Scheduling and SAT

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Outline

1. Introduction
2. Scheduling and SAT Encoding
3. Scheduling and SAT Heuristics
4. Scheduling and SAT Hybrids
5. Conclusion

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Scheduling with Boolean Satisfiability

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- Number of hits for the Google query "Scheduling problem" with ...

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Scheduling with Boolean Satisfiability

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  - [Schaefer]'s dichotomy theorem
- Efficient algorithms (CDCL)
- Successful in Circuit design, Model checking, Planning, ...

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- All the approaches discussed here were developed in the last 5 years
- Recent progress in SAT algorithms opens new research opportunities

Scheduling Problems

Terminology

- Tasks (preemptive, non-preemptive)
- Resources (disjunctive, cumulative, reservoir,...)
- Objectives (makespan, tardiness, flow time,...)
- Side constraints (precedence, time windows, time lags,...)

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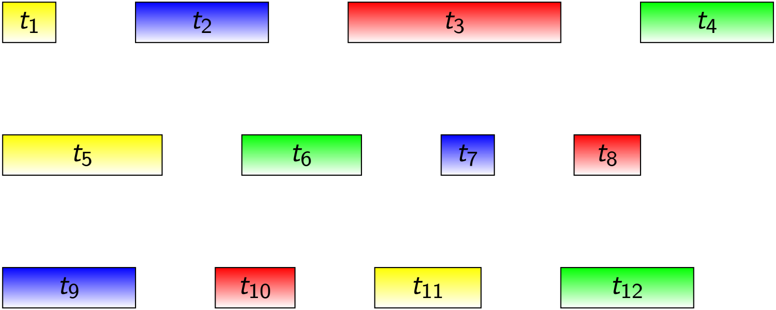
Tip of the iceberg

- SAT-based methods have been applied to a very small subset scheduling problems.
  - Minimization of makespan for non-preemptive tasks and disjunctive resources

Jobshop Scheduling Problem



Problem description

- A set of non-preemptive tasks

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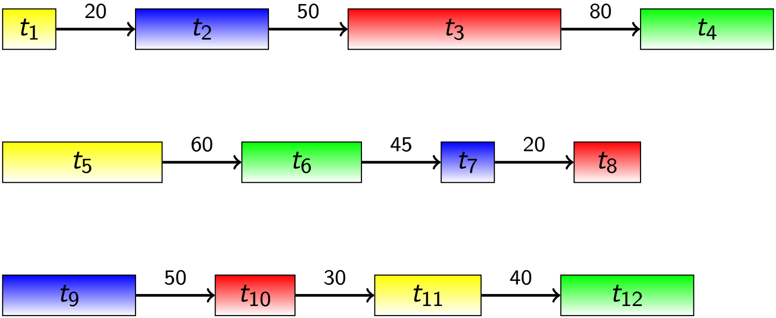

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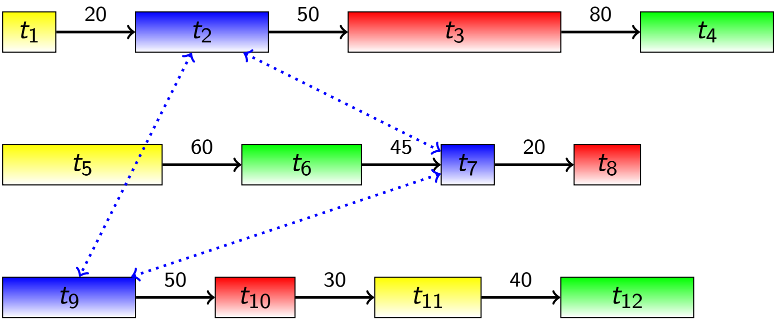

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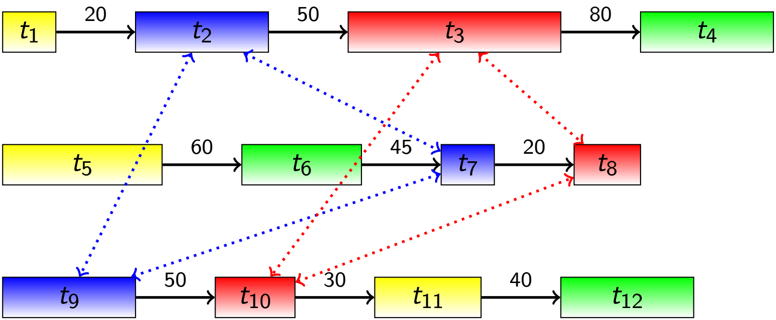

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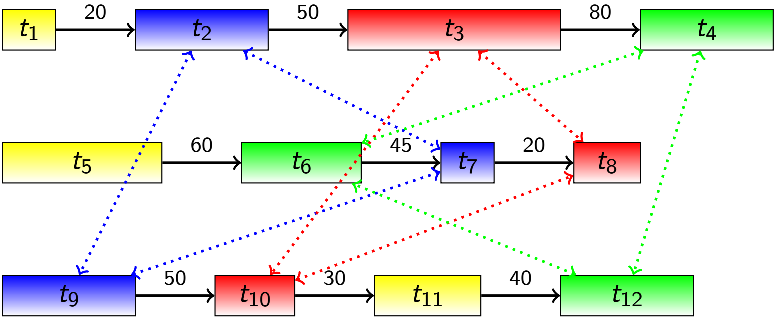

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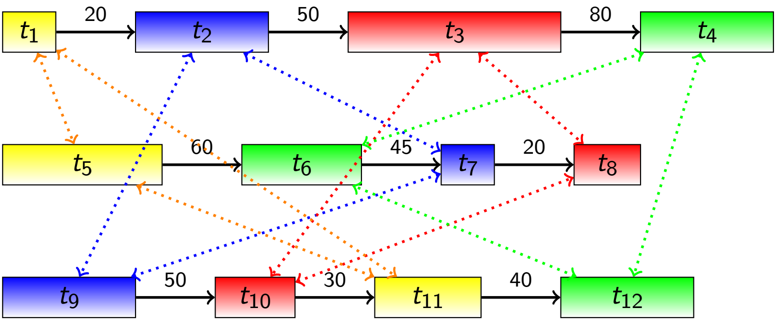

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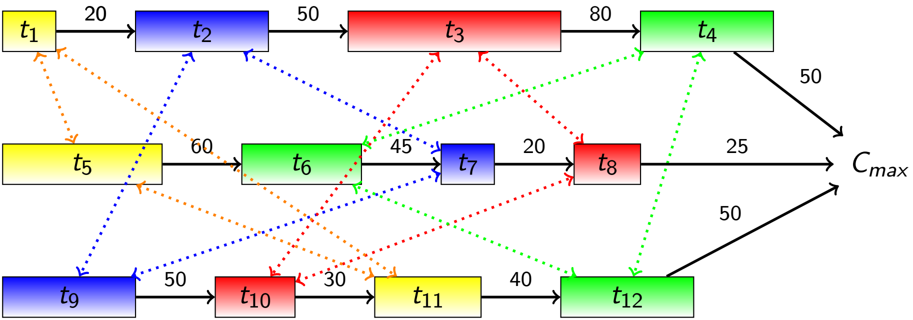

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- Organized in jobs (sequences)
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- Objective: minimize the total duration (C_{max})

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- DPLL: Tree search + Unit propagation
- CDCL: Conflict Driven Clause learning

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  - Extract a learned clause
  - Backjump several levels and unit-propagate the learned clause
- Adaptive branching heuristics (weight conflicting literals)
- And also: restart, simplify the clause base, forget clauses, etc.

CDCL: Example

	f		

$\overline{a} \vee \overline{f} \vee g$

$\overline{a} \vee \overline{b} \vee \overline{h}$

$a \vee c$

$a \vee \overline{i} \vee \overline{l}$

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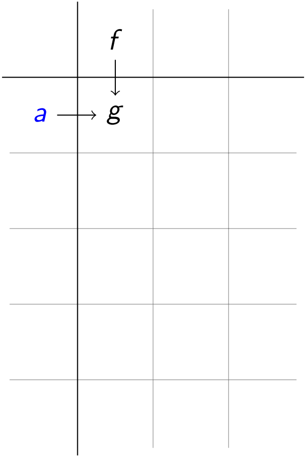

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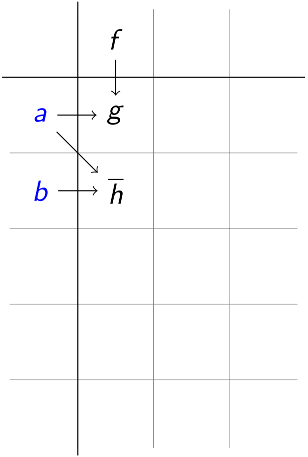

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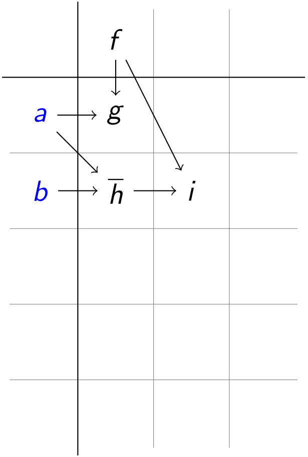

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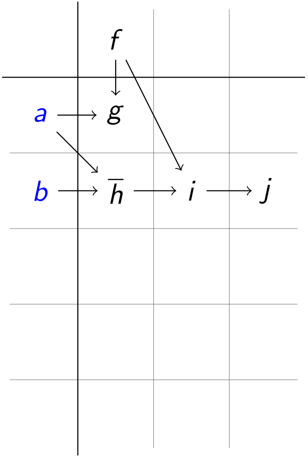

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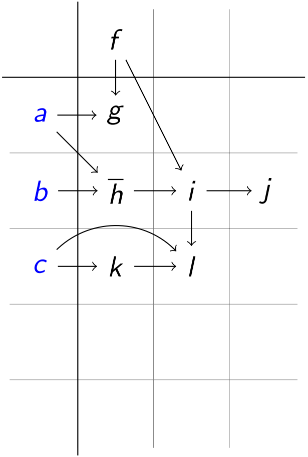

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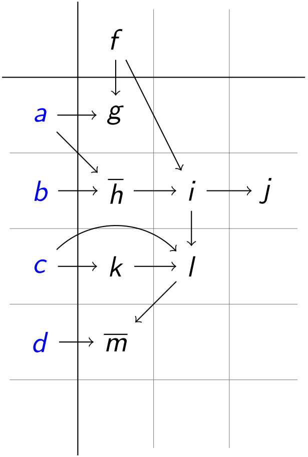

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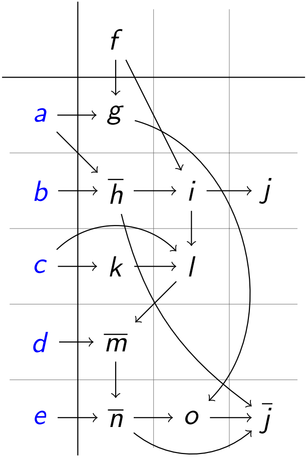

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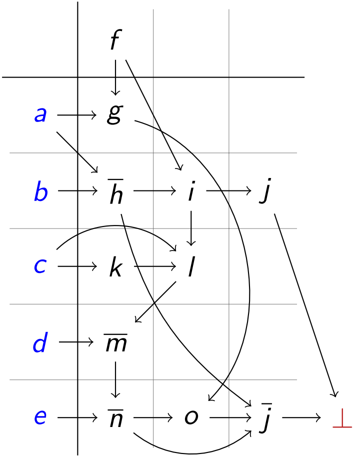

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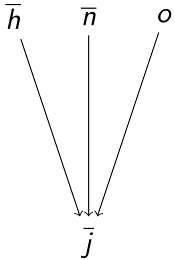
$$
\begin{tikzcd}
\overline{h} \arrow[dr] & \overline{n} \arrow[d] & o \arrow[dl] \\
& \bar{j} &
\end{tikzcd}
$$

$$(h \vee \overline{o} \vee \bar{j} \vee n)$$

CDCL: Example



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$$(h \vee \overline{o} \vee \bar{j} \vee n)$$
$$\equiv$$
$$(\overline{h} \wedge o \wedge \overline{n}) \rightarrow \bar{j}$$

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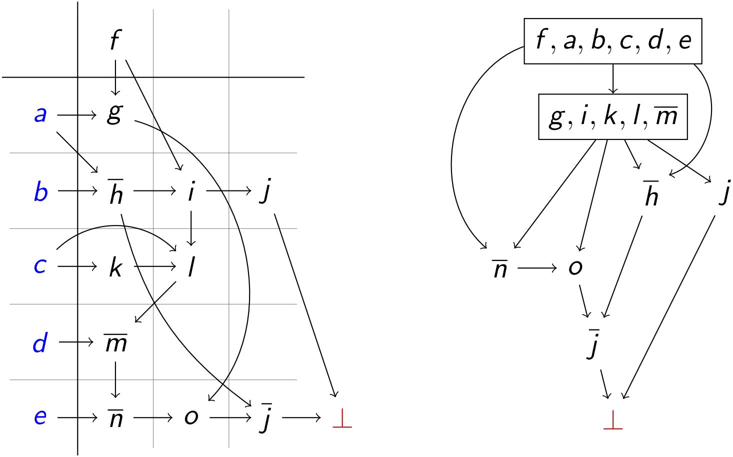

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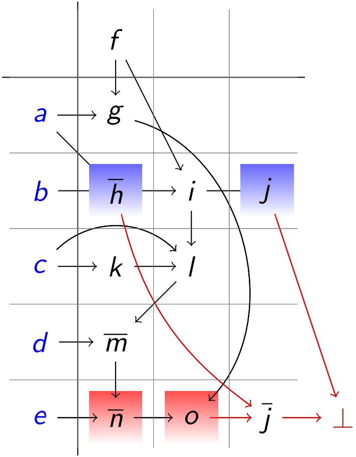

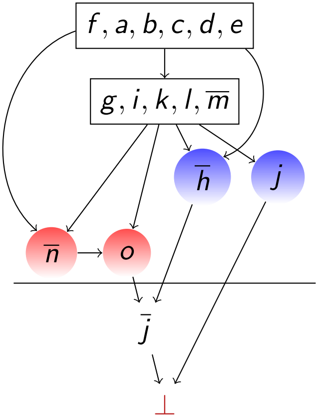

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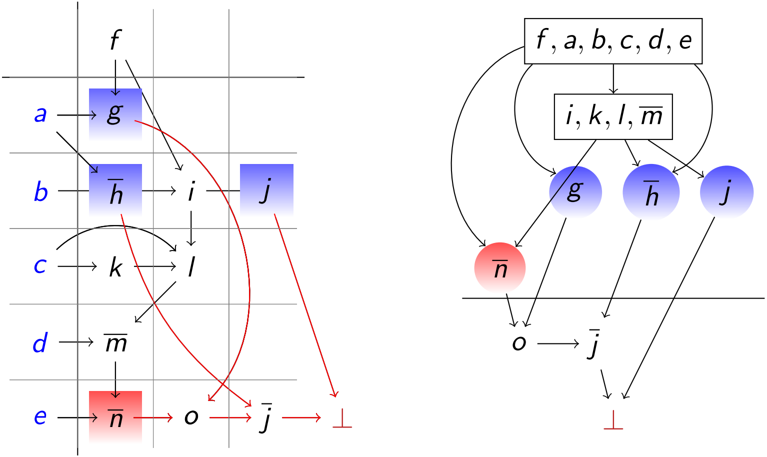

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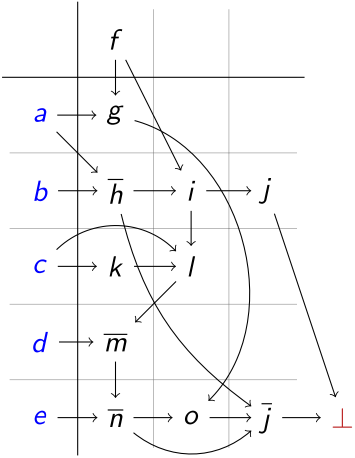

$\overline{a} \vee \overline{f} \vee g$
$\overline{a} \vee \overline{b} \vee \overline{h}$
$a \vee c$
$a \vee \overline{i} \vee \overline{l}$
$a \vee \overline{k} \vee \overline{j}$
$b \vee d$
$b \vee g \vee \overline{n}$
$b \vee \overline{f} \vee n \vee k$
$\overline{c} \vee k$
$\overline{c} \vee \overline{k} \vee \overline{i} \vee l$

$c \vee h \vee n \vee \overline{m}$
$c \vee l$
$d \vee \overline{k} \vee l$
$d \vee \overline{g} \vee l$
$\overline{g} \vee n \vee o$
$h \vee \overline{o} \vee \overline{j} \vee n$
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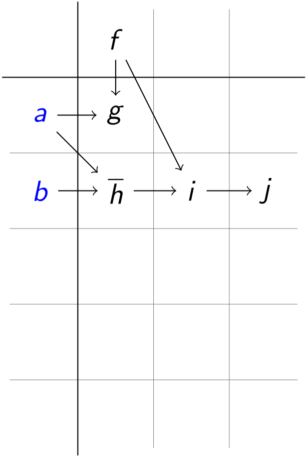

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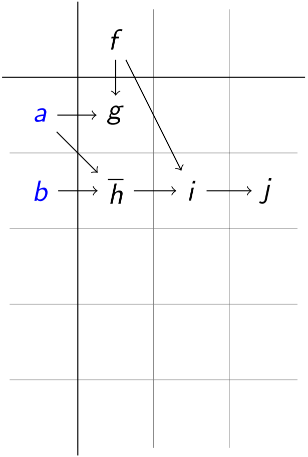

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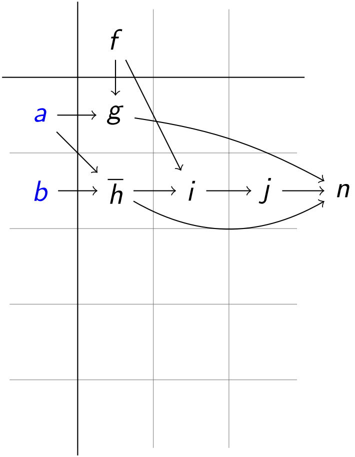

$\overline{a} \lor \overline{f} \lor g$

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$b \lor g \lor \overline{n}$

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- Variable State Independent Decaying Sum (VSIDS)
  - Idea ([Chaff]) weight literals in learned conflicts
  - Decay: favor newer weights
- Weighted degree heuristic
  - On a failure: weight the constraint propagated last
- Activity Based Search
  - On a succes: weight the variables whose domain has changed

Outline

1 Introduction

2 **Scheduling and SAT Encoding**
- Formulation into SAT
- Scheduling by encoding into SAT

3 Scheduling and SAT Heuristics

4 Scheduling and SAT Hybrids

5 Conclusion

CNF encoding

- The way we encode problems into SAT has a huge impact on efficiency
  - Encoding of Planning problems
  - Encoding of CSP (Direct, Log, AC-encoding)
  - Encoding of Pseudo-Boolean (Adder, Sorter)

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- All encodings are based on CSP formulations
  - Some Boolean variables (e.g., relative orders of tasks)
  - Start time variables (Integer variables)
- Integer variables and precedence constraints

Direct Encoding

Domain

- An atom i_v for each pair $(x_i, v \in D(x_i))$
  - $i_v \Leftrightarrow x_i = v$ $\quad \begin{array}{ll} x_i = 1: & 1000 \\ x_i = 2: & 0100 \\ x_i = 3: & 0010 \\ x_i = 4: & 0001 \end{array}$
- Must take at least a value: $i_1 \vee i_2 \vee \ldots \vee i_n$
- Must take at most one value: $\bigwedge_{v \neq w \in D(x_i)} \overline{i_v} \vee \overline{i_w}$

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Complexity

- $O(n^2)$ space: $n(n-1)/2$ binary clauses and one n-ary clause.
- There are different ways to encode the constraints.

Constraints: Tuple Encoding

Example of constraint: $x_i < x_j$

x_j \ x_i	1	2	3	4
1	$\overline{i_1} \vee \overline{j_1}$	$\overline{i_2} \vee \overline{j_1}$	$\overline{i_3} \vee \overline{j_1}$	$\overline{i_4} \vee \overline{j_1}$
2		$\overline{i_2} \vee \overline{j_2}$	$\overline{i_3} \vee \overline{j_2}$	$\overline{i_4} \vee \overline{j_2}$
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Costly (in space) and weak (in propagation)

- $O(n^2)$ binary clauses.
- $\overline{i_4}(x_i \neq 4)$ and $\overline{j_1}(x_j \neq 1)$ are inconsistent, but not unit propagated.

Constraints: AC Encoding [Kasif 90]

Example of constraint: $x_i < x_j$

assignment	atom		support
$x_i = 1$	$\overline{i_1}$	$\vee$	$j_2 \vee j_3 \vee j_4$
$x_i = 2$	$\overline{i_2}$	$\vee$	$j_3 \vee j_4$
$x_i = 3$	$\overline{i_3}$	$\vee$	j_4
$x_i = 4$	$\overline{i_4}$	$\vee$	$\perp$

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Same space complexity, better propagation

- $O(n)$ n-ary clauses
- $\overline{i_4}(x_i \neq 4)$ and $\overline{j_1}(x_j \neq 1)$ are unit clauses.

Order Encoding [Crawford & Backer 94]

Domain

- An atom i_v for each pair $(x_i, v \in D(x_i))$
  - $i_v \Leftrightarrow x_i \leq v$ $\quad \begin{array}{ll} x_i = 1: & 1111 \\ x_i = 2: & 0111 \\ x_i = 3: & 0011 \\ x_i = 4: & 0001 \end{array}$
- Bound propagation:
  - If $x_i \leq v$ then $x_i \leq v + 1$
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Complexity

- $O(n)$ space ($n - 1$ binary clauses)

Constraints: BC Encoding

Example of constraint: $x_i < x_j$

relation	clause
$x_i > 0 \Rightarrow x_j > 1$	$\perp \vee \overline{j_1}$
$x_i > 1 \Rightarrow x_j > 2$	$i_1 \vee \overline{j_2}$
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Better complexity and same propagation on some linear constraints

- $O(n)$ space (n binary clauses)
- $i_3 (x_i \leq 3)$ and $\overline{j_1} (x_j > 1)$ are unit clauses.

Log Encoding [Walsh 00]

Domain

- An atom i_k for each value in $[1, \ldots, \lfloor \log_2 ub \rfloor]$ (assuming $D(x_)) = [0, \ldots, ub]$)
  - $\sum_{k=1}^{ub} 2^k * i_k = v \Leftrightarrow x_i = v$

$x_i = 0$:	00
$x_i = 1$:	01
$x_i = 2$:	10
$x_i = 3$:	11

- For interval domains, no need for extra clauses

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Complexity

- $O(\log_2 n)$ space

Propagation

- Encoding constraints is trickier, and less powerful

Other Encodings

Many more!

- Mix of direct and order encoding [lazy-FD, Numberjack]
- Mix of AC and log encoding [Gavanelli 2007]
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- Mix of direct and order encoding [lazy-FD, Numberjack]
- Mix of AC and log encoding [Gavanelli 2007]
- Mix of order and log encoding [Sugar, Tamura et al. 2006]
  - Log encoding in a base B and order encoding inside a digit
  - Excellent results on scheduling benchmarks! (with CDCL solvers)

Order Encoding, Now and Then

Progress of SAT solvers

- From a few hundreds variables in the 90's to millions now

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- Similar model, hardware of course incomparable, MiniSat
- The hardest instance requires a few 100s conflicts at the most

Closing the Open Shop

Instances

- [Gueret & Prins]: hard for local search, extremely easy for SAT/CP
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results

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  - The two hardest instances were decomposed into 120 subproblems, and required up to 13h to solve
- First approach to close the open shop!

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- [Tamura et al.]'s encoding is better than order encoding
  - However, the huge difference with respect to [Crawford & Backer 94] is due to the solver
- It is now possible to efficiently solve some scheduling problem simply by formulating it as a CNF formula

Outline

A SAT-like Approach [Grimes & Hebrard 09]

- CSP Solver Competition: scheduling benchmarks

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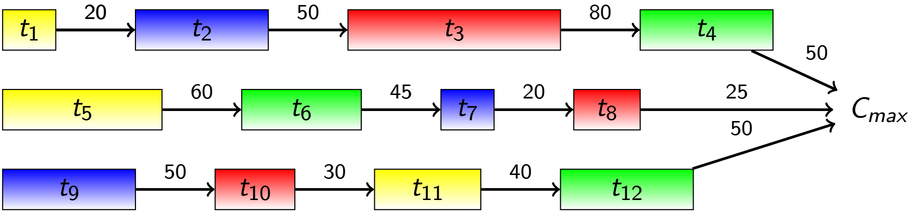
  - Open shop instances closed by [Tamura et al.] can be solved to optimality in a few minutes
- Are adaptive heuristics all that we need to solve disjunctive scheduling problems?

Constraint Model



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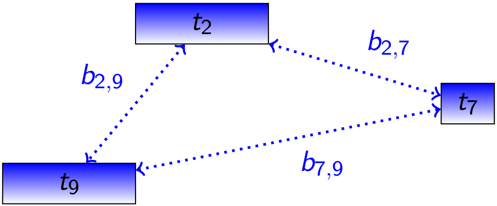

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  - Precedence constraints: $t_i + p_i \leq t_{i+1}$.
- A Boolean Variable standing for the relative order of each pair of conflicting tasks (disjunct):
  - Binary Disjunctive constraints: $b_{ij} = \begin{cases} 0 \Leftrightarrow t_i + p_i \leq t_j \\ 1 \Leftrightarrow t_j + p_j \leq t_i \end{cases}$

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- Almost no problem specific method

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- Restarts + weighted degree “simulates” CDCL behavior?

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Experiment on Jobshop and Variants

- Sequence-dependent setup times
  - Transition between tasks on a machine
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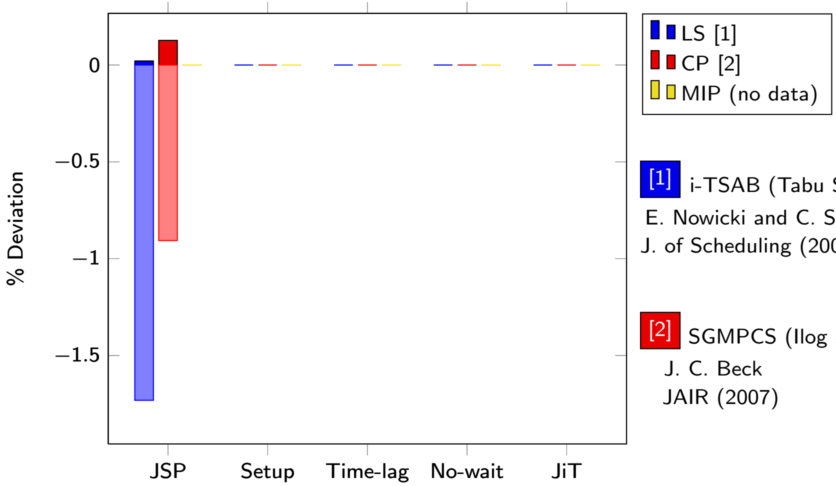
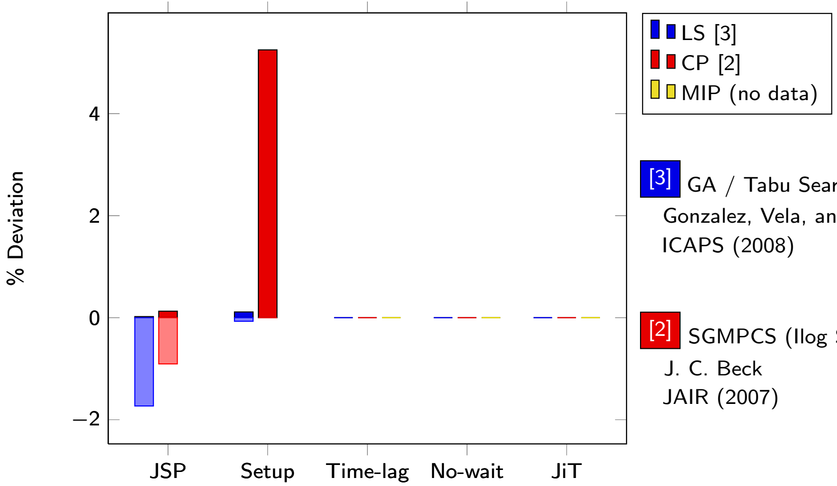
Jobshop - C_{max} - Taillard



[1] i-TSAB (Tabu Search)
E. Nowicki and C. Smutnicki
J. of Scheduling (2005)

[2] SGMPCS (Ilog Scheduler)
J. C. Beck
JAIR (2007)

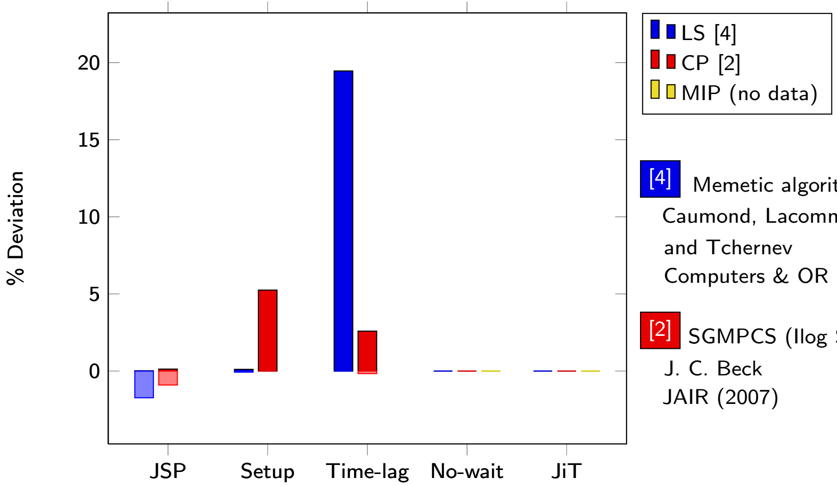
Jobshop with setup times - C_{max} - Brucker & Thiele



[3] GA / Tabu Search
Gonzalez, Vela, and Varela
ICAPS (2008)

[2] SGMPCS (Ilog Scheduler)
J. C. Beck
JAIR (2007)

Jobshop with time lags - C_{max} - Lawrence (modified)



[4] Memetic algorithm
Caumond, Lacomme and Tchernev
Computers & OR (2008)

[2] SGMPCS (Ilog Scheduler)
J. C. Beck
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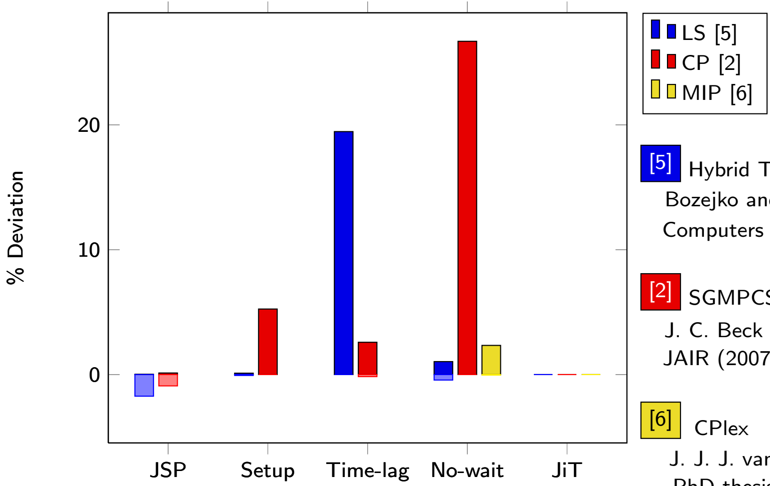
"No-wait" Jobshop - C_{max} - Lawrence

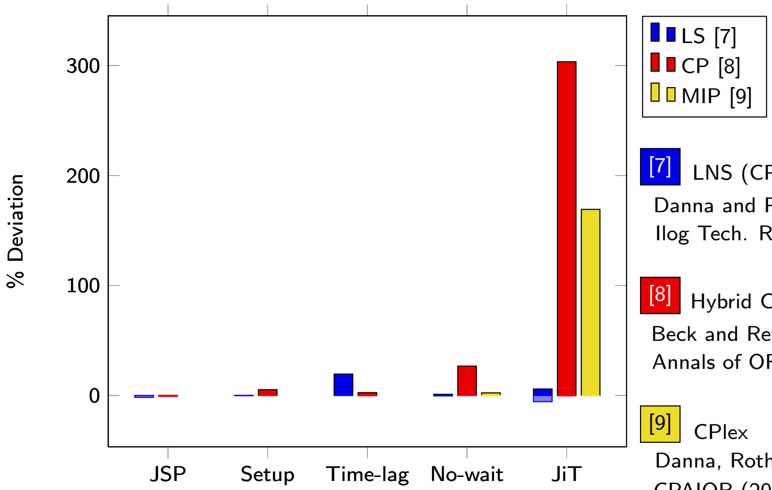


[5] Hybrid Tabu Search
Bozejko and Makuchowski
Computers & IE (2009)

[2] SGMPCS (Ilog Scheduler)
J. C. Beck
JAIR (2007)

[6] CPlex
J. J. J. van den Broek
PhD thesis (2009)

Jobshop - earliness/tardiness - Beck & Refalo; Morton & Pentico



[7] LNS (CP/LP)
Danna and Peron
Ilog Tech. Report (2003)

[8] Hybrid CP/LP
Beck and Refalo
Annals of OR (2003)

[9] CPlex
Danna, Rothberg, Le Pape
CPAIOR (2003)

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    - Even this "pseudo" learning helps!

Outline

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- Lazy Clause Generation
- SAT Modulo Theories

Lazy Clause Generation [Ohrimenko, Stuckey & Codish 07] - [Feydy & Stuckey 09]

Architecture

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  - Search and propagation in CP
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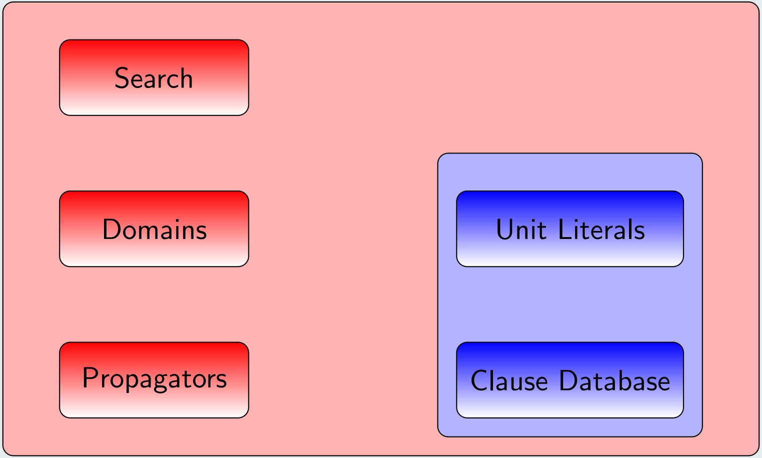
    - Just enough to extract a conflict
  - The SAT formulation is generated lazily (learned during search)

Lazy Clause Generation

Architecture



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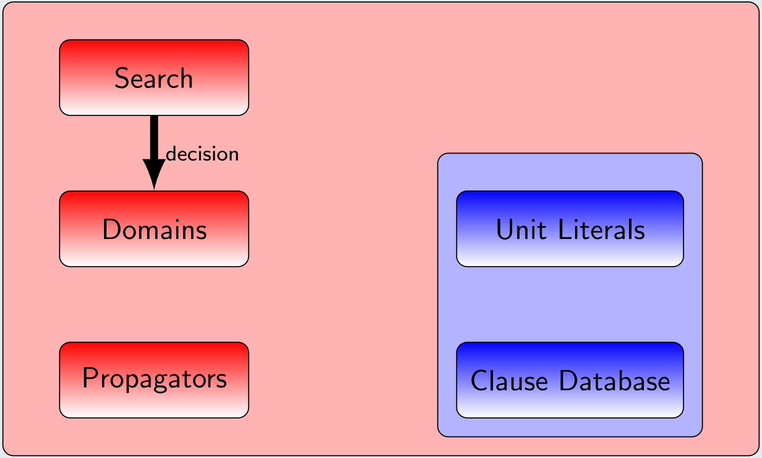

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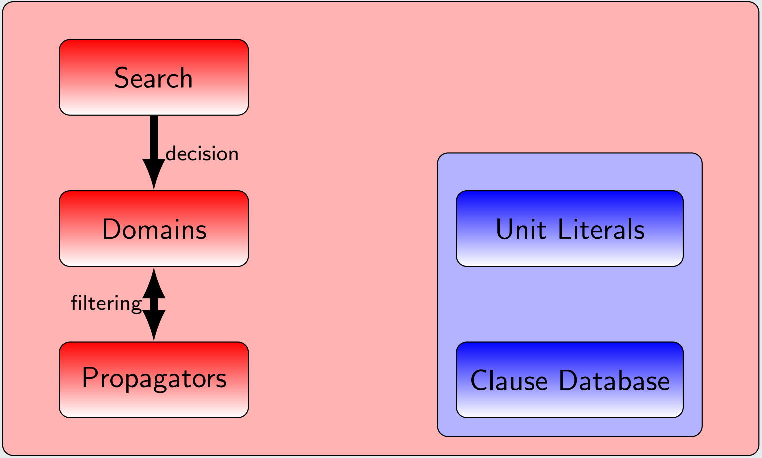

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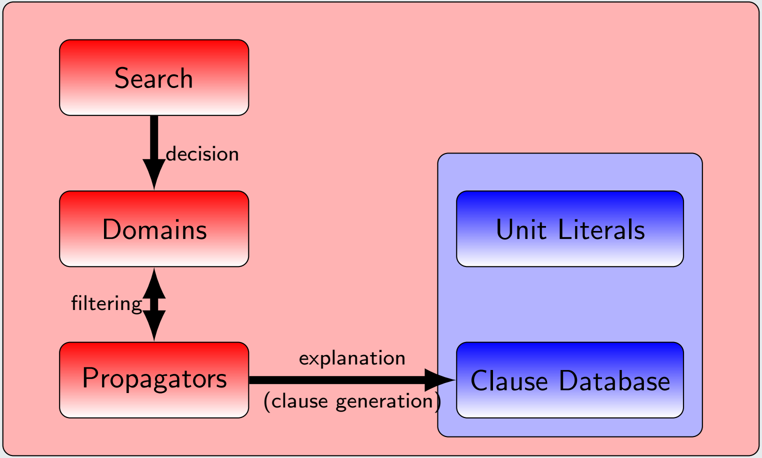

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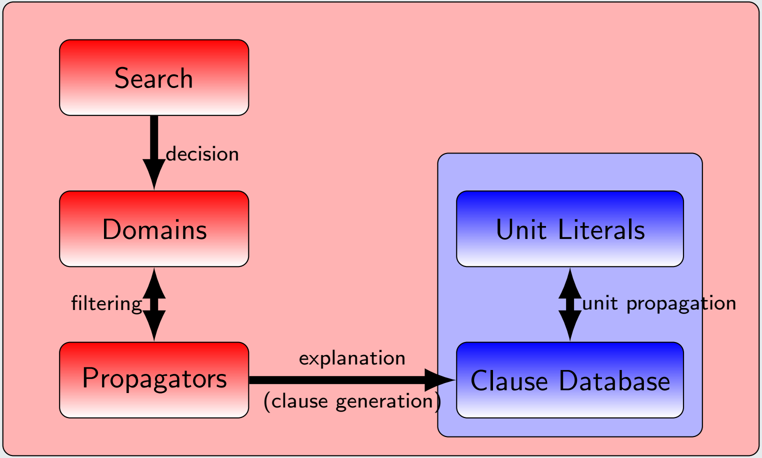

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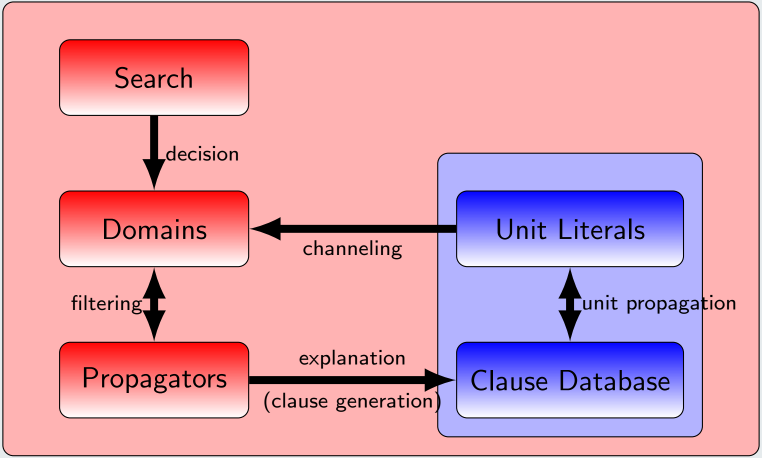

Lazy-FD: Example

$x_i < x_j$

- Initial representation

	CP view	SAT view
$D(x_i)$	$\{1, \ldots, 4\}$	$\overline{i_1} \vee i_2, \overline{i_2} \vee i_3$
$D(x_j)$	$\{2, \ldots, 5\}$	$\overline{j_2} \vee j_3, \overline{j_3} \vee j_4$
constraint	$x_i < x_j$	
constraint	$C(x_i, x_k, \ldots)$	

Lazy-FD: Example

$x_i < x_j$

- Some constraint reduces the domain of x_i to $\{2, \ldots, 5\}$

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$D(x_i)$	$\{2, \ldots, 4\}$	$\overline{i_1} \vee i_2, \overline{i_2} \vee i_3$
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Lazy-FD: Example

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- An explanation clause $T \vee \overline{i_1}$ is produced, and the unit literal $\overline{i_1}$ is propagated

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Lazy-FD: Example

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- The propagator for $x_i < x_j$ is triggered and reduces the domain of x_j

	CP view	SAT view
$D(x_i)$	$\{2, \ldots, 4\}$	$\overline{i_1} \vee i_2, \overline{i_2} \vee i_3$
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constraint	$x_i < x_j$	
constraint	$C(x_i, x_k, \ldots)$	$T \vee \overline{i_1}$

Lazy-FD: Example

$x_i < x_j$

- An explanation clause is also produced

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constraint	$x_i < x_j$	$i_i \vee \overline{j_2}$
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Results on Resource Constrained Project Scheduling Problem (RCPSP) [Schutt, Feydy, Stuckey & Wallace 09]

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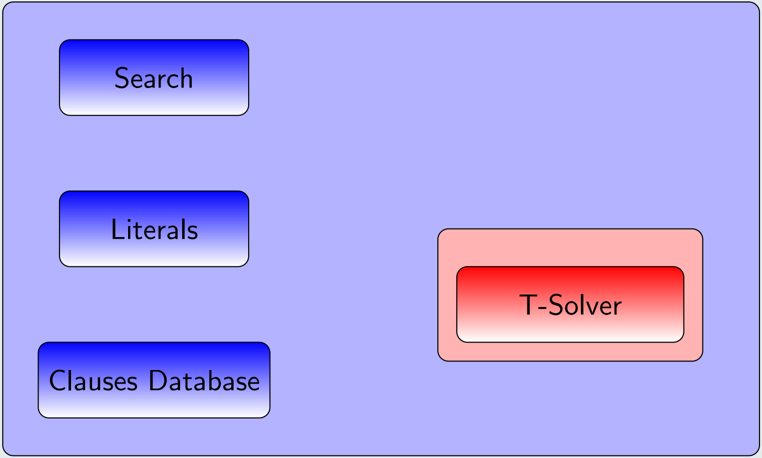
- 54 open instances closed!

SAT Modulo Theories (SMT)

- Framework to hybridize dedicated solvers (Theories, or T-Solvers) with CDCL solvers
  - T-Solver view: a set of propositions each represented by a literal in F
  - CDCL-Solver view: a CNF formula F partially representing the problem
- CDCL-Solver makes decisions and analyzes the conflicts
- T-Solver detects conflicts and/or propagates and generates explanation clauses

SMT Solver

Architecture



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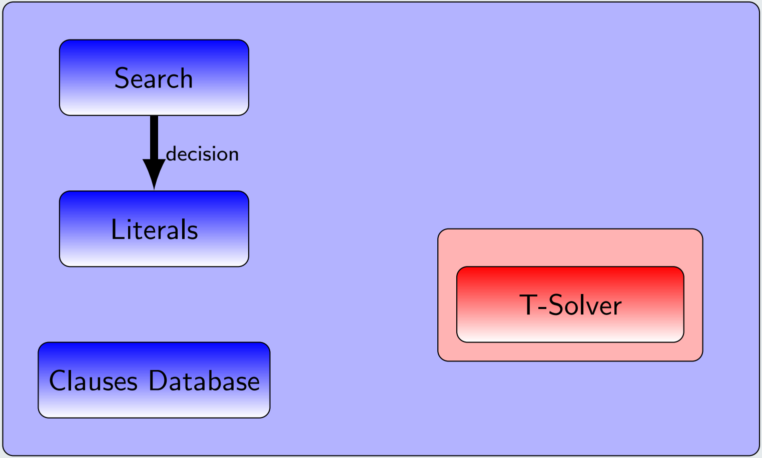

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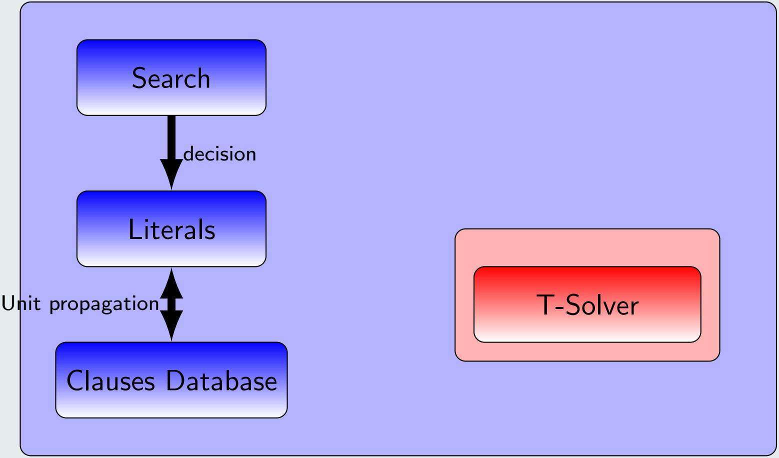

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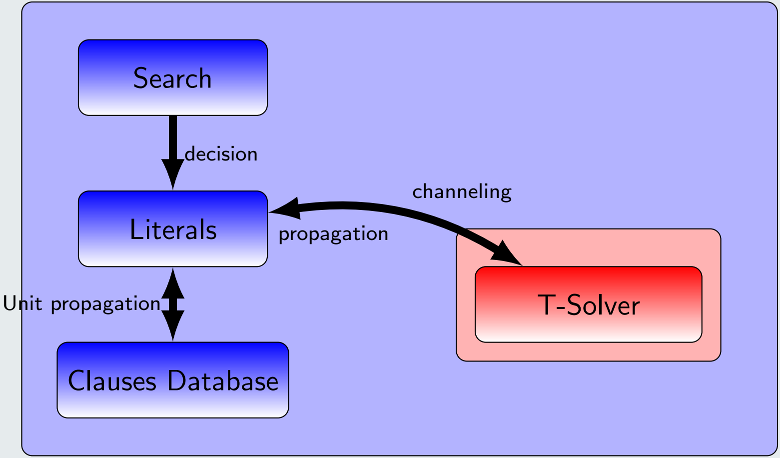

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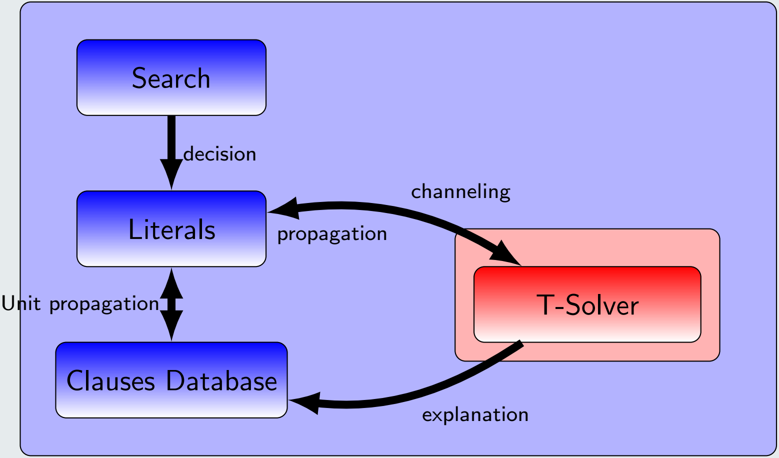

Several Theories

T-Solvers

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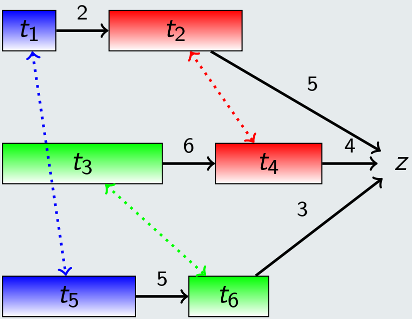
SMT for scheduling

- Satisfiability Modulo Difference Logic.

Example: Jobshop Scheduling

problem



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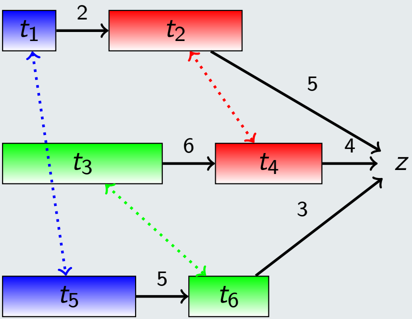

T-Solver view

$$
\begin{array}{ll}
s_1 - s_2 \leq -2 & z - a \leq 15 \\
s_3 - s_4 \leq -4 & \\
s_5 - s_6 \leq -5 & \\
s_2 - z \leq -5 & l_{1\prec 5} \Leftrightarrow s_1 - s_5 \leq -2 \\
s_4 - z \leq -4 & l_{5\prec 1} \Leftrightarrow s_5 - s_1 \leq -5 \\
s_6 - z \leq -3 & l_{2\prec 4} \Leftrightarrow s_2 - s_4 \leq -5 \\
a - s_1 \leq 0 & l_{4\prec 2} \Leftrightarrow s_4 - s_2 \leq -4 \\
a - s_3 \leq 0 & l_{3\prec 6} \Leftrightarrow s_3 - s_6 \leq -6 \\
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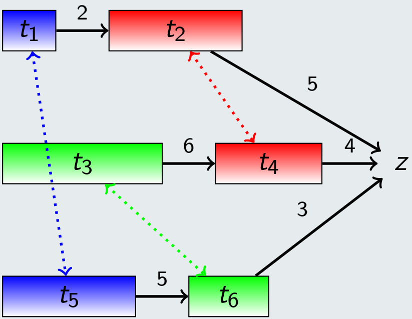

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$a - s_1 \leq 0$
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$z - a \leq 15$

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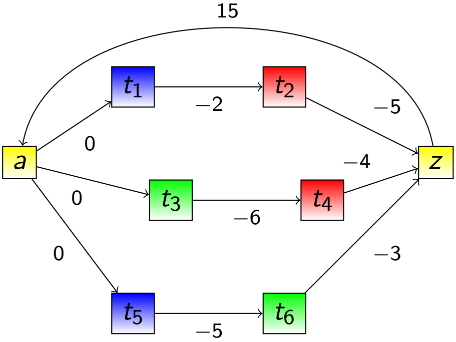
CDCL-Solver view

$l_{1 \prec 5} \vee l_{5 \prec 1}$
$l_{2 \prec 4} \vee l_{4 \prec 2}$
$l_{3 \prec 6} \vee l_{6 \prec 3}$

Example: Jobshop Scheduling

- Reasoning: detection of negative cycles ([Bellman-Ford])



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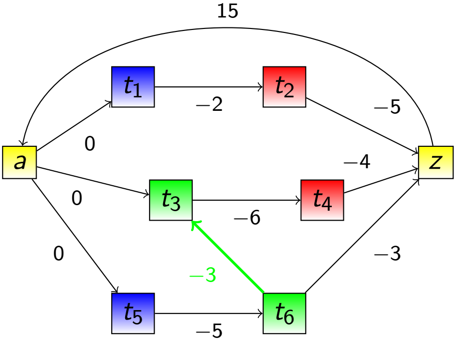

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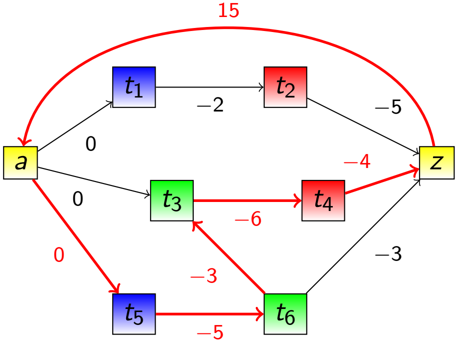

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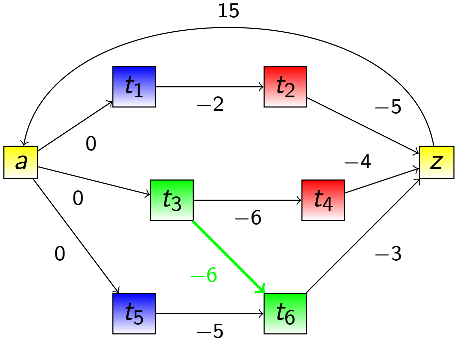

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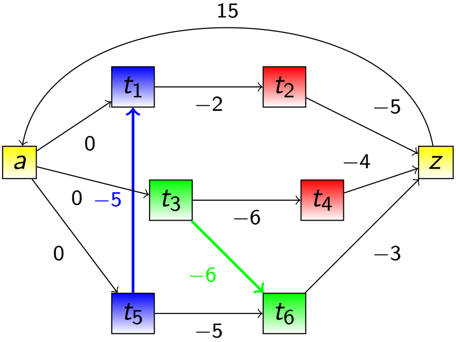

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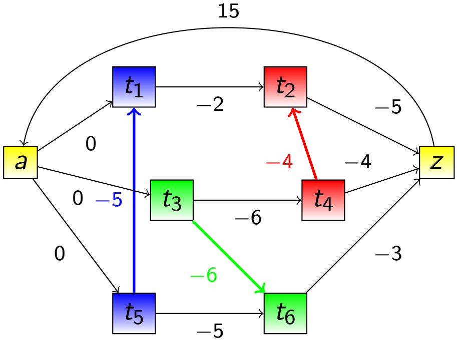

$$l_{1\prec 5} \Leftrightarrow s_1 - s_5 \leq -2$$
$$l_{5\prec 1} \Leftrightarrow s_5 - s_1 \leq -5$$
$$l_{2\prec 4} \Leftrightarrow s_2 - s_4 \leq -5$$
$$l_{4\prec 2} \Leftrightarrow s_4 - s_2 \leq -4$$
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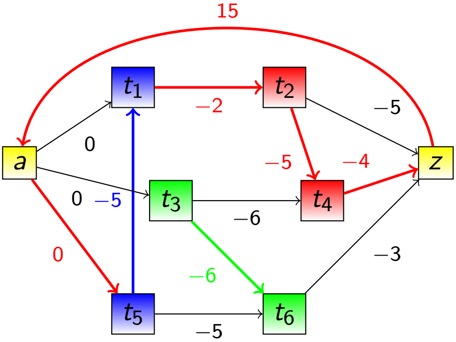

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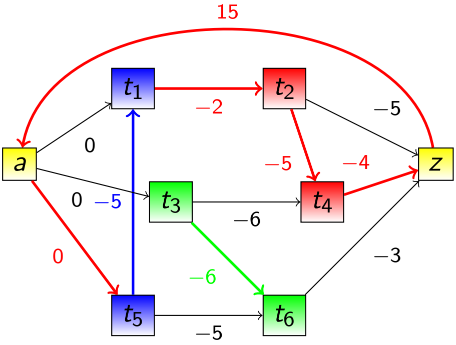

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Learned clause

- $\overline{l_{5\prec 1}} \vee \overline{l_{2\prec 4}}$

Results on Resource Constrained Project Scheduling Problem (RCPSP) [Ansótegui et al. 11]

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Two fomulations

- Time encoding
- Task encoding

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Results

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- State of the art for RCPSP!

Outline

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  - Somehow it does not have the same impact as in SAT
- Hybridization (learning + dedicated reasoning) is the way to go
  - SAT Modulo Theories?
  - CDCL with global constraints and integer domains?
  - Explanation algorithms for global constraints?