From Aerial Vehicles to Aerial Robots through the lens of Tethering and Full Actuation

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For more information about the control methods presented in this talk you can check:

**Tethered platforms:**

**Fully-actuated platforms:**

For more information about our activity on similar topics, refer to: [http://homepages.laas.fr/afranchi/robotics/](http://homepages.laas.fr/afranchi/robotics/)
1. Motivation and Background

2. Tethered Aerial Robots

3. Fully-actuated Aerial Robots

4. Current and Future Works
Motivation and Background
Aerial robots for **physical interaction**

- applications: inspection, maintenance, transportation, manipulation...

Some examples in recently EU-funded projects:

- Seville Univ. (ARCAS)
- DLR (ARCAS)
- CATEC (ARCAS)
- AEROWORKS concept
Challenges of Physically Interactive Aerial Robotics (I)

Floating base

- **active reaction wrench** provided by the thrusters (grounded manipulators have ‘passive’ ground reaction)
- **inaccurate** positioning (because of noisy sensing and external disturbances)
- **dynamic coupling**

Actuators of the base

- additional **aerodynamic layer**
  
  \[ \text{motor torque} \sim \text{propeller acceleration} \]
  
  \[ \downarrow \]
  
  \[ \text{propeller speed} \sim \text{thrust force} \]

- **unmodeled aerodynamics**
Challenges of Physically Interactive Aerial Robotics (II)

Need for a **lightweight payload**
- arms with **weaker motors**
- minimal number of sensors
- flexibility $\Rightarrow$ vibrations

Need to **save energy**
- underactuated configurations (i.e., coplanar propellers)

**Tethered** aerial robots:
- Cable/Bar: *physical connection*
- Modeling, Control, Observation

**Fully Actuated** Aerial Robots:
- *Full-wrench* exertion
- Mech. Design, Modeling, Control
Tethered Aerial Robots
Simplified 2D System Model: Aerial Vehicle

Frames:

- \( \mathcal{F}_W = O_W - \{x_W, y_W, z_W\} \) (World frame)
- \( \mathcal{F}_B = O_B - \{x_B, y_B, z_B\} \) (Vehicle frame)

\( O_B \equiv \text{vehicle center of mass (CoM)} \)

Parameters

- \( m_R \) vehicle mass
- \( J_R \) vehicle rotational inertia

Configuration and inputs

- \( \vartheta \) vehicle attitude (pitch)
- \( f_R \) intensity of the thrust force \( f_R = -f_R z_B \)
- \( \tau_R \) intensity of the torque \( \tau_R y_B \)

Available sensors

- \( a \) onboard accelerometer (see later for definition)
- \( \omega \) onboard gyroscope (\( \equiv \dot{\vartheta} \))
Simplified 2D System Model: Link

Link can be a **bar**, a **taut tie**, or a **compressed strut**

**Passive** rotational **joints** at

- $O_W$ ground fixed point
- $O_B$ vehicle CoM

Parameters and assumptions

- $l$ link length (constant)
- **negligible** mass and inertia w.r.t. $m_R$ and $J_R$
- **negligible** deformation and elasticity

Configuration

- $\varphi$ link elevation
- $f_L$ link internal force
  - $f_L > 0$ **tension** (bar or tie)
  - $f_L < 0$ **compression** (bar or strut)

Available sensors at link-side

- **none**
Motivation

Why an aerial vehicle **linked/tethered** to the ground?

Physically interactive uses

- pull/pushing
- resist strong wind
- landing/take-off from/on
  - a **sloped** surface
  - a **moving** platform (e.g., ship)

Other uses

- enduring **power**
- high-bandwidth communication channel

Some **application** fields

- transportation/manipulation
- inspection and surveillance
- communication relay

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EC-SafeMobil (CATEC)

System Dynamics and Control Problem

System dynamics:

\[
\begin{align*}
\dot{\phi} &= -\frac{g}{l} \cos \phi + \frac{\cos(\phi + \dot{\theta})}{m_R l} f_R \\
\dot{\theta} &= \frac{1}{J_R} \tau_R \\
\dot{f}_L &= -m_R g \sin \phi + m_R l \dot{\phi}^2 + \sin(\phi + \dot{\theta}) f_R
\end{align*}
\]

- \((\phi, \dot{\phi}, \theta, \dot{\theta})\) system state
- \((f_R, \tau_R)\) control inputs
- \((\phi^d, f^d_L)\) desired outputs
- \((a, \omega)\) onboard measurements

From Aerial Vehicles to Aerial Robots: Tethering and Full Actuation
Control Problem

Design a control law for the inputs \((f_R, \tau_R)\) in order to

- asymptotically steer \((\varphi, f_L)\) along a sufficiently smooth desired trajectory \((\varphi^d, f^d_L)\)

Using only

- onboard accelerometer and onboard gyroscope examples
System Dynamics and Control Problem

System dynamics:

\[
\dot{\phi} = -\frac{g}{l} \cos \phi + \frac{\cos(\phi + \dot{\vartheta})}{m_R l} f_R
\]

\[
\ddot{\vartheta} = \frac{1}{J_R} \tau_R
\]

\[
f_L = -m_R g \sin \phi + m_R l \dot{\phi}^2 + \sin(\phi + \vartheta) f_R
\]

- \((\phi, \dot{\phi}, \vartheta, \dot{\vartheta})\) system state
- \((f_R, \tau_R)\) control inputs
- \((\varphi^d, f^d_L)\) desired outputs
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Control Problem

Design a control law for the inputs \((f_R, \tau_R)\) in order to

- asymptotically steer \((\phi, f_L)\) along a sufficiently smooth desired trajectory \((\varphi^d, f^d_L)\)

Using only

- onboard accelerometer and onboard gyroscope

Delocalization of Measurements/Desired Output

Measurements \(\rightarrow\) onboard & proprioceptive
Desired outputs \(\rightarrow\) “off-board” & “exteroceptive”
State-feedback Control: State Space

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} x_2 \\ a_1 \cos x_1 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_2 \cos (x_1 + x_3) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u \\
y &= \begin{bmatrix} \frac{1}{a_2} x_2^2 + \frac{a_1}{a_2} \sin x_1 \\ \sin (x_1 + x_3) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \sin (x_1 + x_3) & 0 \end{bmatrix} u
\end{align*}
\]

where

\[
x = \begin{bmatrix} \phi \\ \dot{\phi} \\ \vartheta \\ \dot{\vartheta} \end{bmatrix}, \quad u = \begin{bmatrix} f_R \\ \tau_R \\ u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} \phi \\ f_L \end{bmatrix}, \quad y_d(t) = \begin{bmatrix} \varphi^d \\ f_L^d \end{bmatrix}
\]

and

\[a_1 = -g/l, \quad a_2 = 1/(m_R l), \quad a_3 = 1/J_R\]
State-feedback Control: State Space

\[ \dot{x} = \begin{bmatrix} x_2 \\ a_1 \cos x_1 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 \cos (x_1 + x_3) \\ 0 \\ 0 \end{bmatrix} u \]  \tag{1}

\[ y = \begin{bmatrix} 1 \\ \frac{x_1}{a_2} x_2 + \frac{a_1}{a_2} \sin x_1 \\ \sin (x_1 + x_3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u \]  \tag{2}

where

\[ x = \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix}, \quad u = \begin{bmatrix} f_R \\ \tau_R \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} \phi \\ f_L \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow y^d(t) = \begin{bmatrix} \phi^d \\ f_L^d \end{bmatrix} = \begin{bmatrix} y_1^d \\ y_2^d \end{bmatrix} \]

and \( a_1 = -g/l, \quad a_2 = 1/(m_R l), \quad a_3 = 1/J_R \)

Temporary assumption (will be relaxed with the observer)

Temporarily assume that \( x \) is fully measurable
State-feedback Control: Dynamic Input-output Linearization

System is not input-output linearizable with static feedback (s-fl)

Redefine a new input as \( \tilde{u} = [\ddot{u}_1 \ u_2]^T = [\dddot{u}_1 \ \dddot{u}_2]^T \)

New system state \( \tilde{x} = [\phi \ \dot{\phi} \ \dot{\theta} \ \ddot{\theta} \ u_1 \ \dddot{u}_1]^T \)

\[ y_{(4)} = b(\tilde{x}) + a_2 \cos(x_1 + x_3) - a_2 a_3 \sin(x_1 + x_3) u_1 \sin(x_1 + x_3) a_3 \cos(x_1 + x_3) u_1 \]

\[ \tilde{E}(\tilde{x}) \tilde{u}, \text{as long as} \ u_1 \neq 0 \]

The system is input-output linearizable with dynamic feedback iff \( u_1 \neq 0 \)

Exact feedback linearization

Total relative degree = dimension of \( \tilde{x} \)

\( \Rightarrow \) The controlled closed-loop system has no internal dynamics

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State-feedback Control: Dynamic Input-output Linearization

System is not input-output linearizable with static feedback \( (s-fl) \) → Redefine a new input as \( \bar{u} = [\ddot{u}_1 \ u_2]^T = [\ddot{u}_1 \ \ddot{u}_2]^T \)

New system state \( \bar{x} = [\phi \ \dot{\phi} \ \dot{\theta} \ \dot{\theta} \ u_1 \ \dot{u}_1]^T \)

Need for further differentiation to see the new input \( \bar{u} \) appear in both output channels

\[
\begin{bmatrix}
    y_1^{(4)} \\
    y_2^{(2)}
\end{bmatrix} = b(\bar{x}) + \begin{bmatrix}
    a_2 \cos(x_1 + x_3) & -a_2a_3 \sin(x_1 + x_3) \ u_1 \\
    \sin(x_1 + x_3) & a_3 \cos(x_1 + x_3) \ u_1
\end{bmatrix} \bar{u} + \underbrace{\bar{E}(\bar{x})}_{\bar{E}(\bar{x})} 
\]

\( \bar{E}(\bar{x}) \)

System is not input-output linearizable with static feedback (s-fl) → Redefine a new input as \( \tilde{u} = [\ddot{u}_1 \ u_2]^T = [\ddot{u}_1 \ \tilde{u}_2]^T \)

New system state \( \tilde{x} = [\phi \ \dot{\phi} \ \vartheta \ \dot{\vartheta} \ u_1 \ \dot{u}_1]^T \)

Need for further differentiation to see the new input \( \tilde{u} \) appear in both output channels

\[
\begin{bmatrix}
    y_1^{(4)} \\
    y_2^{(2)}
\end{bmatrix} = b(\tilde{x}) + 
\begin{bmatrix}
    a_2 \cos(x_1 + x_3) & -a_2 a_3 \sin(x_1 + x_3) u_1 \\
    \sin(x_1 + x_3) & a_3 \cos(x_1 + x_3) u_1
\end{bmatrix} \tilde{u},
\]

(3)

\[
\det(\tilde{E}(\tilde{x})) = \frac{u_1}{lm R J_R}, \text{ as long as } u_1 \neq 0, \text{ the control law } \tilde{u} = E^{-1}(\tilde{x}) [-b(\tilde{x}) + v], \text{ brings the system in the form }
\]

\[
\begin{bmatrix}
    y_1^{(4)} \\
    y_2^{(2)}
\end{bmatrix} = \begin{bmatrix}
    v_1 \\
    v_2
\end{bmatrix} = v
\]

The system is input-output linearizable with dynamic feedback iff \( u_1 \neq 0 \)

---

State-feedback Control: Dynamic Input-output Linearization

System is not input-output linearizable with static feedback (s-fl) → Redefine a new input as \( \tilde{u} = [\ddot{u}_1 \ u_2]^T = [\ddot{u}_1 \ \ddot{u}_2]^T \)

New system state \( \tilde{x} = [\varphi \ \dot{\varphi} \ \vartheta \ \dot{\vartheta} \ u_1 \ \dot{u}_1]^T \)

Need for further differentiation to see the new input \( \tilde{u} \) appear in both output channels

\[
\begin{bmatrix}
  y_1^{(4)} \\
  y_2^{(2)}
\end{bmatrix} = b(\tilde{x}) + \begin{bmatrix}
  a_2 \cos(x_1 + x_3) & -a_2a_3 \sin(x_1 + x_3) \ u_1 \\
  \sin(x_1 + x_3) & a_3 \cos(x_1 + x_3) \ u_1 
\end{bmatrix} \tilde{u},
\]

\( \tilde{E}(\tilde{x}) \)

\[
\det(\tilde{E}(\tilde{x})) = \frac{u_1}{lm_{RJ_R}}, \text{ as long as } u_1 \neq 0, \text{ the control law } \tilde{u} = \tilde{E}^{-1}(\tilde{x})[-b(\tilde{x}) + v], \text{ brings the system in the form }
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\begin{bmatrix}
  y_1^{(4)} \\
  y_2^{(2)}
\end{bmatrix} = \begin{bmatrix}
  v_1 \\
  v_2
\end{bmatrix} = v
\]

The system is input-output linearizable with dynamic feedback iff \( u_1 \neq 0 \)

Exact feedback linearization

Total relative degree = dimension of \( \tilde{x} \)

⇒ The controlled closed-loop system has no internal dynamics \(^1\)

Observer: Sensor Model

**IMU**

- **Gyroscope**: angular rate (angular velocity intensity)
  
  \[ \omega = \dot{\vartheta} \]

- **Accelerometer**: specific acceleration in the body frame
  
  \[ \mathbf{a} = R_W^B (\ddot{p}_B + g \mathbf{z}_W) = [a_x, 0, a_z]^T \]

  \[
  a_x = \cos(\varphi + \vartheta) \left[ l \dot{\varphi}^2 - g \sin \varphi + \frac{f_R}{m_R} \sin(\varphi + \vartheta) \right] \\
  a_z = \sin(\varphi + \vartheta) \left[ l \dot{\varphi}^2 - g \sin \varphi + \frac{f_R}{m_R} \sin(\varphi + \vartheta) \right] - \frac{f_R}{m_R}
  \]

Go to aerial vehicle model
Observer: State Reduction using Gyroscope

Full model

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
a_1 \cos x_1 + a_2 \cos (x_1 + x_3) u_1 \\
0 \\
a_3 u_2 \\
\end{bmatrix}
\]

State reduction

\[
\begin{align*}
\dot{x}_3 &= \omega \\
x_4 &= u_3
\end{align*}
\]

Gyroscope measurement:

one state becomes an input

Reduced model

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
a_1 \cos x_1 + a_2 \cos (x_1 + x_3) u_1 \\
0 \\
u_3 \\
\end{bmatrix}
\]
Nonlinear Observer: High Gain Observer (HGO)\(^2\)

- provides a state estimation \(\hat{x}\) that converges to the actual state value \(x\):
  \[
  \lim_{t \to \infty} \hat{x} = x
  \]
- requires: system in the triangular form

**Desired (Triangular) form**

\[
\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \phi(x, u) + \lambda(u)
\]

\[
w = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x
\]

- \(x\): state vector
- \(u\): control input
- \(w\): measurement
- \(\phi, \lambda\): any nonlinear functions

**Actual form**

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} a_1 \cos x_1 + a_2 \cos (x_1 + x_3) u_1 \\ u_3 \end{bmatrix}
\]

\[
a = \begin{bmatrix} \cos (x_1 + x_3) \left( l x_2^2 - g \sin x_1 + \frac{1}{m_R} \sin (x_1 + x_3) u_1 \right) \\ \sin (x_1 + x_3) \left( l x_2^2 - g \sin x_1 + \frac{1}{m_R} \sin (x_1 + x_3) u_1 \right) - \frac{u_1}{m_R} \end{bmatrix}
\]

- \(x\): state vector
- \(u\): control input
- \(a\): measurements

State and Measurements Transformation needed to get from the actual to the desired form

Observer: State Transformation - part I

Reduced model

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
a_1 \cos x_1 + a_2 \cos (x_1 + x_3) u_1 \\
0
\end{bmatrix}
\]

\[a_x = \cos (x_1 + x_3) \left( lx_2^2 - g \sin x_1 + \frac{1}{m_R} \sin (x_1 + x_3) u_1 \right)\]

\[a_z = \sin (x_1 + x_3) \left( lx_2^2 - g \sin x_1 + \frac{1}{m_R} \sin (x_1 + x_3) u_1 \right) - \frac{u_1}{m_R}\]

State transformation

\[
\begin{align*}
z_1 &= x_1 + x_3 \\
z_2 &= x_2 \\
z_3 &= a_1 \cos x_1 + a_2 \cos (x_1 + x_3) u_1 \quad (= \dot{x}_2 = \dot{z}_2)
\end{align*}
\]

Transformed model (I)

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
a_1 z_2 \sin x_1 + a_2 (\cos z_1) \dot{u}_1 - a_2 (\sin z_1) (z_2 + u_3) u_1
\end{bmatrix}
\]

\[a_x = \cos z_1 \left( lz_2^2 - g \sin x_1 + \frac{1}{m_R} \sin z_1 u_1 \right)\]

\[a_z = \sin z_1 \left( lz_2^2 - g \sin x_1 + \frac{1}{m_R} \sin z_1 u_1 \right) - \frac{u_1}{m_R}\]
Observer: State Transformation - part II

Transformed model (I)

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix}
+ \begin{bmatrix}
u_3 \\
0 \\
a_1 z_2 \sin x_1 + a_2 (\cos z_1) \dot{u}_1 - a_2 (\sin z_1) (z_2 + u_3) u_1
\end{bmatrix}
\]

\[ a_x = \cos z_1 \left( l z_2^2 - \frac{g \sin x_1}{m_R} \sin z_1 u_1 \right) \]

\[ a_z = \sin z_1 \left( l z_2^2 - \frac{g \sin x_1}{m_R} \sin z_1 u_1 \right) - \frac{u_1}{m_R} \]

From the accelerometer

\[
\bar{w}_1 = \text{atan2} \left( \pm \frac{a_x}{\eta}, \pm \frac{a_z + u_1/m_R}{\eta} \right) = z_1 + k\pi
\]

\[ \sin x_1 = \frac{1}{g} \left( \pm \eta + l z_2^2 + \frac{1}{m_R} \sin z_1 u_1 \right) \]

\[ \; \eta = \sqrt{a_x^2 + \left( a_z + \frac{u_1}{m_R} \right)^2} \neq 0 \]

Transformed model (II)

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\sigma(z_1, z_2, z_3, u_1, \dot{u}_1, \eta)
\end{bmatrix}
\begin{bmatrix}
u_3 \\
0 \\
0
\end{bmatrix}
\]

is in **triangular form**!

\[
\bar{w}_1 = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix}
+ k\pi
\]
Simulations: Simultaneous Internal Force and Elevation Control

From tension to compression

Sinusoidal trajectories

M. Tognon and A. Franchi, “Nonlinear observer-based tracking control of link stress and elevation for a tethered aerial robot using inertial-only measurements”, in 2015 IEEE Int. Conf. on Robotics and Automation, Seattle, WA, 2015, pp. 3994–3999
M. Tognon, A. Testa, E. Rossi, and A. Franchi, “Exploiting a passive tether for robust takeoff and landing on slopes: Methodology and experiments”, in 2016 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Daejeon, South Korea, 2016
Fully-actuated Aerial Robots
Underactuation vs. Full-actuation in Aerial Robots

**Underactuated**
- *position-only* control (coupled position and orientation)
- *force-only* control in interaction
- + only (low) internal drag (*efficient*)
- + lower complexity

**Fully-actuated**
- + *full-pose* control (independent control of position and orientation)
- + *full-wrench* control in interaction
- – internal wrench (*wasted* energy)
- – higher complexity

Force Disturbance along in X, Y, Z axis

Applied
Multi-rotor aerial platforms are essentially made of two elements:

- **a rigid body** $\rightarrow$ rigid body dynamics

\[
\begin{bmatrix} m\ddot{p}^W_B \\ J\ddot{\omega}^W_B \end{bmatrix} = - \begin{bmatrix} mge_3 \\ \omega_B^W \times J\omega_B^W \end{bmatrix} + \begin{bmatrix} f^W \\ \tau^B \end{bmatrix}
\]

where $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

- **a set of propellers** attached to the body $\rightarrow$ total input wrench

Wrenches of the single propellers: $f^W_B = R_B^S_i \begin{bmatrix} 0 \\ 0 \\ c_f \end{bmatrix} \left[ w_i | w_i \right]_{u_i}$, $i = 1, \ldots, n$

Total input wrench: $f^W = R_B^W \sum_{i=1}^n f^B_i = R_B^W F_1 \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = R_B^W F_1 u$ (5)

$\tau^B = \sum_{i=1}^n p_{B,S_i}^B \times f^B_i + \sum_{i=1}^n \tau_i^B = F_2 \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = F_2 u$ (6)
Putting (5) and (6) in (4):

\[
\begin{bmatrix}
\dot{m}
\dot{p}_B^W \\
J \dot{\omega}_B^W 
\end{bmatrix} = - \begin{bmatrix}
m \dot{g} e_3 \\
\omega_B^W \times J \omega_B^W 
\end{bmatrix} + \begin{bmatrix}
R_B^W & 0 \\
0 & I 
\end{bmatrix} \begin{bmatrix}
F_1 \\
F_2 
\end{bmatrix} u,
\] where \( u = \begin{bmatrix}
w_1 \\
w_2 \\
\vdots \\
w_n 
\end{bmatrix} \)
Putting (5) and (6) in (4):

\[
\begin{bmatrix}
m\ddot{p}^W_B \\
J\dot{\omega}^W_B
\end{bmatrix} = - \begin{bmatrix}
mg e_3 \\
\omega^W_B \times J\omega^W_B
\end{bmatrix} + \begin{bmatrix}
R^W_B & 0 \\
0 & I
\end{bmatrix}\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} u, \quad \text{where } u = \begin{bmatrix}
w_1 |w_1| \\
\vdots \\
w_n |w_n|
\end{bmatrix}
\]

- all propellers are coplanar \(\Rightarrow F_1\) is rank deficient

\[
F_1 = cf \begin{bmatrix}
0 & \cdots & 0 \\
0 & \cdots & 0 \\
1 & \cdots & 1
\end{bmatrix} = cf \begin{bmatrix}
0^T \\
0^T \\
1^T
\end{bmatrix}
\]

- the control force is

\[
cf R^W_B \begin{bmatrix}
0 \\
0 \\
1^T u
\end{bmatrix}
\]

- it can be arbitrarily oriented only changing the whole-body orientation \(R^W_B\)
- the propeller speeds \(u\) control only the amplitude of the force
Putting (5) and (6) in (4):

\[
\begin{bmatrix}
m\ddot{p}_B^W \\
J\dot{\omega}_B^W
\end{bmatrix} = - \begin{bmatrix}
\omega_B^W \times J\omega_B^W
\end{bmatrix} + \begin{bmatrix}
R_B^W & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} u,
\]

where \( u = \begin{bmatrix} w_1 |w_1| \\
\vdots \\
w_n |w_n| \end{bmatrix} \)

- If coplanarity assumption is relaxed then \( \Rightarrow F_1 \) can be made full-rank

\[
F_1 = c_f \begin{bmatrix}
* & * & \cdots & * \\
* & * & \cdots & * \\
* & * & \cdots & * \\
\end{bmatrix}
\]

- the control force is

\[
R_B^W F_1 u = c_f R_B^W \begin{bmatrix}
* & * & \cdots & * \\
* & * & \cdots & * \\
* & * & \cdots & *
\end{bmatrix} u
\]

- using \( u \), both orientation and amplitude of the force can be decided independently of the whole-body orientation \( R_B^W \)
Examples of Fully-actuated platforms (I)

quadrotor + tilting propellers


Examples of Fully-actuated platforms (I)

**quadrotor + tilting propellers**


**planar hexarotor with tilted propellers**

Examples of Fully-actuated platforms (II)

4+4 orthogonal rotors


Examples of Fully-actuated platforms (II)

4+4 orthogonal rotors\(^5\)


Inverse Dynamics Approach

Given a reference pose (6D) trajectory:

- \( p_{Br}^W(t) \) (position of the CoM)
- \( R_{Br}^W(t) \) (orientation of the main body)

Dynamics:

\[
\begin{bmatrix}
  mI & 0 \\
  0 & J
\end{bmatrix}
\begin{bmatrix}
  \ddot{p}_B^W \\
  \dot{\omega}_B^W
\end{bmatrix}
= - \begin{bmatrix}
  \omega_B^W \times J \omega_B^W \\
  \frac{m ge_3}{\omega_B^W} \\
\end{bmatrix}
+ \begin{bmatrix}
  R_B^W & 0 \\
  0 & I
\end{bmatrix}
\begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix}
\]

Inverse dynamics:

\[
u = \begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix}^+ \begin{bmatrix}
  R_B^W & 0 \\
  0 & I
\end{bmatrix} \left( \begin{bmatrix}
  mI & 0 \\
  0 & J
\end{bmatrix} \left( \begin{bmatrix}
  \ddot{p}_{Br}^W \\
  \dot{\omega}_{Br}^W
\end{bmatrix} + v \right) + \begin{bmatrix}
  \frac{m ge_3}{\omega_B^W} \\
  \omega_B^W \times J \omega_B^W
\end{bmatrix} \right)
\]

Exactly linearized error system

\[
\begin{bmatrix}
  \ddot{p}_B^W - \ddot{p}_{Br}^W \\
  \dot{\omega}_B^W - \dot{\omega}_{Br}^W
\end{bmatrix}
= v
\]

then use any linear-systems control law for \( v \) to steer \( p_B^W \rightarrow p_{Br}^W(t) \) and \( R_B^W \rightarrow R_{Br}^W(t) \)
Inverse Dynamics Approach

Given a reference pose (6D) trajectory:
- $p^W_{Br}(t)$ (position of the CoM)
- $R^W_{Br}(t)$ (orientation of the main body)

Dynamics:

\[
\begin{bmatrix}
    mI & 0 \\
    0 & J
\end{bmatrix}
\begin{bmatrix}
    \ddot{p}^W_B \\
    \dot{\omega}^W_B
\end{bmatrix} = - \begin{bmatrix}
    mge_3 \\
    \omega^W_B \times J\omega^W_B
\end{bmatrix} + \begin{bmatrix}
    R^W_B & 0 \\
    0 & I
\end{bmatrix}
\begin{bmatrix}
    F_1 \\
    F_2
\end{bmatrix} u,
\]
where $u = \begin{bmatrix}
w_1 |w_1| \\
\vdots \\
w_n |w_n|
\end{bmatrix}
$

Inverse dynamics:

\[
u = \begin{bmatrix}
    F_1 \\
    F_2
\end{bmatrix}^+ \begin{bmatrix}
    R^W_B & 0 \\
    0 & I
\end{bmatrix} \left( \begin{bmatrix}
    mI & 0 \\
    0 & J
\end{bmatrix} \left( \begin{bmatrix}
    \ddot{p}^W_{Br} \\
    \dot{\omega}^W_{Br}
\end{bmatrix} + v \right) + \begin{bmatrix}
    mge_3 \\
    \omega^W_B \times J\omega^W_B
\end{bmatrix} \right) + \mathcal{N} \left( \begin{bmatrix}
    F_1 \\
    F_2
\end{bmatrix}^+ \right)\]

Exactly linearized error system

\[
\begin{bmatrix}
    \ddot{p}^W_B - \ddot{p}^W_{Br} \\
    \dot{\omega}^W_B - \dot{\omega}^W_{Br}
\end{bmatrix} = v
\]

then use any linear-systems control law for $v$ to steer $p^W_B \rightarrow p^W_{Br}(t)$ and $R^W_B \rightarrow R^W_{Br}(t)$
Applications of the Inverse Dynamics Approach: Quadrotor w/ Tilt. Prop.

quadrotor + tilting propellers

The wrench exerted by the propellers has several limitations

- **maximum speed** $\sim \frac{\text{maximum motor torque}}{\text{propeller drag}}$
  (considered in this talk)

- **only positive speeds** due to non-symmetric propeller shape
  (considered in this talk)

- **maximum/minimum speed rate** $\sim \frac{\text{maximum/minimum motor torque}}{\text{motor/propeller inertia}}$
  (non-considered in this talk)
The wrench exerted by the propellers has **several limitations**

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**Inverse dynamics:**

- desired wrench obtained by **matrix (pseudo)inversion**
- set of feasible forces not considered
- the **smaller** the cant angles the **larger** the input **forces**

**Inverse dynamics approach may lead to unfeasible propeller speeds** (\( >> 0 \) or \( < 0 \))
Set of Feasible Forces

How to **overcome the drawbacks** of the previous approach?

Using a novel method presented here\(^7\) \(^8\)

Let’s look at the dynamics while following any trajectory \(\mathbf{p}_B^W(t)\) with \(\mathbf{R}_B^W(t)\)

\[
\begin{bmatrix}
\dot{\mathbf{p}}_B^W + m g e_3 \\
\mathbf{J} \dot{\mathbf{\omega}}_B^W + \mathbf{\omega}_B^W \times \mathbf{J} \mathbf{\omega}_B^W
\end{bmatrix} = \begin{bmatrix} \mathbf{R}_B^W \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix} \mathbf{u},
\]
where \(e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\), \(\mathbf{u} \in \mathcal{U}\) (admissible inputs)

It is interesting to **analyze** the set of admissible input forces when

- the input torque is **constrained**, i.e., \(\mathbf{F}_2 \mathbf{u} = \mathbf{\tau}\) for a given \(\mathbf{\tau}\)
- the **propeller speeds** are **feasible**, i.e., \(\mathbf{u} \in \mathcal{U}\)

\[\mathcal{U}_1(\mathbf{\tau}) = \{ \mathbf{u}_1 = \mathbf{F}_1 \mathbf{u} \text{ s.t. } \mathbf{F}_2 \mathbf{u} = \mathbf{\tau} \text{ and } \mathbf{u} \in \mathcal{U} \}\]


Set of **admissible input forces** (in body frame)

\[
\mathcal{U}_1(\tau) = \{ u_1 = F_1 u \text{ s.t. } F_2 u = \tau \text{ and } u \in \mathcal{U} \}
\]
Set of **admissible input forces** (in body frame)

\[ \mathcal{U}_1(\tau) = \{ u_1 = F_1 u \text{ s.t. } F_2 u = \tau \text{ and } u \in \mathcal{U} \} \]

Octorotor for \( \tau = 0 \) (approximation)
Set of \textbf{admissible input forces} (in body frame)

\[ \mathcal{U}_1(\tau) = \{ u_1 = F_1 u \text{ s.t. } F_2 u = \tau \text{ and } u \in \mathcal{U} \} \]

Hexarotor for $\tau = 0$ for different cant angles $\alpha$
Hierarchical Approach: Position Controller

Position **tracking errors**

\[ e_p = p_B - p_r, \quad \text{and} \quad e_v = \dot{p}_B - \dot{p}_r. \] (7)

**Reference force** vector

\[ f_r = m\ddot{p}_r + mg_e_3 - K_p e_p - K_v e_v, \] (8)

where \( K_p \) and \( K_v \) are positive diagonal gain matrixes

**Remark**

If \( u \) could always be chosen such that:

\[ \mathbf{R}_B^W \mathbf{F}_1 u = f_r = m\ddot{p}_r + mg_e_3 - K_p e_p - K_v e_v, \]

then \( e_p \rightarrow 0 \) and \( e_v \rightarrow 0 \) exponentially.

However, this is not always possible, due to the **input saturation**

**Idea**

**Relax the orientation tracking** if the position tracking is not possible
Position-Tracking-Compatible Orientations

$$\mathcal{R}(f_r) = \{ R \in SO(3) \mid \exists u \in \mathcal{U}, \ RF_1 u = f_r \land F_2 u = 0 \}$$  \hspace{1cm} (9)

Set of orientations of the main body that allow to exert $f_r$ on the CoM while ensuring

- **propeller speeds feasibility**, i.e., $u \in \mathcal{U}$
- a **given input torque**, e.g., $F_2 u = 0$

---

Position–orientation compatibility

Simultaneous tracking of both $p_r(t)$ and $R_r(t)$ is possible

$$\iff R_r(t) \in \mathcal{R}(f_r(t))$$
Position-Tracking-Compatible Orientations

\[ \mathcal{R}(f_r) = \{ R \in SO(3) \mid \exists u \in U, \ RF_1 u = f_r \land F_2 u = 0 \} \] (9)

Set of orientations of the main body that allow to exert \( f_r \) on the CoM while ensuring

- propeller speeds feasibility, i.e., \( u \in U \)
- a given input torque, e.g., \( F_2 u = 0 \)

Position–orientation compatibility

Simultaneous tracking of both \( p_r(t) \) and \( R_r(t) \) is possible

\[ \iff \]

\[ R_r(t) \in \mathcal{R}(f_r(t)) \]

Non-compatibility \( \Rightarrow \) relax the orientation tracking

- compute a new desired orientation \( R_d \in SO(3) \)
- reference control torque \( \tau_r = \omega_B \times J \omega_B - K_{Re} e_R - K_{\omega} \omega_B \)

where \( e_R \) is the orientation error in \( SO(3) \) defined as

\[ e_R = \frac{1}{2} (R_d^T R_B - R_B^T R_d)^\vee \]
Control algorithm in pills

At every $t$, given $p_r, R_r$:

1. compute $f_r = m\ddot{p}_r + mg e_3 - K_p e_p - K_v e_v$
Control algorithm in pills

At every $t$, given $p_r, R_r$:

1. compute $f_r = m\ddot{p}_r + mg e_3 - K_p e_p - K_v e_v$

2. solve $R_d = \arg\min_{R \in \mathcal{R}(f_r)} \text{dist}(R, R_r)$
Control Algorithm

Control algorithm in pills

At every $t$, given $p_r, R_r$:

1. compute $f_r = m\ddot{p}_r + mge_3 - K_pe_p - K_ve_v$
2. solve $R_d = \arg\min_{R \in \mathcal{R}(f_r)} \text{dist}(R, R_r)$
3. compute $\tau_r = \omega_B \times J\omega_B - K_R e_R - K_\omega \omega_B$, to track $R_d$
Control algorithm in pills

At every $t$, given $p_r, R_r$:

1. compute $f_r = m\ddot{p}_r + mge_3 - K_pe_p - K_ve_v$
2. solve $R_d = \arg\min_{R \in \mathcal{R}(f_r)} \text{dist}(R, R_r)$
3. compute $\tau_r = \omega_B \times J\omega_B - K_R e_R - K_\omega \omega_B$, to track $R_d$
4. compute $u$ to implement $\tau_r$ and $f_r$
Exploit the 6 DoFs for position and orientation independent regulation
Experiments: Linearly Increasing Position Acceleration
Current and Future Works
Physical Interaction with a Rigidly-attached Tool

- momentum-based external \textit{wrench observer}
- 6D \textit{admittance control} at the \textit{tooltip}

Rope-pulling
- unstable operation for a co-planar multirotor
- (3D orientation dynamics made stiffer than 3D translation one)

Peg-in-hole
- unstable operation for a co-planar multirotor
- aerial manipulators with a fully-actuated base

preliminary simulation
control of a ‘truly’ redundant aerial manipulator
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For related work, visit http://homepages.laas.fr/afranchi/robotics/

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Questions?