Order-Disorder Oscillations in the Populations of Faulty Repulsive Agents

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Abstract

We study the temporal evolution of populations including thousands of mobile repulsive agents. In the first part, the agents follow the rules of a deterministic automaton. We show that, the population may evolve toward different types of high-symmetry steady distributions, depending on the agent-agent repulsion law, and on the presence (or absence) of borders around the playground. We observe the formation of hexagonal-condensed lattices that maximize the potential function of the agent distribution. In the second part, agents are faulty. They violate the repulsion law and follow the rule of a non-deterministic automaton. We observe fast and random oscillations between the ordered phase and a strongly disorganized state when the number of agents is small (say for population involving a few tens of agents). These oscillations are essentially averaged and disappear in the large populations, resulting in a partially-ordered homogeneous distribution.

1. Introduction

In this work, we analyze the evolution of populations of monokinetic repulsive agents (MA), moving through a 2-dimension grid. Ants, mini-robots, or even people are possible examples of such monokinetic agents. The populations that we consider typically include from a few tens to a few thousands of agents. The evolution of the system is calculated from the evolution of each agent, using the simulator that we describe below. We study the spatio-temporal evolution of populations, starting from a random distribution generated in a rectangular terrain. We analyze two cases:
- Case 1: Each agent applies a deterministic repulsion law with respect to the other agents, following the transition rules defined in its state automaton. We show in section 3 that, at high agent density, the agents may get localized close to the vertices of an hexagonal lattice (CHL). This emergence of a geometric order out of the initial random distribution occurs when the repulsion law is independent of the distance fort the agents detected inside the sense disk of each agent. In section 4, we introduce a potential function to analyze the formation of order and to justify the CHL, which maximizes the potential function for the agent-agent repulsion law under consideration.
- Case 2: Each agent is faulty and randomly violates with probability \( p_F \) the agent-agent repulsion mechanism defined in the previous case. It follows a non deterministic automaton. This erratic behavior might result for instance from some malfunctioning, illness, tiredness… or deliberate perturbation of the agents. We show in section 5 that the small populations including a few tens of agents no more evolve to a steady state, but "chaotically" fluctuate between global order and full disorder. These global oscillations progressively disappear when increasing the size of the population, because the "local" transitions cannot propagate through the whole system.

2. Multi-Agent Simulator

The spatio-temporal evolution of the populations of mobile state automata provides a unified formalism for numerous problems, with possible application to biology [1], epidemiology, distributed robotics [2], economy and statistical physics. Some previous works originating from robotics considered a small number of agents (typical a few tens) [3,4,5,6]. However one of the most difficult problems consists in modeling and understanding the collective behavior of large populations, say comprising from thousands to dozens of thousands (or even more) elements. E. Yoshisa et al. used kinetic equations (i.e., a statistical approach neglecting local fluctuations) to describe the diffusion of information in the huge populations of robots by direct robot contact [7,8]. Unfortunately, the kinetic coefficients in their equations were completely phenomenological. One work was published in 1998 [9], and another one using reaction-diffusion equations in 1999 [10]. Recently, the principle of transposing stochastic rate equations to agents was addressed in [11].

All results reported in this communication are based on a multi-agent simulator (MASS) written in our laboratory over 4 years. MASS calculates the collective properties from the behavior of each agent, which follows the rules of its state automaton. Thus, the simulator cannot...
describe huge populations of agents (say populations including millions of agents...). Usually, it works smoothly and interactively for populations comprising up to a few tens of thousands of agents, depending on the complexity of the agent state automaton.

MASS is a project, the full description of which is beyond the scope of this communication. However, an extended documentation (explaining how using and programming MASS) and the simulator can be downloaded from the web site http://www.agentdynamics.org. This URL also includes several animated images, which display the transient dynamics of the agent population corresponding to the studies discussed in this work. The simulator works in the Windows environment and exhibits a highly interactive interface necessary to trace collective effects in the "large" populations. The operation is based on the following paradigm:

1) All agents of a population execute the same mission, defined by the same state automaton. However, each agent can evolve separately its state policies.
2) Each agent has an internal timer, which determines its next action time (NAT), and enables asynchronous activation. Simply, a scheduler scans all agent NATs and activates the agent with the shortest NAT.
3) The agent activated by the scheduler scans its environment (in a circular manner, starting from the eight neighbouring cells, and then increasing the distance to the analyzed cells up to the sense radius) to identify the other agents and the obstacles inside a sense disk. From this construction, the agent recognizes a configuration of agents and obstacles (i.e., an environment state of its automaton). Then, it chooses and executes an action in accordance with the rules defined by its state automaton, simply a move in the present study. In the current version, the agent moves on a square lattice, one step away in any of its eight surrounding cells.
4) Finally, the agent increments its NAT and returns the control to the scheduler, which again activates the agent with the shortest NAT (step 2).

3. Order emergence at high agent density

In this section, we consider the spatio-temporal evolution of a population including a few thousands of repulsive agents, which are initially randomly generated in a rectangular terrain. The number of agents is an adjustable simulation parameter, ranging typically from a few tens to a few thousands. We shall show that at high agent density, the initial random distribution evolves toward a spatial state of high symmetry that critically depends on the agent-agent repulsion mechanism and on the presence (or the absence) of borders to constrain the agent moves. Here, we consider that each agent selects a move direction depending on the positions of the other surrounded agents detected inside a sense disk of radius R. In the following we use the notation \( \vec{r}_{ij} = M_i M_j \), where \( M_i \) and \( M_j \) are two points in the plane. Let us consider that the agent is initially in \( M_0 \). When it is activated by the scheduler, it moves to \( M_1 \). The agent shift \( r_{01} \) is defined by the following equation:

\[
\vec{r}_{01} = \delta \sum_{\alpha \in C(0,R)} K^{\alpha 2} \frac{\vec{r}_{0\alpha}}{\left| \vec{r}_{0\alpha} \right|}
\]

The summation \( \alpha \) is restricted to all agents inside the sense disk \( C(R, r_0) \) of radius \( R \) centered on the agent position \( r_0 \). \( n \) is a free coefficient, which characterizes the interaction between two agents. In what follows, we shall particularly consider the case \( n=1 \), which defines a move direction

\[
\vec{r}_{01} = \delta \sum_{\alpha \in C(0,R)} \frac{K^{\alpha 2} \vec{r}_{0\alpha}}{\left| \vec{r}_{0\alpha} \right|}
\]

similar to that of charged particles interacting through an electrostatic potential. We shall also particularly consider the case \( n=3 \), which defines a repulsion law independent of the distance to the agents detected in the sense disk. Note that there is no reason to reject this kind of interaction as we consider macroscopic systems which are governed by a state automaton, so that any interaction law is programmable. \( K \) is a coefficient homogeneous to a length to force the right hand side of Eq. 1 to be homogeneous to a distance. The move amplitude is proportional to \( \delta \), which we assume small enough to make the agent move \( r_{01} \) small compared to the distance \( r_{0\alpha} \) to any other agent inside the sense disk. \( \delta \) is negative for the repulsion case.

Figs 1a and 1b below show from one example, how the moves followed by an agent, critically depend on the interaction parameter \( n \) in Eq. 1.

![Fig. 1: Dependence of the move direction versus the exponent n in the interaction law (see Eq. 1). (a): n=1; (b): n=3 (electrostatic mode)](image-url)
(electrostatic-like repulsion). This difference leads to very different evolutions of the agent population as will be shown below. Each agent applies the rules defined by the 2-state automaton represented in Table 1.

<table>
<thead>
<tr>
<th>State</th>
<th>Definition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>The agent discovers no agent inside its sense disk</td>
<td>Move straight forward if there is no obstacle. Otherwise, execute a Snell’s reflection on the obstacles. Borders (if any) are viewed as obstacles.</td>
</tr>
<tr>
<td>State 2</td>
<td>The agent discovers other agents inside its sense disk</td>
<td>Move one cell away in the direction defined by Eq. 1. Note that this move is in practice approximated by the closest move into one of the 8 cells surrounding the agent.</td>
</tr>
</tbody>
</table>

Table 1: State-action table of the agent automaton

We run numerous simulations, with different numbers of agents. It is possible to "remove" the borders by considering cyclic boundary conditions, so that an agent crossing a border (say the border $x=0$) is re-injected in the terrain through the opposite edge (say the border $X=L_{\text{Max}}$). The pertinent parameters are the density $\sigma=N/S$ of agents per surface units ($N$ being the number of agents, $S=L_XL_Y$ the surface of the terrain rectangular terrain), the sense radius $R$ of agents, and the coefficient $n$, which characterizes the repulsive law of the state automaton (see Eq. 1). In all simulations, we trace the agent moves, following an initial random distribution (both in position and in move direction). Each agent moves applies the rules of its state automaton.

### 3.1. Low-density limit

In the low-density limit (i.e., when $\pi NR^2<<S$), agent-agent collisions are rare and no spectacular effect is revealed by the simulator. Basically, each agent follows a quasi-free rectilinear trajectory, with, from time to time, a collision that changes its direction, in accordance with Eq. 1.

### 3.2. High-density limit when $n=1$

The high density occurs when $\pi NR^2 >> S$, i.e., when the sense disks of the agents strongly overlap. Again, the simulator reveals no spectacular effect when $n=1$ (see Eq. 1). The agents move all straight between collisions, as in the low density case, and occupy uniformly the terrain as displayed in Fig. 2. The average agent-agent distance is $a = \sqrt{S/N}$. The simulation conditions here are $N=1600$, $L_x=450$ (terrain width), $L_y=350$ (terrain height), $R=40$ (sense radius).

### 3.3. High-density limit when $n=-1$

The long-time behavior is radically different from that described in the previous paragraphs when $n=-1$. When cyclic boundary conditions apply (i.e., no border), the agents "condense" around the vertices of a hexagonal lattice as shown in Fig. 3. Each vertex includes about 15-16 agents. The simulation conditions are exactly those considered for Fig. 2. The distance $d$ separating two vertices is slightly larger than the agent sense radius $R$, typically $d=1.1R$. Changing the total number of agents simply changes the number of agents condensed per vertex and very little the distance separating two vertices. MASS shows the transient regime (i.e., the formation of the lattice from the initial disorder) and shows the local fluctuations of the agent dynamics leading to variations of the number of agents condensed in the different nodes.

Of course, it is impossible to show here the population dynamics due to the intrinsic limitations of a paper publication. So, we recommend visiting the site http://www.agentodynamics.org to watch several examples of temporal evolutions calculated by the simulator.
Additional figures will be presented in the final paper and during the conference.

4. Symmetry and potential function

In this section, we provide a simple explanation of the formation of the condensed hexagonal lattice reported in Fig. 3. We follow an approach usual in physics, which consists in defining a (dimensionless) potential function \( S \), here:

\[
S = \sum_{\alpha} \sum_{\beta \in C(\alpha, R)} \alpha \beta (r_{\alpha \beta}) \quad \text{where} \quad V(r) = \frac{K^\alpha}{r^n}
\]

The summation \( \alpha \) runs over all agents in the system. Now, let us consider that the agent located in \( M_0 \) moves to \( M_1 \) in accordance with Eq. 1. Following a few lines of algebra, it may be shown that if the move \( r_{01} \) is small compared to the distance to the other agents in the sense disk, the potential function changes from \( S_0 \) to \( S_1 \) following the equation:

\[
S_1 = S_0 + \frac{n}{\delta} \frac{\delta}{K^2} |r_{01}|^2
\]

Thus, the infinitesimal move of any agent induces a uniform increase (or decrease, depending on the sign of \( n \)) of the potential function, which in fact was chosen for this collective property. Remember that \( |r_{01}| \) is proportional to \( \delta^2 \) so that the second term of the right-hand side is actually a small quantity linear in \( \delta \). As we consider repulsions \( \delta \) is negative. There are two classes of behaviors:

- Case \( n>0 \) : In that case, \( n / \delta \) is negative. Thus, \( S_1<S_0 \), and the moves of the different agents minimize the potential \( S \). Obviously, this minimization cannot be accompanied by a grouping of the agents in the same place that would lead to the divergence of several terms contributing to the potential (see Eq. 2).
- Case \( n<0 \) : In that case, \( n / \delta \) is positive. Thus, \( S_1>S_0 \), and the repeated moves of the different agents now maximize \( S \). When \( n=-1 \), the potential reduces to:

\[
S = \frac{1}{K} \sum_{\alpha} \sum_{\beta \in C(\alpha, R)} |r_{\alpha \beta}|
\]

There is a crucial difference with the case \( n>0 \). Here, several agents can occupy the same site (or be infinitely close) without preventing the maximization of \( S \), whereas the “condensation” of agents on the same site is contradictory with the minimization of the potential when \( n>0 \), because of the divergence of the potential, which scales as \( 1/r^n \).

However, the property that the agents can occupy the same site when \( n=-1 \) does not tell us which geometry is expected for the final steady state distribution (SSD). Finding the SSD from the microscopic binary potential is surely a very difficult task. Fortunately, the crystal theory teaches us that the steady-state distribution is necessarily a high-symmetry state and suggests the formation of a 2-dimension cubic lattice or a hexagonal lattice. Thus, we calculated the final value of the potential (Eq. 4) for these distributions. The values of the potential for the different final distributions are:

- Condensed hexagonal lattice: \( V_H = \frac{3\sqrt{3}R^2N^2}{K} \) (all agents condense as displayed in Fig. 3).
- Condensed square lattice: \( V_S = \frac{4R^2N^2}{K} \) (all agents would condense on the vertices of a square lattice).
- Homogeneous distributed population at high density \( \pi R^2 N >> S \). \( V_S = \frac{\pi R^2 N^2}{KS} \).

Although we have not demonstrated that the condensed hexagonal lattice is the distribution which maximizes the potential function when \( n=-1 \), the comparison of the potential values for the high-symmetry lattices strongly suggests that it is, because \( V_H>V_S>>V_S \), and because it is difficult to imagine a high-symmetry steady distribution different from those we just considered.

5. Chaotic oscillations in the presence of faulty agents.

The simulations that we described in the previous sections reduce to analyzing the symmetry of the final state, following the spatiotemporal evolution of a random initial distribution of agents, which possibly evolves to a high-symmetry steady-state when \( n=1 \). In this section, we study the temporal evolution of the distribution when all agents are potentially faulty.

Making each agent faulty is easy: we simply replace the agent automaton described in Table 1 by the non-deterministic automaton displayed in Table 2. The difference is that an agent in state 2 now chooses the repulsion with probability \( p \). In other words, it randomly violates the repulsion law with probability \( (1-p) \) when it is surrounded by other agents, and moves all straight in that case. As we stressed in the introduction, this erratic behavior might result for instance from some malfunctioning, illness, tiredness… of the agents, and \( p=(1-p) \) may be viewed as the agent failure probability.
The number of agents is small (typically less than hundred), the population randomly exhibits fast transitions between ordered and disordered phases. In other words, the population is either fully ordered in a high-symmetry state, or fully disordered. The repetition of transitions seems is random process. As it is impossible to represent the state, or fully disordered. The repetition of transitions achieved when \( p=1 \) and \( n=1 \). Now, if \( p=0 \) (fully faulty agents), there is no agent-agent repulsion and therefore no order. What happens in the intermediate regime, when \( 0<p<1 \)? The question of the stability of the long-term distribution arises in the presence of faulty agents: Does the violation of the repulsion law generate some segregation between ordered and disordered zones that would permanently coexist in the playground?

We conducted many simulations, considering non-deterministic agents enclosed in a rectangle. The idea was to change the system size. Thus, we changed the number \( N \) of agents and the terrain size \( L_xL_y \), keeping constant the agent density \( N/(L_xL_y) \) and the agent sense radius. Concretely, we considered several populations including from 85 to 12240 agents, and several values of the parameter \( p \), which characterizes the violation of the repulsion law. Simulation clearly show that, when the number of agents is small (typically less than hundred), the population randomly exhibits fast transitions between ordered and disordered phases. In other words, the population is either fully ordered in a high-symmetry state, or fully disordered. The repetition of transitions seems is random process. As it is impossible to represent this temporal evolution in the framework of a paper publication, we suggest visiting the WEB page http://www.agentodynamics.org, which includes an animated image showing the transitions when the number of agents is small \( (N=85) \).

To quantify these simulations, we took advantage of the following property: In the disordered regime, collisions are rare, as most agents move all-straight. Contrarily, in the high-symmetry phase (Fig. 3), each agent is localized close to a vertex and it frequently changes its move direction to stay close to a vertex, because the localization is dynamic and it does not stop. Thus, an interesting parameter is the number \( N_f \) of straight-moving agents (SMA), which keep the same move direction over two consecutive activations. If there is no organization, most agents move freely and the fraction \( N_f/N \) of SMAs is high, i.e., close to 1. Contrarily, when the agents are condensed around vertices, they change very often their move direction and \( N_f/N \) is low.

Simulations show that when \( n=1 \) and \( p=1 \) (i.e. no fault), the population condenses in the high-symmetry state displayed in Fig. 3. In that case, the fraction \( N_f/N \) of straight-moving agents is small and fluctuates between 5 and 10%. We have not plot \( (N_f/N)(t) \), as it reduces to a noisy flat line. Now, let us consider the evolution of the small population \( (N=85) \) of faulty agents displayed Fig. 4, for \( p=0.8 \) and \( p=0.4 \).

The low values of \( N_f/N \) correspond to the existence of a high-symmetry state (HSS) similar to the steady state achieved when \( p=1 \). However this phase is metastable, as it regularly explodes into a disordered regime characterized by a high values of \( N_f/N \). The explosion of the HSS exactly occurs during the fast vertical variation from a low to a high value of \( N_f/N \) in Fig. 4. In other words, the population is either completely ordered (when \( N_f/N \) fluctuates around 0.1) or strongly disorganized, with \( N_f/N \) typically larger that 0.5 when \( p=0.8 \), or even close to 1 when \( p=0.4 \).

Fig. 5 shows the evolution of the fraction of SMAs in the case \( p=0.4 \), when enlarging considerably the terrain size and the number of agents. The dimensions are multiplied by the factor 6 in Fig. 5a (and 12 in Fig. 5b), keeping constant the agent density \( N/(L_xL_y) \) and the agent radius \( R=20 \). Again, we observe sharp transitions of the fraction of SMAs between low and high values. However, the amplitude of the fluctuations of \( N_f/N \) diminishes when increasing the size of the population. The reason is that the ordered state (when \( N_f/N \) fluctuates around 0.1) and the disordered states (when \( N_f/N \) fluctuates around 1) coexist.

<table>
<thead>
<tr>
<th>State 1</th>
<th>The agent discovers no agent inside its sense disk</th>
<th>Move straight forward (there are no obstacles in the reported simulation).</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 2</td>
<td>The agent discovers other agents inside its sense disk</td>
<td>1) Move one cell away in the direction defined by Eq. 1 with probability ( p ) or 2) Move straight forward with probability ( (1-p) ).</td>
</tr>
</tbody>
</table>

Table 2: State-action table of the non-deterministic automaton

Fig. 4: Time dependence of the fraction of straight-moving agents for \( p=0.8 \) and \( p=0.4 \). The number of agents is \( N=85 \) and \( n=1 \). Terrain: \( L_x=135 \), \( L_y=110 \); Agent radius: \( R=20 \).
in different points of the playground, because the "local" transitions (displayed in Fig. 4) cannot propagate over the whole system. The fraction of SMA tends to fluctuate around an average value (around 0.6-0.7).

It is important to stress that, even when we considered as much as 12240 agents (see Fig. 5b), the simulator deduces the global evolution of the population from the execution of the state-automaton rules of each agent. Considering the extreme complexity of describing transient non-equilibrium regimes, these results show the interest of developing fast multiagent simulators.

### 6. Conclusion

This work calculates the evolution of populations comprising from a few tens to thousands of monokinetic agents. Each agent follows the rules of its finite state automaton. We used this property to define an agent-agent repulsion law that we adjust with the parameter \( n \) in Eq. 1. The simulations reveal very different collective behaviors depending on the agent-agent repulsion law. In the high-density limit (when \( \pi NR^2 \gg S \)), we observe that the cyclic-boundary system evolves to a condensed hexagonal lattice represented in Fig. 3, when the repulsion law is independent on the distance (\( n=-1 \)). Using a potential function, we showed that this lattice is more stable than the square lattice, i.e., the other high-symmetry lattice that could be considered for the steady state distribution. In the second part, we showed that when agents are randomly faulty and violate the repulsion law, the distributions including a few tens of agents exhibits fast random transitions between completely-ordered states and strongly disorganized states. For large systems, order and disorder coexist in different places. In conclusion, the simulations reveal the complexity of collective dynamics, and how the collective behavior of multiagent systems may become quasi-unpredictable depending on the fraction of faulty elements.

### References


