Robust rendezvous planning under maneuver execution errors.
A worst case approach

C. Louembet, D. Arzelier, G. Deaconu

1LAAS-CNRS

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Rendezvous

Robustify open-loop maneuver plan to GNC systems errors:

Navigation syst. Uncertainties on the measured state
Thrusters actuation syst.
  - Errors on impulses firing time
  - Errors on impulses execution

Objectives

Robust and convex optimisation for worst-case approach
  - Desensitize the maneuver plan to GNC errors
  - Provide tractable algorithms (polynomial complexity)
  - Provide deterministic and guaranteed feasibility certificate
Rendezvous guidance problem

Convex and Robust Optimisation

Handling the GNC system errors
   Errors on impulses firing time
   Thrust mis-execution
   Navigation errors

Conclusion
Rendezvous guidance problem

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Orbital rendezvous definition

Orbital Rendezvous consists of:

- Keplerian relative motion
- Usage of ergols thrusters:
  - the control is modeled by impulsive signals,
  - Instantaneous velocity change,
  - Sequence of coasting arc limited by thruster impulses;

Rendez-vous problem

- Steering the chaser spacecraft from a state $A$ to state $B$ in fixed time;
- Assuming some operating constraints (Actuators bounds, safety constraints ...);

Rendezvous guidance problem $\Leftrightarrow$ Optimal control problem
Formulation of the RdV guidance problem

Deterministic and exact final condition rendezvous exact

- Minimize a given objective $J(\cdot)$ Consumption under dynamic constraints:

$$\min_{\Delta \tilde{V}, \nu_i} J(\cdot)$$

w.r.t.

$$\frac{d \tilde{X}(\nu)}{d\nu} = \tilde{A}_{TH} \tilde{X}(\nu) + \tilde{B}_{TH} \sum_i \Delta \tilde{V}_i \delta(\nu - \nu_i)$$

$$\begin{cases} 
\tilde{X}(\nu_1) = \tilde{X}_1, & \tilde{X}(\nu_f) = \tilde{X}_f \\
-\Delta \tilde{v}_i \leq \Delta \tilde{V}(\nu_i) \leq \Delta \tilde{v}_i, & \forall \ i = 1, \cdots, N
\end{cases}$$

(1)

\begin{itemize}
  \item $\nu$ excentric anomaly (independant variable)
  \item $\nu_i$ impulses firing time
  \item $\tilde{X} \in \mathbb{R}^6$ State vector in $\nu$
  \item $\Delta \tilde{V}_i \in \mathbb{R}^3$ $i^{th}$ impulse out of $N$
  \item $(\tilde{A}_{TH}, \tilde{B}_{TH})$ Tschauner-Hempel relative motion dynamics
  \item $\Delta \tilde{v}$ Impulse saturation
\end{itemize}
Formulation of the RdV guidance problem

Deterministic and exact final condition rendezvous exact

Direct methodology: Linear Programming [Waespy, 1970]

- Fixing the number and time of impulse $\nu_i$
- Replacing the the dynamic equation by its transition equation
- Minimizing $J(\cdot)$ linear in $\Delta \tilde{V}$

$$\min_{\Delta \tilde{V}} \quad J(\cdot)$$

w.r.t.

$$\left\{ \begin{array}{l}
\tilde{X}_f = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + B\Delta \tilde{V} \\
-\Delta \tilde{V}_i \leq \Delta \tilde{V}(\nu_i) \leq \Delta \tilde{V}_i, \forall \ i = 1, \ldots, N
\end{array} \right.$$  \hspace{1cm} (2)

$\Phi_{ya}$: Yamanaka-Ankersen Transition matrix

$\Delta \tilde{V}_i \in \mathbb{R}^{3N}$: Impulse vector

$Z \in \mathbb{R}^{3N}$: slack variables vector

$B = [\Phi_{ya}(\nu_f, \nu_1)B \ldots \Phi_{ya}(\nu_f, \nu_N)B]$
Disturbed Rendezvous problem

The rendezvous condition

$$\tilde{X}(\nu_f) = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + \underbrace{[\Phi_{ya}(\nu_f, \nu_1)B \ldots \Phi_{ya}(\nu_f, \nu_N)B]}_{B}\Delta \tilde{V}$$

Under the following disturbance

Navigation errors: $\tilde{X}_1 \in \mathcal{X}_{nav}$

Impulse time errors $\nu \in \mathcal{V}$

Thrust errors: $\Delta \tilde{V} \in \mathcal{U}$
Disturbed Rendezvous problem

The rendezvous condition

\[ \tilde{X}(\nu_f) = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + \left[ \Phi_{ya}(\nu_f, \nu_1)B \ldots \Phi_{ya}(\nu_f, \nu_N)B \right] \Delta \tilde{V} \]

Under the following disturbance

Navigation errors: \( \tilde{X}_1 \in \mathcal{X}_{\text{nav}} \)

Impulse time errors \( \nu \in \mathcal{V} \)

Thrust errors: \( \Delta \tilde{V} \in \mathcal{U} \)

\( \tilde{X}(\nu_f) \) belongs to a subset \( \mathcal{X}_f \) that should be included in a tolerance set \( \mathcal{T} \)

\[ \tilde{X}(\nu_f) \in \mathcal{X}_f \subset \mathcal{T} \]

\( \Rightarrow \) The rendezvous condition must be relaxed
RDV with relaxed final conditions

Exact final condition: Linear Programming

\[
\begin{align*}
\min_{\Delta \tilde{V}} J(\cdot) \\
\begin{cases}
A_{RdV} \Delta \tilde{V} = b_{RdV} \\
A_{\Delta V} \Delta \tilde{V} \leq b_{\Delta V}
\end{cases}
\end{align*}
\]

where

\[
A_{RdV} = B \\
b_{RdV} = \tilde{X}_f - \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1
\]

\[
A_{\Delta V} = \begin{bmatrix} \mathbb{I}_{3N} \\ -\mathbb{I}_{3N} \end{bmatrix},
\]

\[
b_{\Delta V} = \begin{bmatrix} \Delta \tilde{v}_1 \\ \Delta \tilde{v}_1 \end{bmatrix}
\]
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RDV with relaxed final conditions

Polytopic final condition: Linear Programming

\[
\begin{align*}
\min_{\Delta \tilde{V}} & \quad J(\cdot) \\
\text{w.r.t.} & \quad A\Delta \tilde{V} \leq b
\end{align*}
\]

where

\[
A = \begin{bmatrix}
HB \\
\mathbb{I}_{3N} \\
-\mathbb{I}_{3N}
\end{bmatrix}, \quad b = \begin{bmatrix}
K - H \left( \Phi_{ya}(\nu_f, \nu_1) \tilde{X}_1 \right) \\
\Delta \tilde{v} \\
\Delta \tilde{v}
\end{bmatrix}
\]
RDV with relaxed final conditions

Ellipsoidal final condition: Conic quadratic Programming

$$\begin{align*}
\min_{\Delta \tilde{V}} & \quad J(\cdot) \\
\text{w.r.t.} & \quad \left\{ \left\| A\Delta \tilde{V} + b \right\|_2 \leq 1, \right. \\
& \quad A_{\Delta V} \Delta \tilde{V} \leq b_{\Delta V} \right. 
\end{align*}$$

where

$$A = RB$$

$$b = R \left( -\tilde{X}_f + \Phi_y a(\nu_f, \nu_1)\tilde{X}_1 \right)$$

$$A_{\Delta V} = \begin{bmatrix} I_{3N} \\ -I_{3N} \end{bmatrix}, \quad b_{\Delta V} = \begin{bmatrix} \Delta \tilde{V} \\
\Delta \tilde{V} \end{bmatrix}$$
Objective functions $J(\cdot)$ and budget condition

$J(\cdot)$ must be linear in the decision variable

$\Delta V$ cost Minimize the propellant consumption

$$J(\cdot) = \|\Delta \tilde{V}\|_1$$

_Tolerance subset needs to precised before computation_
**Objective functions** $J(\cdot)$ and budget condition

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*Tolerance subset needs to precised before computation*

**Robustness cost** Minimize the tolerance subset

- **Box case** $\|K\|_1$
- **Ellipsoid case** $\log \det R^{-1}$

*Tolerance subset is part of decision variables*
Objective functions $J(\cdot)$ and budget condition

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**$\Delta V$ cost**  Minimize the propellant consumption

\[ J(\cdot) = \|\Delta \tilde{V}\|_1 \]

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**Robustness cost**  Minimize the tolerance subset

- **Box case** $\|K\|_1$
- **Ellipsoid case** $\log \det R^{-1}$

*Tolerance subset is part of decision variables*

**$\Delta V$ budget**  $\Delta V$ consumption can be restricted

\[ \|\Delta \tilde{V}\|_1 \leq M_{\Delta V} \]
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Convex and Robust Optimisation

Uncertain program \((P_\Delta)\)

\[
\begin{align*}
\min_{x} & \quad f_0(x) \\
\text{w.r.t.} & \quad f_i(x, \Delta_i(u_i)) \leq 0, \quad \forall \Delta_i \in V_i, \ i = 1, \ldots, m \\
\end{align*}
\]

\(x \in \mathbb{R}^n\) Optimisation variables

\(f_0, f_i\) Program structure (cost and constraint functions)

\(\Delta_i\) Problem data

\(u_i \in U_i \subset \mathbb{R}^{k_i}\) Disturbance variables

\(V\) Uncertainty set \(V = V_1 \times \cdots \times V_m\)

\(U\) Disturbance set \(U = U_1 \times \cdots \times U_m\)
### Convex and Robust Optimisation

#### Uncertain program \((P_{\Delta})\)

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#### Paradigms

- \(x\) must be obtained by solving (3) without the exact knowledge of datas \(\Delta_i\);

- Results are valid only for datas \(\Delta_i \in V_i\);

- No comprimises on constraints \(f_i(x, \Delta_i(u_i)) \leq 0\) are tolerated while \(\Delta_i \in V_i\).
Convex and Robust Optimisation

Uncertain program \((P_\Delta)\)

Minimize \(f_0(x)\)

With respect to \(f_i(x, \Delta_i(u_i)) \leq 0, \ \forall \Delta_i \in \mathcal{V}_i, \ i = 1, \ldots, m\) \hspace{1cm} (3)

Definitions

Robust feasible solution \(x\) is a feasible robust solution of the uncertain program \((P_{\Delta \in \mathcal{U}})\) if and only if \(f_i(x, \Delta_i) \leq 0\) for all \(\Delta\)'s realizations

Guaranteed cost the guaranteed is the worst case cost for a given robust feasible solution \(x\): \(\max_{\Delta_i} \{f_0(x) : \Delta_i \in \mathcal{V}_i, \forall i\}\).
**Convex and Robust Optimisation**

### Uncertain program \((P_\Delta)\)

\[
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\min_x & \quad f_0(x) \\
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\]  

(3)

### Definitions

**Robust feasible solution** \(x\) is a feasible robust solution of the uncertain program \((P_{\Delta \in \mathcal{U}})\) if and only if \(f_i(x, \Delta_i) \leq 0\) for all \(\Delta\)'s realizations

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\max_{\Delta_i} \{ f_0(x) : \Delta_i \in \mathcal{V}_i, \forall i \}.
\)

### Robust counterpart

\[
\begin{align*}
\min_x & \quad \max_{\Delta_i \in \mathcal{V}_i} f_0(x) \\
\text{sous} & \quad f_i(x, \Delta_i(u_i)) \leq 0, \quad \forall \Delta_i \in \mathcal{V}_i, \ i = 1, \ldots, m
\end{align*}
\]  

(4)
Robust Linear Programming

Robust counterpart to uncertain LP

\[
\begin{align*}
\min & \quad \max_{x} \quad \gamma^{T} x \\
\text{sous} & \quad Ax \leq b, \quad \forall (A, b) \in \mathcal{V}
\end{align*}
\]

- \((A, b)\) depend linearly in the disturbance variables \(u_i\)

\[
\mathcal{V} = \left\{ [A; b] = [A^0, b^0] + \sum_{j=1}^{k} u_j [A^j, b^j], \quad u \in \mathcal{U} \subset \mathbb{R}^k \right\}
\]

- Disturbance variables \(u_i\) belong to convex set \(\mathcal{U}\)
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Robust & convex counterpart [BenTal, 2009]

<table>
<thead>
<tr>
<th>Type of (\mathcal{U})</th>
<th>Robust Counterpart</th>
</tr>
</thead>
<tbody>
<tr>
<td>polytopic/interval</td>
<td>Linear Prog.</td>
</tr>
<tr>
<td>ellipsoidal</td>
<td>Conic Quadratic Prog.</td>
</tr>
<tr>
<td>ellipsoid intersection</td>
<td>Conic Prog.</td>
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<tr>
<td>Conic</td>
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<td>Unstructured norm-bounded</td>
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Errors on impulses firing time

- Uncertain firing time:
  \[ \nu_i \in [\nu_i^* - \delta \nu_i, \nu_i^* - \delta \nu_i] = [\nu_i] \]
  \[ \Rightarrow \Phi_{ya}(\nu_f, \nu_i) \in [\Phi_{ya}(\nu_i)] \]

Interval Analysis Computation
[Moore66,Jaulin01]

- Polytopic arrival set:
  \( H (B \Delta \tilde{V} + \Phi_{ya}(\nu_f, \nu_1)\tilde{X}(\nu_1) - \tilde{X}_f) \leq K \)

- Uncertainties on transition matrix:
  \[ \Phi_{ya}(\nu_f, \nu_i) = \Phi_{ya}^*(\nu_f, \nu_i) + u_i \delta \Phi_i, |u_i| \leq 1, i = 1, \ldots, N \]

- Uncertain polytopic RDV with \( \|u\|_{\infty} \leq 1 \)
  \[ H \left( \Phi_{ya}(\nu_f, \nu_1)\tilde{X}(\nu_1) + \sum_{i=1}^{N} (\Phi_{ya}^*(\nu_f, \nu_i) + u_i \delta \Phi_i) B \Delta \tilde{V}(\nu_i) \right) \leq K \]
Errors on impulses firing time

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- Uncertain polytopic RDV with \[ \|u\|_\infty \leq 1 \]
  \[ H \left( \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + \sum_{i=1}^{N} \left\{ (\Phi_{ya}(\nu_f, \nu_i) + u_i \delta \Phi_i) B \Delta \tilde{V}(\nu_i) \right\} \right) \leq K \]

- Robust polytopic RDV
  \[ H \left( \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + \sum_{i=1}^{N} \Phi_{ya}(\nu_f, \nu_i) B \Delta \tilde{V}(\nu_i) + \max_{u_i} \left\{ u_i \delta \Phi_i B \Delta \tilde{V}(\nu_i) \right\} \right) \leq K \]
  \[ \Downarrow \]
  \[ H \left( \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + \sum_{i=1}^{N} \Phi_{ya}(\nu_f, \nu_i) B \Delta \tilde{V}(\nu_i) + \max_{u_i} \left\{ u_i \delta \Phi_i B \Delta \tilde{V}(\nu_i) \right\} \right) \leq K \]
Errors on impulses firing time

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- Polytopic arrival set:
  \[ H \left( B \Delta \tilde{V} + \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) - \tilde{X}_f \right) \leq K \]

Robust Counterpart, LP [Ben-Tal & Nemirovski 00]

\[
\begin{align*}
\min_{\Delta \tilde{V}, Z} & \quad \sum_{i=1}^{N} K_i \\
\text{w.r.t.} & \\
\left\{ \begin{array}{l}
H\Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + H \sum_{i=1}^{N} \Phi_{ya}^*(\nu_f, \nu_i) B \Delta \tilde{V}(\nu_i) + |\delta \Phi_i B| Z_i \leq K \\
\Delta \tilde{V}_i \leq Z_i, \quad -\Delta \tilde{V}_i \leq Z_i \\
\begin{bmatrix} Z_{3i+1}, & Z_{3i+2}, & Z_{3i+3} \end{bmatrix}^T \leq \Delta \tilde{V}_{i+1}, \forall \ i = 0, \ldots, N - 1 \\
\|\Delta \tilde{V}\|_1 \leq \sum_{i=1}^{3N} Z_i \leq M \Delta V \end{array} \right. 
\end{align*}
\]

(5)
Errors on impulses firing time: numerical examples

ATV mission: 1 second errors

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<th>Initial anomaly</th>
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<td>6763 km</td>
<td>0.0052</td>
<td>0</td>
<td>7200 s</td>
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<td>$X_1 = [-30000, 5000, 8.154, 0]^T$</td>
<td>$X_1 = [-1000, 0, 0, 0]^T$</td>
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Table: ATV mission
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Errors on impulses firing time: numerical examples

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Errors on impulses firing time: numerical examples

Proba 3 mission: 1 second errors

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\[ X_1 = [-5000 \ 0 \ 0 \ 0]^T \]

\[ X_1 = [-20 \ 0 \ 0 \ 0]^T \]

Table: Proba3 mission
Errors on impulses firing time: numerical examples

Proba 3 mission: 1 second errors

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Impulsion mis-execution

Errors description

- Uncertain orientation: where $M_{rot}$ is the Cardan rotation matrix with small angles ($|\psi|, |\theta|, |\phi| \leq \beta$)

$$M_{rot} = \begin{bmatrix}
1 & -\psi_i & \theta_i \\
\psi_i & 1 & -\phi_i \\
-\theta_i & \phi_i & 1
\end{bmatrix}$$

- Uncertain amplitude: $|\lambda_i| \leq \varepsilon$

$$\Delta \tilde{V}_i = (1 + \lambda_i)M_{rot}\Delta V_i^*$$
Impulsion mis-execution

- Mis-impulsion: $\Delta \tilde{V}_i = (1 + \lambda_i) M_{rot} \Delta V_i^*$
- Polytopic arrival set:
  $$H \left( B \Delta \tilde{V} + \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) - \tilde{X}_f \right) \leq K$$

- Uncertainties on the $B$ matrix
  $$B = \left[ \Phi_{ya}(\nu_f, \nu_1) B M_1 \ldots \Phi_{ya}(\nu_f, \nu_N) B M_N \right]$$
  where $M_i = (1 + \nu_i) M_{rot}$
  $$M_i \in [M] = I_3 + [\begin{array}{ccc} \epsilon & (1 + \epsilon)\beta & (1 + \epsilon)\beta \\ (1 + \epsilon)\beta & \epsilon & (1 + \epsilon)\beta \\ (1 + \epsilon)\beta & (1 + \epsilon)\beta & \epsilon \end{array}]$$

- Uncertain polytopic RDV with $\|u\|_\infty \leq 1$:
  $$H \Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + HBM(u))\Delta \tilde{V} \leq K$$

- Robust polytopic RDV condition
  $$\Phi_{ya}(\nu_f, \nu_1) \tilde{X}(\nu_1) + HBM \Delta \tilde{V} + \sum_{i=1}^{N} \sum_{j=1}^{4} \left| H \Phi_{ya}(\nu_f, \nu_i) BM^j \Delta \tilde{V}_i \right| \leq K$$
Impulsion mis-execution

- Mis-impulsion: \( \Delta \tilde{V}_i = (1 + \lambda_i)M_{\text{rot}} \Delta V_i^* \)
- Polytopic arrival set:
  \[
  H \left( B \Delta \tilde{V} + \Phi_{\text{ya}}(\nu_f, \nu_1) \tilde{X}(\nu_1) - \tilde{X}_f \right) \leq K
  \]

Robust counterpart, LP

\[
\min_{\Delta \tilde{V}} \sum_{i=1}^{6} K_i
\]

w.r.t.

\[
\begin{cases}
\Phi_{\text{ya}}(\nu_f, \nu_1) \tilde{X}(\nu_1) + H B \Delta \tilde{V} + \sum_{i=1}^{N} \sum_{j=1}^{4} |H \Phi_{\text{ya}}(\nu_f, \nu_i) B M| Z_i \leq K \\
\Delta \tilde{V}_i \leq Z_i \\
-\Delta \tilde{V}_i \leq Z_i \\
[Z_{3i+1}, Z_{3i+2}, Z_{3i+3}]^T \leq \Delta \tilde{V}_{i+1}, \forall i = 0, \cdots, N - 1 \\
\| \Delta \tilde{V} \|_1 \leq \sum_{i=1}^{3N} Z_i \leq M_{\Delta V}
\end{cases}
\]
**Impulses mis-execution : numerical examples**

**ATV mission: Magnitude errors 0.1% Orientation errors 1°**

<table>
<thead>
<tr>
<th>Semi-major axis</th>
<th>Eccentricity</th>
<th>Initial anomaly</th>
<th>mission duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>6763 km</td>
<td>0.0052</td>
<td>0</td>
<td>2767 s</td>
</tr>
<tr>
<td>$X_1 = [-30000, 5000, 8.154, 0]^T$</td>
<td>$X_1 = [-1000, 0, 0, 0]^T$</td>
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**Table: ATV mission**
**Impulses mis-execution : numerical examples**

**ATV mission: Magnitude errors 0.1% Orientation errors 1°**

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**Table: ATV mission**
Impulses mis-execution: numerical examples

Proba 3 mission: Magnitude errors 0.1% Orientation errors $1^\circ$

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<td>0</td>
<td>141888 s</td>
</tr>
<tr>
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Table: Proba3 mission

Conclusion
Impulses mis-execution : numerical examples

Proba 3 mission: Magnitude errors 0.1% Orientation errors 1°

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$X_1 = [-5000 0 0 0]^T$

$X_1 = [-20 0 0 0]^T$

Table: Proba3 mission
Navigation errors

- Ellipsoidal uncertainties on initial state:
  \[ \tilde{X}_1 = \tilde{X}_1^* + P_u u, \quad u^T u \leq 1 \]

- Arrival ellipsoidal tolerance subset:
  \[ \| R (B\Delta \tilde{V} - \tilde{X}(\nu_f) + \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1) \|_2 \leq 1 \]

Minimal ellipsoidal tolerance subset

- The tolerance subset of minimum volume is independent from the optimized plan;
- The minimal tolerance subset can be computed from the uncertainties set on initial state and the propagation time:
  \[ Q^{-1} = P_u^T \Phi_{ya}(\nu_f, \nu_1) \Phi_{ya}(\nu_f, \nu_1)^T P_u \]
- The optimized plan only control the center of the propagated domain.
- The tolerance can be optimized by a feedback MPC approach

\[ \text{[Deaconu13]}^1 \]

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1[Deaconu13], G. Deaconu et al., Minimizing the effects of the navigation uncertainties on the spacecraft rendezvous precision Journal of Guidance, Control, and Dynamics, accepted in 2013
Rendezvous guidance problem

Convex and Robust Optimisation

Handling the GNC system errors
- Errors on impulses firing time
- Thrust mis-execution
- Navigation errors

Conclusion
Concluding remarks

Obtained results

- Tractable robust counterparts;
- Desensitized maneuver plan for thrusters system errors

\[ \tilde{X}(\nu_f) = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + B\Delta\tilde{V} \]

- Certified tolerance set;
- Robustness price may be expensive.

Comments

- LP tools (not yet suitable for embedded CPU);
- Algorithms provide certified results for risk management;
Thank You For Your Attention