PhD Thesis Proposal

June 3, 2017

1 Essential Information

Title	Input-Output Stability and Induced Gains for Switched/Hybrid Systems
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Salary:	≈ 1500 euros per month (after taxes)
Key Dates:	*Before June 09*: Contact the supervisors *June 30*: Selection complete *Fall 2017*: PhD studies begin *Fall 2020*: PhD studies end

2 Scientific Context

Switched systems form a simpler class of hybrid systems where the continuous dynamics interact with discrete variables. Such systems appear in the design of discontinuous control laws, and also as a tool for analysis in certain design problems. Studying stability of such systems has helped researchers generalize classical stability tools to the framework of discontinuous, and hybrid, dynamical systems. Now that the stability techniques for switched systems are quite mature, it is natural to study further fundamental properties of switched systems. This thesis proposes a set of problems to carry forward the field of switched systems in this direction.

For linear and nonlinear time-invariant systems, input-output gains have been studied in a lot of depth and can be related to several fundamental properties associated with the dynamical system. Two approaches of our particular interest are based on using the input-to-state stability (ISS) notion [9], or the $\mathcal{L}_p$-induced gains [5]. Also, the computation of such gains has been extensively used in the design of controllers. It is thus very natural to study the computation of input-output gains for switched dynamical systems. The underlying problems in studying this topic allows the student to acquire knowledge of the fundamental stability concepts in the theory of dynamical systems, and moreover the computational aspect will allow the student to learn and develop familiarity with numerical methods. From the application point of view, the ISS estimates have proved extremely useful in designing controllers in the presence of measurement or communication errors [10]. Similarly, the input-output induced gains also appear in the controller design problems while incorporating disturbance attenuation [1, 4].

Analysis of input-output gains for switched dynamical systems has been a topic of interest in the control community for almost a decade now. It was initiated by the open problem formulated in [3, Problem 4.1], and was partially solved by the same author in [2], under certain restrictive conditions on the subsystem dynamics and for a class of switching signals with sufficiently large dwell-time. The ISS estimates for switched systems under a dwell-time assumption has been studied in [12].

Because the approaches adopted in the literature for approaching this problem have been developed under strong assumptions on the system dynamics, we aim to explore alternative methods that may allow generalizing the existing results. It is important to note that the proposed topic builds upon stability results for switched systems, which is by now a very mature topic [6]. This thesis will also concentrate on using optimization tools that can be used for computing the gains and how such methods can be used for stabilization of switched systems.

Part of the tools used for this thesis will rely upon the recent works involving $\mathcal{L}_p$ norms for hybrid dynamical systems [7] and also the representation of the corresponding dwell time and average dwell time properties therein reported. Indeed, when using a hybrid formalism, certain peculiar features of switching systems subject to some constraints on the allowable switching signals can be well represented and studied (see also [8]). An interesting outcome of this study could then be to develop suitable tools for establishing (global) asymptotic stability properties of some suitable interconnection of switched systems, under some assumptions on the switching signal, an ISS or input/output stability property, and then also a suitable detectability condition (see, for example [11] for some results in this direction).

Thus, the study of input-output stability notions with ISS estimates, or induced gains, is a fundamental problem in the theory of switched systems. Developing good understanding on how to compute such gains will lead to systematic design of stability certificates and then also controllers for switched systems.

3 Contribution of the Thesis

A switched system is described by a collection of dynamical subsystems $\{f_p, G_p, H_p, p \in \mathcal{P}\}$, where $\mathcal{P}$ is a finite index/parameter set. In addition, there is a piecewise constant switching signal $\sigma : [0, \infty) \to \mathcal{P}$, which determines the active subsystem at the given time instant. Mathematically, the evolution of the state trajectory of such systems is described by the following equations:

$$\dot{x} = f_\sigma(x) + G_\sigma(x)u \tag{1a}$$

$$y = H_\sigma(x). \tag{1b}$$

In this thesis, we are interested in studying the effect of the exogenous input u on the trajectories of the system when the underlying system dynamics are switching. The computation of input-output gains builds upon stability concepts of switched systems. The theory for stability of switched systems is already a mature topic and forms the cornerstone in the analysis and design of modern hybrid dynamical systems.

As seen from the literature, there are many open questions related to computing the ISS estimates, or induced input-output gains, that can be asked in the context of switched systems depending on what assumptions are imposed on the subsystem dynamics and the switching signals. The use of tools from the hybrid dynamical systems context will produce novel research results, that will constitute the theoretical contribution of the thesis.

Also, the computation of nonconservative gain estimates has not been much addressed in the switched systems community from the computational viewpoint, which could be done by way of suitable convex optimization tools In particular, the thesis would primarily focus on finding relaxed conditions on system data under which the proposed input-output gains are finite. After developing a solid knowledge of these conditions, the student would be advised to develop numerical techniques for computing the gains. An interesting direction of this PhD thesis would be then to apply the optimization techniques developed in the MAC team for the computation of optimized induced gain estimates for switched systems.

Once a good understanding has been developed for the class of switched systems for which the problem can be solved, and algorithms have been implemented, several research directions could stem from this work. One could next ask the question of designing switching signals that could generate a desired input-output behavior. Designing controllers with appropriate induced gains for switched systems would also be an interesting direction. From the applications viewpoint, designing appropriate switching controllers for regulation and stabilization of electrical circuits is certainly an interesting research direction.

To summarize, the problem of computing ISS estimates and induced gains for switched systems is theoretically interesting and also numerically challenging, which promises to provide a PhD student with appropriate training for analysis, control, and simulation of switched dynamical systems. Several interesting problems that can stem out from this work are also promising for advancing the field of control of switched systems. The tools used for this analysis may well see also significant contributions to the field of input-output properties of hybrid dynamical systems and stability analysis of interconnected switched or hybrid systems.

References

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