from the set of increasing and positive functions \(F\) to the set of convex functions \(F_{\text{acs}}\).
A function $f$ is approximated by a container $[f, \overline{f}]_\mathcal{L}$ if $f \preceq f \preceq \overline{f}$ and $[f, \overline{f}]_\mathcal{L} = [\overline{f}]_\mathcal{L}$. This means that $f$ necessarily belongs to the grey zone of the figure, and by denoting $\text{Cvx}$ the convex hull of a function, that $\forall f \in [f, \overline{f}]_\mathcal{L} : \overline{f} = \text{Cvx}(f)$. Handling such containers amounts doing computations modulo $\mathcal{L}$. We thus obtain the equivalence class of the non-approximated result $f$. Therefore, even throughout the computations, the extremal points of $\overline{f}$ truly belong to the exact function $f$, and the asymptotic slope of $\overline{f}$ is the one of $f$.

Such a container belongs to the following set:

$$F \triangleq \{ [f, \overline{f}]_\mathcal{L} | f \in \mathcal{F}_{acv}, \overline{f} \in \mathcal{F}_{acx}, \sigma(f) = \sigma(\overline{f}) \}.$$ 

Its bounds $f$ and $\overline{f}$ are non-decreasing, piecewise affine and ultimately affine functions. They are in addition concave for the lower bound (set $\mathcal{F}_{acv}$), and convex for the upper bound (set $\mathcal{F}_{acx}$). Moreover, their asymptotic slopes $\sigma(f)$ and $\sigma(\overline{f})$ are equals, so are the slopes of their ultimately affine parts.

According to the computations, let us first recall that the elementary operations of the Network Calculus are:

- sum: $(f \oplus g)(t) = \min \{f(t), g(t)\}$,
- inf-convolution: $(f \ast g)(t) = \min_{\tau \geq 0} \{f(\tau) + g(t - \tau)\}$,
- subadditive closure: $f^*(t) = \min_{r \geq 0} f^r(t)$ with $f^0(t) = e$.

On the set $F$ of containers, these operations are now denoted $[\circ, \ast, \star] \in \{[\oplus], [\ast], [\star]\}$ and redefined as inclusion functions such that for $f = [f, \overline{f}]_\mathcal{L} \in F$, $g = [g, \overline{g}]_\mathcal{L} \in F$, $\forall f \in f$, and $\forall g \in g$:

$$[f \circ g] \in F,$$

$$[f \ast g] \in f \ast g.$$

Thanks to the convexity characteristics of the bounds of a container, the computation algorithms of these inclusion functions are of linear complexity depending on the input size for the sum $[\oplus]$, the inf-convolution $[\ast]$ and the upper bound of the subadditive closure $[\star]$, whereas the algorithm for the computation of the lower bound of $[\star]$ is of quasi-linear complexity depending on the input size.

Finally, it is interesting to have an idea of the performance of this toolbox by the following method. First, an exact system $A$ is approximated by a container $A$ ($A \in A$). Then, the subadditive closures of both the exact system $A^*$ and the container $A^{[\star]}$ are computed, and the result obtained with the exact system is approximated by another container: $A^* \in B$. At last, the pessimism of the toolbox is given by comparing $B$ (obtained from the exact system), and $A^{[\star]}$ (obtained from the approximated system). After experiments, we reach a pessimism of about 30%.

References