Protocentric Aerial Manipulators: Flatness Proofs and Simulations

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Technical Attachment to:

”Differential Flatness and Control of Protocentric Aerial Manipulators with Any Number of Arms and Mixed Rigid-/Elastic-Joints”

2016 IEEE/RSJ International Conference on Intelligent Robots and Systems, Daejeon, South Korea, October 2016

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INTRODUCTION

This document is a technical attachment to [1] for explicit proofs of the propositions (Sec. I), and the detailed computation of the system dynamics (Sec. II). Furthermore, here we also provide extensive simulation results for [1] in Section III.

Aerial manipulators are an example of aerial vehicles physically interacting with the external environment. For the reader interested in this rapidly expanding and broad topic we also suggest the reading of [2], where a force nonlinear observer for aerial vehicles is proposed, of [3], where an IDA-PBC controller is used for modulating the physical interaction of aerial robots, of [4], [5] where fully actuated platforms for full wrench exertion are presented, of [6]–[8] where the capabilities of exerting forces with a tool are studied, and of [1], [9], [10] where aerial manipulators with elastic-joint arms are modeled and their controllability properties discovered. Another example of physical interaction are tethered aerial vehicles, the interested reader is referred to [11], [12], where flatness, controllability and observability is studied, to [13] where the case of a moving base is thoroughly analyzed, to [14], [15] where the case of multiple tethered vehicles is investigated, and to [16] where a passive tether is used for robust landing on sloped surfaces.

I. PROOFS OF THE PROPOSITIONS

Recall the proposition in Case R (Sec. III) of [1]:

Proposition 1. $\mathbf{y} = [\mathbf{p}_0^T\ \mathbf{q}_r^T]^T \in \mathbb{R}^{(n+2)}$ *is a flat output of the system described in* (7) *of [1]. It is necessary that* $\mathbf{y}$ *is 4 times differentiable. Equivalently, the total relative degree is* $r = 4n+8$.

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This work has been partially funded by the European Union’s Horizon 2020 research and innovation programme under grant agreement No 644271 AEROARMS

A. Proof of Proposition 1

Consider the CoM position of the overall system

$$\mathbf{p}_c = \frac{1}{m_s}\left(m_0\mathbf{p}_0 + \sum_{j=1}^{m}\left(\sum_{i=1}^{n^j}(m_{ij}\mathbf{p}_{ij} + m_{m_{ij}}\mathbf{p}_{m_{ij}})\right)\right). \quad (1)$$

Hence, the implicit dependency of $\mathbf{p}_c$ can be given as $\mathbf{p}_c = \mathbf{p}_c(\mathbf{p}_0,\mathbf{q}_r) = \mathbf{p}_c(\mathbf{y})$. Notice that

$$m_s\ddot{\mathbf{p}}_c = \begin{bmatrix}-\sin(\theta_0)\\ -\cos(\theta_0)\end{bmatrix}u_t + \begin{bmatrix}0\\ m_s g\end{bmatrix}. \quad (2)$$

We then define the vector $\mathbf{w} = \mathbf{w}(\mathbf{y},\dot{\mathbf{y}},\ddot{\mathbf{y}}) = \ddot{\mathbf{p}}_c - [0\ g]^T = [w_x\ w_z]^T \in \mathbb{R}^2$, which is a function of flat outputs. It is clear that $\mathbf{w} = -\frac{u_t}{m_s}[\sin(\theta_0)\ \cos(\theta_0)]^T$. Therefore $\theta_0 = \text{atan2}(-w_x,-w_z)$ and $u_t = m_s||\mathbf{w}||$. Furthermore, differentiating $\theta_0(w_x,w_z)$ we obtain $\dot{\theta}_0(w_x,w_z,\dot{w}_x,\dot{w}_z)$ and $\ddot{\theta}_0(w_x,w_z,\dot{w}_x,\dot{w}_z,\ddot{w}_x,\ddot{w}_z)$, which are all functions of the derivatives of $\mathbf{p}_c$ from the second up to the fourth order.

Now considering the rotational dynamics of the last link of each manipulator, we can retrieve the n^μ-th motor torque as

$$\tau_{n^\mu} = \mathbf{m}_{0n^\mu}^T(\theta_{0n^\mu})\ddot{\mathbf{p}}_0 + \sum_{l=1}^{n^\mu-1} m_{ln^\mu}(\theta_{0l^\mu},\theta_{0n^\mu})\ddot{\theta}_{0l^\mu} + \\ + \mathbb{J}_{n^\mu}\ddot{\theta}_{0n^\mu} + c_{r_{n^\mu}}(\mathbf{q}_{r^\mu},\dot{\mathbf{q}}_{r^\mu}) + g_{r_{n^\mu}}(\theta_{0n^\mu}), \quad (3)$$

where $c_{r_{n^\mu}}$ and $g_{r_{n^\mu}}$ are the n^μ-th elements of vectors $\mathbf{c}_r$ and $\mathbf{g}_r$, which are corresponding to the Coriolis and gravitational forces acting on the center of the n-th link of the μ-th manipulator, respectively (See Sec. II-C for the details). Hence, τ_{n^μ} can be represented solely as a function of the flat outputs $\mathbf{y}$ and of its time derivatives $\dot{\mathbf{y}}$, $\ddot{\mathbf{y}}$.

Starting from the last joint, we can recursively compute all the joint torques of the μ-th arm as functions of the flat outputs up to their final derivative. This means we can write the control torque of the ν-th joint of the μ-th manipulator in form of $\tau_{\nu^\mu} = \tau_{\nu^\mu}(\mathbf{y},\dot{\mathbf{y}},\ddot{\mathbf{y}})$, and $\nu^\mu = \{1,2,\cdots,n^\mu\}$, using

$$\tau_{\nu^\mu} = \tau_{\nu^\mu+1} + \mathbf{m}_{0\nu^\mu}^T(\theta_{0\nu^\mu})\ddot{\mathbf{p}}_0 + c_{r_{\nu^\mu}}(\mathbf{q}_{r^\mu},\dot{\mathbf{q}}_{r^\mu}) + \mathbb{J}_{\nu^\mu}\ddot{\theta}_{0\nu^\mu} + \\ + g_{r_{\nu^\mu}}(\theta_{0\nu^\mu}) + \sum_{l=1,l\neq\nu^\mu}^{n^\mu} m_{l\nu^\mu}(\theta_{0l^\mu},\theta_{0\nu^\mu})\ddot{\theta}_{0l^\mu}, \quad (4)$$

where it is clear that for $\nu^\mu = n^\mu$, $\tau_{\nu^\mu+1} = 0$, because the $(n^\mu+1)$-th motor does not exist. In this way, one can compute all the input torques of all the manipulators, until the very first ones, as functions of sole flat outputs and their finite numbers of derivatives. Hence, τ_{1^μ} will also be the sole function of the flat outputs as well. Then we can utilize τ_{1^μ} in the third equation of the system dynamics to compute u_r:

$$u_r = \mathbb{J}_0\ddot{\theta}_0 + \sum_{j=1}^{m} \tau_{1^j} - d_{G_x} u_t. \tag{5}$$

Notice that θ_0 and u_t have been computed above as functions of the flat output and a finite number of its derivatives only. Hence this holds for u_r too. Since $\ddot{\theta}_0$ is a function of $\ddddot{\mathbf{y}}$, then so is u_r, implying the relative degree of the system is four times the dimension of $\mathbf{y}$, i.e. $r = 4(2+n) = 8+4n$ (see also Sec. V of [1]). This concludes the the proof. $\square$

B. Proof of Proposition 2

Recall the proposition in Case E (Sec. IV) of [1]:

Proposition 2. $\mathbf{y} = [\mathbf{p}_0^T\ \mathbf{q}_{r_r}^T]^T \in \mathbb{R}^{(n+2)}$ *is a flat output of the system described in* (12) *of [1].*

The proof is analogous to that of Proposition 1. Knowing the fact that the CoM position of the overall system, given in (1) and its dynamics as in (2) are sole functions of the flat outputs and derivatives, we can write again $\theta_0 = \text{atan2}(-w_z, -w_x)$ and $u_t = m_s||\mathbf{w}||$. Furthermore, differentiating $\theta_0(w_x, w_z)$ we obtain $\dot{\theta}_0(w_x, w_z, \dot{w}_x, \dot{w}_z)$ and $\ddot{\theta}_0(w_x, w_z, \dot{w}_x, \dot{w}_z, \ddot{w}_x, \ddot{w}_z)$, which are all functions of the derivatives of $\mathbf{p}_c$ from the second up to the fourth order.

Assume that all elastic joints are linear. Consider the μ-th manipulator, and let's focus on the torque input of its last joint, i.e., τ_{n^μ}. If this last joint is rigid, then its expression is identical to (3), while $\theta_{m_{0n^\mu}}$ is clearly undefined. If instead this last, i.e., n^μ-th joint is elastic, then we first need to compute $\theta_{m_{0n^\mu}}$. This can be written from its link-side dynamics as

$$\theta_{m_{0n^\mu}} = \theta_{0n^\mu} + \frac{1}{k_{e_{n^\mu}}}\Bigg(\mathbf{m}_{0n^\mu}^T(\theta_{0n^\mu})\ddot{\mathbf{p}}_0 + g_{r_{n^\mu}} + c_{r_{n^\mu}}(\mathbf{q}_{r^\mu}, \dot{\mathbf{q}}_{r^\mu}) + \\ + \sum_{l=1}^{n^\mu-1} m_{ln^\mu}(\theta_{0l^\mu}, \theta_{0n^\mu})\ddot{\theta}_{0l^\mu} + (\mathbb{J}_{n^\mu} - J_{m_{n^\mu}})\ddot{\theta}_{0n^\mu}\Bigg). \tag{6}$$

Notice the similarity between this and (3). Hence, $\theta_{m_{0n^\mu}}$ is represented as a function of flat outputs and derivatives. Then, τ_{n^μ} is available from the last equation of the system dynamics of the μ-th manipulator:

$$\tau_{n^\mu} = J_{n^\mu}\ddot{\theta}_{m_{0n^\mu}} + k_{e_{n^\mu}}\theta_{m_{0n^\mu}} - k_{e_{n^\mu}}\theta_{0n^\mu}, \tag{7}$$

where $\theta_{m_{0n^\mu}}$ and its derivatives are available from (6). Hence, we see that τ_{n^μ} can always be represented as a function of flat outputs and derivatives, together with θ_{0n^μ} even when the n^μ-th joint (last joint of the μ-th manipulator) is elastic.

Now, let's focus on the generic link $\nu^\mu < n^\mu$. We can proceed recursively from top to bottom, assuming we have already computed $\tau_{\nu^\mu+1}$. If the ν^μ-th link is rigid, then its expression is identical to (4) and $\theta_{m_{0\nu^\mu}}$ is not defined. If it is elastic, we first need to compute $\theta_{m_{0\nu^\mu}}$. This can be done with:

$$\theta_{m_{0\nu^\mu}} = \theta_{0\nu^\mu} + \frac{1}{k_{e_{\nu^\mu}}}\Bigg(\tau_{\nu^\mu+1} + \mathbf{m}_{0\nu^\mu}^T(\theta_{0\nu^\mu})\ddot{\mathbf{p}}_0 + \\ + \sum_{l=1, l\neq\nu^\mu}^{n^\mu} m_{l\nu^\mu}(\theta_{0l^\mu}, \theta_{0\nu^\mu})\ddot{\theta}_{0l^\mu} + \\ + (\mathbb{J}_{\nu^\mu} - J_{m_{\nu^\mu}})\ddot{\theta}_{0\nu^\mu} + g_{r_{\nu^\mu}}(\theta_{0\nu^\mu}) + c_{r_{\nu^\mu}}(\dot{\mathbf{q}}_{r^\mu}, \mathbf{q}_{r^\mu})\Bigg). \tag{8}$$

Again, notice the similarity with (4). We observe that (8) can also be employed for $\nu^\mu = n^\mu$, simply setting the non-existing $\tau_{n^\mu+1}$ equal to zero. Then, τ_{ν^μ} can be easily computed from

$$\tau_{\nu^\mu} = J_{\nu^\mu}\ddot{\theta}_{m_{0\nu^\mu}} + k_{e_{\nu^\mu}}\theta_{m_{0\nu^\mu}} - k_{e_{\nu^\mu}}\theta_{0\nu^\mu}, \tag{9}$$

which can be directly employed also for $\nu^\mu = n^\mu$.

Until this point, we showed that all control torques of the μ-th robotic arm, regardless of their connection type (rigid or elastic), can be represented as sole functions of flat outputs and their derivatives. This means that the equations above are valid for the each robotic arm. Now, finally from the third equation of the system dynamics we retrieve

$$u_r = \mathbb{J}_0\ddot{\theta}_0 + \sum_{j=1}^{m} \tau_{1^j} - d_{G_x} u_t, \tag{10}$$

in which τ_{1^μ} is utilized from either (4) or (9), depending on the type of the actuation. Moreover $\ddot{\theta}_0$ and u_t are available from previous computations. Hence, PVTOL torque is also represented using only the flat outputs. This concludes the sketch of the proof. $\square$

Remark 1. *Notice the different relative degree of the dependencies of τ_{ν^μ} given in* (9) *on the flat outputs for different values of ν^μ. Assume for instance that both the $(n^\mu-1)$-th and the n^μ-th link are elastic. Then from bottom to top,*

- *First: from* (7)*, we see that τ_{n^μ} is a function of $\ddot{\theta}_{m_{0n^\mu}}$; while $\theta_{m_{0n^\mu}}$ is a function of $\ddot{\mathbf{p}}_0$ and $\ddot{\mathbf{q}}_{r^\mu}$, making τ_{n^μ} itself a function of $\ddddot{\mathbf{p}}_0$ and $\ddddot{\mathbf{q}}_{r^\mu}$.*
- *Second: from* (7)*, $\tau_{n^\mu-1}$ is a function of $\ddot{\theta}_{m_{0(n^\mu-1)}}$. But in* (8)*, from recursion, $\theta_{m_{0(n^\mu-1)}}$ is a function of τ_{n^μ}, making $\ddot{\theta}_{m_{0(n^\mu-1)}}$, and thus $\tau_{n^\mu-1}$, a function of $\ddot{\tau}_{n^\mu}$. Knowing from the fist step above τ_{n^μ} is a function of $\ddddot{\mathbf{p}}_0$ and $\ddddot{\mathbf{q}}_{r^\mu}$, we find $\tau_{n^\mu-1}$ as a function of $\mathbf{p}_0^{(6)}$ and $\mathbf{q}_{r^\mu}^{(6)}$, which are the* sixth *time derivatives.*

In general, for a fully elastic manipulator, an increase of two relative degrees per link is to be expected.

Remark 2. *We notice that the orientation of the PVTOL, i.e., θ_0, is not part of the flat outputs, conceivably due to the under-actuation of the flying robot. This motivated us to use the absolute representation of the manipulator joint angles, which makes the control torques appear recursively in the manipulator dynamics. Notice from the remark above that while this is not a problem for Case R, for Case E this increases the relative degrees.*

Hence it is worth noting that using a fully actuated aerial robot might be beneficial if manipulators with compliant *actuators are to be used for specific tasks, e.g., safe physical interaction. This does not apply of course if the robotic arm is rigidly actuated. Further study on this remark is in the scope of our future studies.*

From Remark 1, it is possible to compute the relative degree of the overall system. Recalling that k^μ is the number of elastic joints in link μ, and defining $\tilde{k}^\mu = \max(1, k^\mu)$, then

$$r = 4 + 4\max_{\mu} \tilde{k}^\mu + \sum_{\mu=1}^{m} (2 + 2\tilde{k}^\mu) n^\mu \tag{11}$$

where it can be seen a quadratic dependence on the number of elastic joints. The term $\max_{\mu} \tilde{k}^\mu$ returns the value $\tilde{k}^\mu$ for the manipulator arm with the highest number of elastic joint. For a better understanding, let us give the following examples:

Example 1. *Protocentric Aerial Manipulator (PAM) with m number of manipulator arms, each having only rigid actuators. Notice that this actually corresponds to Case R. Since there are no compliant actuators, $k = k^\mu = \max_{\mu} \tilde{k}^\mu = 0$, and so $\tilde{k}^\mu = 1$. Then* (11) *becomes $r = 4+4+4n = 8+4n$, which is a perfect match to Sec. I-A.*

Example 2. *PAM with m number of manipulator arms, each having some rigid actuators and each having only one compliant actuator. This means that $k^\mu = 1$. Then $\tilde{k}^\mu = 1$, and it means $r = 4+4+4n = 8+4n$. This means that if each manipulator has only one elastic joint, then the total relative degree is the same with the case if all joints were rigid (this result is in line with that of [9]).*

Example 3. *PAM with 2 number of manipulator arms with mixed rigid-/elastic- joints. Let's say for the first arm it is $n^1 = 5, k^1 = 4$ and for the second one it is $n^2 = 7, k^2 = 3$. This is a highly complicated PAM, with two arms in total 12 actuators and links, and 7 compliant joints. Then we see that $\max_{\mu} \tilde{k}^\mu = 4$. Hence the total relative degree of the system is* $r = 4 + 4*4 + (2+2*4)*5 + (2+2*3)*7 = 126$.

II. Computation of the System Dynamics

A. Time-Varying Coordinates of the system components

Let us start with the positions and the orientations of the elements of the robotic system, which consist of a PVTOL equipped with m fully actuated manipulators; the μ-th manipulator has n^μ DoFs. Then the absolute orientation of the ν^μ-th joint will be $\theta_{0\nu^\mu} = \theta_0 + \sum_{i=1}^{\nu^\mu} \theta_{i^\mu}$. For example, the absolute orientation of the second link of the third manipulator will be $\theta_{02^3} = \theta_0 + \theta_{1^3} + \theta_{2^3}$. Moreover, the rotation matrix corresponding to $\theta_{0\nu^\mu}$ will be $\mathbf{R}(\theta_{0\nu^\mu}) = \mathbf{R}_0(\theta_0)\prod_{i=1}^{\nu^\mu} \mathbf{R}_i(\theta_{i^\mu})$, where for $\theta_* \in \mathbb{R}$ it is

$$\mathbf{R}_* = \begin{pmatrix} \cos\theta_* & \sin\theta_* \\ -\sin\theta_* & \cos\theta_* \end{pmatrix} \in \mathrm{SO}(2).$$

Then we can write the following distance vectors

$$\begin{aligned}
\mathbf{p}_G &= \mathbf{p}_0 + \mathbf{R}_0 \mathbf{d}_G \\
\mathbf{p}_{n^\mu} &= \mathbf{p}_0 + \mathbf{R}_{01^\mu}\bar{\mathbf{d}}_{1^\mu} + \cdots + \mathbf{R}_{0(n^\mu-1)}\bar{\mathbf{d}}_{(n^\mu-1)} + \mathbf{R}_{0n^\mu}\mathbf{d}_{n^\mu} \\
&= \mathbf{p}_0 + \underbrace{\sum_{i=1}^{n^\mu-1} \mathbf{R}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu}}_{:=0,\quad if\ n^\mu=1} + \mathbf{R}_{0n^\mu}\mathbf{d}_{n^\mu} \\
\mathbf{p}_{e^\mu} &= \mathbf{p}_0 + \sum_{i=1}^{n^\mu} \mathbf{R}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu},
\end{aligned}$$

where $\bar{\mathbf{d}}_i = \mathbf{d}_i + \tilde{\mathbf{d}}_i$, $i = \{1, 2, ..., n^\mu\}$, and for the motors,

$$\begin{aligned}
\mathbf{p}_{m_{n^\mu}} &= \mathbf{p}_0 + \mathbf{R}_{01^\mu}\bar{\mathbf{d}}_{1^\mu} + \mathbf{R}_{02^\mu}\bar{\mathbf{d}}_{2^\mu} + \cdots + \mathbf{R}_{0(n^\mu-1)}\bar{\mathbf{d}}_{(n^\mu-1)} \\
&= \mathbf{p}_0 + \underbrace{\sum_{i=1}^{n^\mu-1} \mathbf{R}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu}}_{:=0,\quad if\ n^\mu=1}.
\end{aligned}$$

Notice that this is due to the Assumption A.3 of [1]. The following gives the linear velocities for the ν^μ-the link and motor, and the end-effector

$$\begin{aligned}
\dot{\mathbf{p}}_{\nu^\mu} &= \dot{\mathbf{p}}_0 + \underbrace{\sum_{i=1}^{\nu^\mu-1} \bar{\mathbf{R}}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu}\dot{\theta}_{0i^\mu}}_{:=0,\quad if\ \nu^\mu=1} + \bar{\mathbf{R}}_{0\nu^\mu}\mathbf{d}_{\nu^\mu}\dot{\theta}_{0\nu^\mu} \\
\dot{\mathbf{p}}_{m_{\nu^\mu}} &= \dot{\mathbf{p}}_0 + \underbrace{\sum_{i=1}^{\nu^\mu-1} \bar{\mathbf{R}}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu}\dot{\theta}_{0i^\mu}}_{:=0,\quad if\ \nu^\mu=1} \\
\dot{\mathbf{p}}_{e^\mu} &= \dot{\mathbf{p}}_0 + \sum_{i=1}^{n^\mu} \bar{\mathbf{R}}_{0i^\mu}\bar{\mathbf{d}}_{i^\mu}\dot{\theta}_{0i^\mu},
\end{aligned}$$

where from the definition $\nu^\mu = \{1, 2, \cdots, i, \cdots, n^\mu\}$, and $\bar{\mathbf{R}}_* = \frac{\partial \mathbf{R}_*}{\partial \theta_*}$.

B. Energies and the Inertia Matrix

Now we can write the energy of the system. We start by considering rigid manipulators. The kinetic energy is

$$K = \underbrace{\frac{1}{2}m_0\dot{\mathbf{p}}_0^T\dot{\mathbf{p}}_0 + \frac{1}{2}J_0\dot{\theta}_0^2}_{\text{PVTOL base}} + \underbrace{K_m}_{\text{manipulators}}, \tag{12}$$

where

$$K_m = \sum_{j=1}^{m}\left(\frac{1}{2}\sum_{i=1}^{n^j}\left(m_{ij}\dot{\mathbf{p}}_{ij}^T\dot{\mathbf{p}}_{ij} + m_{m_{ij}}\dot{\mathbf{p}}_{m_{ij}}^T\dot{\mathbf{p}}_{m_{ij}} + (J_{ij} + J_{m_{ij}})\dot{\theta}_{0i^j}^2\right)\right).$$

The potential energy is

$$V = -gm_0\mathbf{p}_0.\mathbf{e}_2 - g\sum_{j=1}^{m}\left(\sum_{i=1}^{n^j}\left[m_{ij}\mathbf{p}_{ij} + m_{m_{ij}}\mathbf{p}_{m_{ij}}\right]\right).\mathbf{e}_2, \tag{13}$$

where $\mathbf{e}_2 = [0\ 1]^T$. Let us now write the well known Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{f} = \mathbf{G}\mathbf{u}, \tag{14}$$

where $L=K-V$, can be computed using above. It is clear that using $K=\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$ we can find the inertia matrix as

$$\mathbf{M}=\begin{pmatrix}\mathbf{M}_p & * \\ \mathbf{M}_{pr} & \mathbf{M}_r\end{pmatrix}=\mathbf{M}^T\in\mathbb{R}^{(3+n)\times(3+n)}, \tag{15}$$

where

$$\mathbf{M}_p=\begin{pmatrix}m_s & * & * \\ 0 & m_s & * \\ 0 & 0 & J_0\end{pmatrix}=\mathbf{M}_p^T\in\mathbb{R}^{3\times 3}$$

is the pvtol-side inertia matrix; the sum m_s of all masses is given by

$$m_s=m_0+\sum_{j=1}^{m}\Big(\sum_{i=1}^{n^j}m_{ij}+m_{m_{ij}}\Big).$$

The arm-side inertia matrix is

$$\mathbf{M}_r=\begin{pmatrix}\mathbf{M}_{r^1} & * & \cdots & * \\ 0 & \mathbf{M}_{r^2} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}_{r^m}\end{pmatrix}=\mathbf{M}_r^T\in\mathbb{R}^{n\times n},$$

where for the μ-th manipulator it is

$$\mathbf{M}_{r^\mu}=\begin{pmatrix}\mathbb{J}_{1^\mu} & * & \cdots & * \\ m_{12^\mu}(\theta_{01^\mu},\theta_{02^\mu}) & \mathbb{J}_{2^\mu} & \cdots & * \\ \vdots & \vdots & \ddots & * \\ m_{1n^\mu}(\theta_{01^\mu},\theta_{0n^\mu}) & m_{2n^\mu}(\theta_{02^\mu},\theta_{0n^\mu}) & \cdots & \mathbb{J}_{n^\mu}\end{pmatrix}$$
$$=\mathbf{M}_{r^\mu}^T\in\mathbb{R}^{n^\mu\times n^\mu}.$$

For the ν-th joint of the μ-th manipulator it is

$$\mathbb{J}_{\nu^\mu}=J_{\nu^\mu}+J_{m_{\nu^\mu}}+m_{\nu^\mu}\boldsymbol{\alpha}_{\nu^\mu}^T\boldsymbol{\alpha}_{\nu^\mu}+\underbrace{\sum_{i=\nu^\mu+1}^{n^\mu}(m_i+m_{m_i})\bar{\boldsymbol{\alpha}}_{\nu^\mu}^T\bar{\boldsymbol{\alpha}}_{\nu^\mu}}_{:=0,\quad \text{if}\quad \nu=n^\mu},$$

$$\nu^\mu=\{1,...,n^\mu\}$$

and for the coupling between the ν-th and ξ-th joints of the μ-th manipulator it is

$$m_{\nu\xi^\mu}(\theta_{0\nu^\mu},\theta_{0\xi^\mu})=m_{\xi^\mu}\bar{\boldsymbol{\alpha}}_{\nu^\mu}^T\boldsymbol{\alpha}_{\xi^\mu}+\underbrace{\sum_{i=\xi^\mu+1}^{n^\mu}(m_i+m_{m_i})\bar{\boldsymbol{\alpha}}_{\nu^\mu}^T\bar{\boldsymbol{\alpha}}_{\xi^\mu}}_{:=0,\quad if\ \ \xi^\mu=n^\mu},$$

$$\begin{aligned}\nu^\mu&=\{1,...,n^\mu-1\},\\ \xi^\mu&=\{1,...,n^\mu\},\ \nu^\mu<\xi^\mu\\ \boldsymbol{\alpha}_{\nu^\mu}(\theta_{0\nu^\mu})&=\bar{\mathbf{R}}(\theta_{0\nu^\mu})\mathbf{d}_{\nu^\mu}\\ \bar{\boldsymbol{\alpha}}_{\nu^\mu}(\theta_{0\nu^\mu})&=\bar{\mathbf{R}}(\theta_{0\nu^\mu})\bar{\mathbf{d}}_{\nu^\mu}.\end{aligned}$$

Notice that $\mathbb{J}_{\nu^\mu}$ is state independent. The coupling term between the PVTOL and the arm side inertia is given with the following,

$$\mathbf{M}_{pr}=\Big(\mathbf{M}_{pr^1}^T\quad \mathbf{M}_{pr^2}^T\quad\cdots\quad\mathbf{M}_{pr^m}^T\Big)^T\in\mathbb{R}^{n\times 3},$$

where for the μ-th manipulator it is

$$\mathbf{M}_{pr^\mu}=\begin{pmatrix}\mathbf{m}_{01^\mu}^T(\theta_{01^\mu}) & 0 \\ \mathbf{m}_{02^\mu}^T(\theta_{02^\mu}) & 0 \\ \mathbf{m}_{03^\mu}^T(\theta_{03^\mu}) & 0 \\ \vdots & \vdots \\ \mathbf{m}_{0n^\mu}^T(\theta_{0n^\mu}) & 0\end{pmatrix}\in\mathbb{R}^{n^\mu\times 3}.$$

For the ν-th joint of the μ-th manipulator:

$$\mathbf{m}_{0\nu^\mu}(\theta_{0\nu^\mu})=m_{\nu^\mu}\boldsymbol{\alpha}_{\nu^\mu}+\underbrace{\sum_{i=\nu^\mu+1}^{n^\mu}(m_i+m_{m_i})\bar{\boldsymbol{\alpha}}_{\nu^\mu}}_{:=0,\quad if\ \ \nu^\mu=n^\mu}\in\mathbb{R}^{2\times 1}.$$

This completes the computation of the generalized inertia matrix.

C. Gravitational and Coriolis/Centrifugal Forces

Since $L=K-V$, we can write

$$\frac{d}{dt}\Big(\frac{\partial L}{\partial\dot{\mathbf{q}}}\Big)-\frac{\partial L}{\partial\mathbf{q}}=\mathbf{M}\ddot{\mathbf{q}}+\underbrace{\dot{\mathbf{M}}\dot{\mathbf{q}}-\frac{\partial L}{\partial\mathbf{q}}}_{\mathbf{c}+\mathbf{g}}=\mathbf{M}\ddot{\mathbf{q}}+\underbrace{\dot{\mathbf{M}}\dot{\mathbf{q}}-\frac{\partial K}{\partial\mathbf{q}}}_{\mathbf{c}}+\underbrace{\frac{\partial V}{\partial\mathbf{q}}}_{\mathbf{g}}=\mathbf{f}.$$

Hence from the fact that $\mathbf{g}=\frac{\partial V}{\partial\mathbf{q}}$ and using (13) we can find the gravitational forces as presented in [1].

The Coriolis/centrifugal forces are shown above as

$$\mathbf{c}=\dot{\mathbf{M}}\dot{\mathbf{q}}-\frac{\partial K}{\partial\mathbf{q}}. \tag{16}$$

For the first term of the right side of the equality, we have the following

$$\dot{\mathbf{M}}=\begin{pmatrix}\dot{\mathbf{M}}_p & * \\ \dot{\mathbf{M}}_{pr} & \dot{\mathbf{M}}_r\end{pmatrix}=\dot{\mathbf{M}}^T\in\mathbb{R}^{(3+n)\times(3+n)},$$

$$\dot{\mathbf{M}}_{pr}=\begin{pmatrix}\dot{\mathbf{M}}_{pr^1}\\ \dot{\mathbf{M}}_{pr^2}\\ \vdots\\ \dot{\mathbf{M}}_{pr^m}\end{pmatrix}\in\mathbb{R}^{n\times 3},$$

where

$$\dot{\mathbf{M}}_p=\mathbf{0}_3,\quad \dot{\mathbf{M}}_{pr^\mu}=\begin{pmatrix}\dot{\mathbf{m}}_{01^\mu}(\theta_{01^\mu})^T & 0 \\ \dot{\mathbf{m}}_{02^\mu}(\theta_{02^\mu})^T & 0 \\ \dot{\mathbf{m}}_{03^\mu}(\theta_{03^\mu})^T & 0 \\ \vdots & \vdots \\ \dot{\mathbf{m}}_{0n^\mu}(\theta_{0n^\mu})^T & 0\end{pmatrix}\in\mathbb{R}^{n^\mu\times 3},$$

and

$$\dot{\mathbf{M}}_r=\begin{pmatrix}\dot{\mathbf{M}}_{r^1} & * & \cdots & * \\ 0 & \dot{\mathbf{M}}_{r^2} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \dot{\mathbf{M}}_{r^m}\end{pmatrix}=\dot{\mathbf{M}}_r^T\in\mathbb{R}^{n\times n},$$

where

$$\dot{\mathbf{M}}_{r^\mu}=\begin{pmatrix}0 & * & \cdots & * \\ \dot{m}_{12^\mu}(\theta_{01^\mu},\theta_{02^\mu}) & 0 & \cdots & * \\ \vdots & \vdots & \ddots & * \\ \dot{m}_{1n^\mu}(\theta_{01^\mu},\theta_{0n^\mu}) & \dot{m}_{2n^\mu}(\theta_{02^\mu},\theta_{0n^\mu}) & \cdots & 0\end{pmatrix}$$
$$=\dot{\mathbf{M}}_{r^\mu}^T\in\mathbb{R}^{n^\mu\times n^\mu}. \tag{17}$$

For the second term of the equality in (16), we put the kinetic energy in the following form

$$K=\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}=K_0+K_1+K_2, \tag{18}$$

where

$$
\begin{aligned}
K_0 &= \frac{1}{2} m_s \dot{\mathbf{p}}_0^T \dot{\mathbf{p}}_0 + \frac{1}{2} J_0 \dot{\theta}_0^2 + \sum_{j=1}^{m} \frac{1}{2} \sum_{i=1}^{n^j} \mathbb{J}_{i^j} \dot{\theta}_{0i^j}^2 \\
K_1 &= \sum_{j=1}^{m} \dot{\mathbf{p}}_0^T \sum_{i=1}^{n^j} \mathbf{m}_{0i^j} \dot{\theta}_{0i^j} \\
K_2 &= \sum_{j=1}^{m} \sum_{l=1}^{n^j-1} \Big(\sum_{i=l+1}^{n^j} m_{li^j} \dot{\theta}_{0l^j} \dot{\theta}_{0i^j} \Big).
\end{aligned}
\tag{19}
$$

Now remember that $\frac{\partial K}{\partial \mathbf{q}} = \frac{\partial K_0}{\partial \mathbf{q}} + \frac{\partial K_1}{\partial \mathbf{q}} + \frac{\partial K_2}{\partial \mathbf{q}}$. It is clear that $\frac{\partial K_0}{\partial \mathbf{q}} = \mathbf{0}_{(n+3)\times 1}$. Moreover notice the following equality

$$
\frac{\partial K_1}{\partial \mathbf{q}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{\mathbf{p}}_0^T \dot{\mathbf{m}}_{01^1} \\ \vdots \\ \dot{\mathbf{p}}_0^T \dot{\mathbf{m}}_{0n^1} \\ \vdots \\ \vdots \\ \dot{\mathbf{p}}_0^T \dot{\mathbf{m}}_{01^m} \\ \vdots \\ \dot{\mathbf{p}}_0^T \dot{\mathbf{m}}_{0n^m} \end{bmatrix} = \begin{pmatrix} \dot{\mathbf{M}}_p \\ \dot{\mathbf{M}}_{pr} \end{pmatrix} \dot{\mathbf{q}}_p. \tag{20}
$$

Recalling that

$$
\dot{\mathbf{M}}\dot{\mathbf{q}} = \begin{pmatrix} \dot{\mathbf{M}}_p \\ \dot{\mathbf{M}}_{pr} \end{pmatrix} \dot{\mathbf{q}}_p + \begin{pmatrix} \dot{\mathbf{M}}_{pr}^T \\ \dot{\mathbf{M}}_r \end{pmatrix} \dot{\mathbf{q}}_r, \tag{21}
$$

we have

$$
\begin{aligned}
\mathbf{c} &= \dot{\mathbf{M}}\dot{\mathbf{q}} - \frac{\partial K}{\partial \mathbf{q}} \\
&= \begin{pmatrix} \dot{\mathbf{M}}_{pr}^T \\ \dot{\mathbf{M}}_r \end{pmatrix} \dot{\mathbf{q}}_r - \frac{\partial K_2}{\partial \mathbf{q}} = \begin{bmatrix} \sum_{j=1}^{m} \sum_{i=1}^{n^j} \bar{\mathbf{m}}_{0i^j} \dot{\theta}_{0i^j}^2 \\ 0 \\ \mathbf{c}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r) \end{bmatrix} \in \mathbb{R}^{(3+n)\times 1},
\end{aligned}
\tag{22}
$$

where $\bar{\mathbf{m}}_{0i^j} = \frac{\partial \mathbf{m}_{0i^j}}{\partial \theta_{0i^j}} \in \mathbb{R}^{2\times 1}$; noticing that $\frac{\partial \bar{\mathbf{R}}_*}{\partial \theta_*} = -\mathbf{R}_*$:

$$
\begin{aligned}
\bar{\mathbf{m}}_{0\nu^\mu}(\theta_{0\nu^\mu}) = m_{\nu^\mu} \tilde{\boldsymbol{\alpha}}_{\nu^\mu} + \underbrace{\sum_{i=\nu^\mu+1}^{n^\mu} (m_i + m_{m_i}) \bar{\tilde{\boldsymbol{\alpha}}}_{\nu^\mu}}_{:=0,\quad if\ \ \nu^\mu = n^\mu} &\in \mathbb{R}^{2\times 1}, \\
\nu^\mu &= \{1, ..., n^\mu\} \\
\tilde{\boldsymbol{\alpha}}_{\nu^\mu}(\theta_{0\nu^\mu}) &= -\mathbf{R}(\theta_{0\nu^\mu}) \mathbf{d}_{\nu^\mu} \\
\bar{\tilde{\boldsymbol{\alpha}}}_{\nu^\mu}(\theta_{0\nu^\mu}) &= -\mathbf{R}(\theta_{0\nu^\mu}) \bar{\mathbf{d}}_{\nu^\mu},
\end{aligned}
$$

and $\mathbf{c}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r) \in \mathbb{R}^n$ is the arm side Coriolis forces in the form of

$$
\mathbf{c}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r) = [\mathbf{c}_{r^1}^T(\mathbf{q}_{r^1}\ \dot{\mathbf{q}}_{r^1}) \ \cdots, \mathbf{c}_{r^m}^T(\mathbf{q}_{r^m}\ \dot{\mathbf{q}}_{r^m})]^T \in \mathbb{R}^n.
$$

Now, from (19) we can write $K_2 = \sum_{j=1}^{m} K_{2j}$ and for the μ-th manipulator it is $K_{2^\mu} = \frac{1}{2} \dot{\mathbf{q}}_{r^\mu}^T \mathbf{B}_{r^\mu} \dot{\mathbf{q}}_{r^\mu}$, where

$$
\begin{aligned}
\mathbf{B}_{r^\mu} &= \mathbf{M}_{r^\mu} - \mathrm{diag}\{\mathbb{J}_{1^\mu}, \mathbb{J}_{2^\mu}, \cdots, \mathbb{J}_{\mathrm{n}^\mu}\} \\
&= \begin{pmatrix} 0 & * & \cdots & * \\ m_{12^\mu}(\theta_{01^\mu}, \theta_{02^\mu}) & 0 & \cdots & * \\ \vdots & \vdots & \ddots & * \\ m_{1n^\mu}(\theta_{01^\mu}, \theta_{0n^\mu}) & m_{2n^\mu}(\theta_{02^\mu}, \theta_{0n^\mu}) & \cdots & 0 \end{pmatrix} \\
&= \mathbf{B}_{r^\mu}^T \in \mathbb{R}^{n^\mu \times n^\mu}.
\end{aligned}
$$

Then we can write for the μ-th component of $\mathbf{c}_r$

$$
\begin{aligned}
\mathbf{c}_{r^\mu}(\mathbf{q}_{r^\mu}, \dot{\mathbf{q}}_{r^\mu}) &= \dot{\mathbf{M}}_{r^\mu} \dot{\mathbf{q}}_{r^\mu} - \frac{\partial K_{2^\mu}}{\partial \mathbf{q}_{r^\mu}} \\
&= \dot{\mathbf{M}}_{r^\mu} \dot{\mathbf{q}}_{r^\mu} - \frac{1}{2} \frac{\partial \left(\dot{\mathbf{q}}_{r^\mu}^T \mathbf{B}_{r^\mu} \dot{\mathbf{q}}_{r^\mu} \right)}{\partial \mathbf{q}_{r^\mu}},
\end{aligned}
\tag{23}
$$

where $\dot{\mathbf{M}}_{r^\mu}$ is available from (17). Then we computed the followings

$$
\dot{\mathbf{M}}_{r^\mu} \dot{\mathbf{q}}_{r^\mu} = \begin{bmatrix} 0 + \sum_{i=2}^{n^\mu} \dot{m}_{1i^\mu} \dot{\theta}_{0i^\mu} \\ \dot{m}_{12^\mu} \dot{\theta}_{01^\mu} + \sum_{i=3}^{n^\mu} \dot{m}_{2i^\mu} \dot{\theta}_{0i^\mu} \\ \dot{m}_{13^\mu} \dot{\theta}_{01^\mu} + \dot{m}_{23^\mu} \dot{\theta}_{02^\mu} + \sum_{i=4}^{n^\mu} \dot{m}_{3i^\mu} \dot{\theta}_{0i^\mu} \\ \vdots \\ \sum_{i=1}^{n^\mu-1} \dot{m}_{in^\mu} \dot{\theta}_{0i^\mu} + 0 \end{bmatrix}, \tag{24}
$$

and

$$
\frac{\partial K_{2^\mu}}{\partial \mathbf{q}_{r^\mu}} = \begin{bmatrix} 0 + \sum_{i=2}^{n^\mu} {}^1\bar{m}_{1i^\mu} \dot{\theta}_{0i^\mu} \dot{\theta}_{01^\mu} \\ \left({}^2\bar{m}_{12^\mu} \dot{\theta}_{01^\mu} + \sum_{i=3}^{n^\mu} {}^2\bar{m}_{2i^\mu} \dot{\theta}_{0i^\mu} \right) \dot{\theta}_{02^\mu} \\ \left({}^3\bar{m}_{13^\mu} \dot{\theta}_{01^\mu} + {}^3\bar{m}_{23^\mu} \dot{\theta}_{02^\mu} + \sum_{i=4}^{n^\mu} {}^3\bar{m}_{3i^\mu} \dot{\theta}_{0i^\mu} \right) \dot{\theta}_{03^\mu} \\ \vdots \\ \sum_{i=1}^{n^\mu-1} {}^{n^\mu}\bar{m}_{in^\mu} \dot{\theta}_{0i^\mu} \dot{\theta}_{0n^\mu} + 0 \end{bmatrix}, \tag{25}
$$

where ${}^k\bar{m}_{kl} = \frac{\partial m_{kl}(\theta_{0k}, \theta_{0l})}{\partial \theta_{0k}}$ (note that here $k = \nu^\mu$ and $l = \xi^\mu$), and

$$
\begin{aligned}
{}^k\bar{m}_{kl}(\theta_{0k}, \theta_{0l}) &= m_l \bar{\tilde{\boldsymbol{\alpha}}}_k^T \boldsymbol{\alpha}_l + \underbrace{\sum_{i=l+1}^{n} (m_i + m_{m_i}) \bar{\tilde{\boldsymbol{\alpha}}}_k^T \bar{\boldsymbol{\alpha}}_l}_{:=0,\quad if\ \ l=n} \qquad \text{if } k < l \\
{}^k\bar{m}_{kl}(\theta_{0k}, \theta_{0l}) &= m_k \boldsymbol{\alpha}_l^T \tilde{\boldsymbol{\alpha}}_k + \underbrace{\sum_{i=k+1}^{n} (m_i + m_{m_i}) \bar{\boldsymbol{\alpha}}_l^T \bar{\tilde{\boldsymbol{\alpha}}}_k}_{:=0,\quad if\ \ k=n} \qquad \text{if } k > l \\
\boldsymbol{\alpha}_k(\theta_{0k}) &= \bar{\mathbf{R}}(\theta_{0k}) \mathbf{d}_k \\
\tilde{\boldsymbol{\alpha}}_k(\theta_{0k}) &= -\mathbf{R}(\theta_{0k}) \mathbf{d}_k \\
\bar{\boldsymbol{\alpha}}_k(\theta_{0k}) &= \bar{\mathbf{R}}(\theta_{0k}) \bar{\mathbf{d}}_k \\
\bar{\tilde{\boldsymbol{\alpha}}}_k(\theta_{0k}) &= -\mathbf{R}(\theta_{0k}) \bar{\mathbf{d}}_k,
\end{aligned}
\tag{26}
$$

with $\bar{\mathbf{R}}_* = \frac{\partial \mathbf{R}_*}{\partial \theta_*}$ and we have used $\frac{\partial \bar{\mathbf{R}}_*}{\partial \theta_*} = -\mathbf{R}_*$. Now utilizing (24) and (25) in (23), one can write the ν-th element of

$\mathbf{c}_{r^\mu}$ as

$$c_{r_{\nu^\mu}} = \sum_{i=1}^{\nu^\mu-1} \left(\dot{m}_{i\nu^\mu} - {}^{\nu^\mu}\bar{m}_{i\nu^\mu}\,\dot{\theta}_{0\nu^\mu}\right)\dot{\theta}_{0i} + \\ + \sum_{i=\nu^\mu+1}^{n} \left(\dot{m}_{\nu^\mu i} - {}^{\nu^\mu}\bar{m}_{\nu^\mu i}\dot{\theta}_{0\nu^\mu}\right)\dot{\theta}_{0i},$$

which is equivalent to

$$\begin{aligned} c_{r_{\nu^\mu}} &= \sum_{i=1\ldots n^\nu,\ i\neq\nu^\mu} \left(\dot{m}_{i\nu^\mu} - {}^{\nu^\mu}\bar{m}_{i\nu^\mu}\,\dot{\theta}_{0\nu^\mu}\right)\dot{\theta}_{0i} \\ &= \sum_{i=1\ldots n^\mu,\ i\neq\nu^\mu} {}^{i}\bar{m}_{i\nu^\mu}\,\dot{\theta}_{0i}^2. \end{aligned} \tag{27}$$

This actually means that due to the A.2 of [1] there are no Coriolis forces appearing from the motion of one arm to another. Moreover, since we choose the absolute orientations as the generalized coordinates, $c_{r_{\nu^\mu}}$ contains only the centrifugal terms.

D. Control Input Matrix

In the paper, [1], we have presented the generalized body-fixed forces $\mathbf{f} = \mathbf{G}\mathbf{u}$, which appear in Equation (14) of this technical report. The corresponding control input matrix is

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_p & \mathbf{0}_{2\times n} \\ \mathbf{G}_d & \mathbf{G}_r \end{pmatrix} \in \mathbb{R}^{(n+3)\times(n+2)}, \mathbf{G}_p = \begin{pmatrix} -\sin(\theta_0) & 0 \\ -\cos(\theta_0) & 0 \end{pmatrix}$$

$$\mathbf{G}_d = \begin{pmatrix} d_{G_x} & 1 \\ \mathbf{0}_{n\times 1} & \mathbf{0}_{n\times 1} \end{pmatrix} \in \mathbb{R}^{(n+1)\times 2}, \mathbf{G}_r = \begin{pmatrix} \mathbf{G}_{rp} \\ \mathbf{G}_{rr} \end{pmatrix} \in \mathbb{R}^{(n+1)\times n},$$

where

$$\mathbf{G}_{rp} = \begin{bmatrix} -1 & \mathbf{0}_{1\times(n^1-1)} & \cdots & -1 & \mathbf{0}_{1\times(n^m-1)} \end{bmatrix} \in \mathbb{R}^{1\times n},$$

$$\mathbf{G}_{rr} = \begin{pmatrix} \mathbf{G}_{rr^1} & \cdots & * \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{G}_{rr^m} \end{pmatrix} \in \mathbb{R}^{n\times n},$$

$$\mathbf{G}_{rr^\mu} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \in \mathbb{R}^{n^\mu\times n^\mu}.$$

E. Clarifying Examples for Case E:

For the Case E of [1] we have defined and used sets for generating the system dynamics, based on the one we did for Case R. There, we have defined the matrices $\mathbf{S}_{N^\mu} \in \mathbb{R}^{n^\mu\times n^\mu}$ and $\mathbf{S}_{K^\mu} \in \mathbb{R}^{k^\mu\times n^\mu}$. Let us fix the idea by giving the following examples:

Example 4. *Say that* $N^\mu = \{1,2,3,4\}$ *and* $K^\mu = \{2,4\}$. *Define a vector, say* $\mathbf{v}^\mu = [v_{1^\mu}\ v_{2^\mu}\ v_{3^\mu}\ v_{4^\mu}]^T \subset N^\mu$. *Then the matrix*

$$\mathbf{S}_{N^\mu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{S}_{N^\mu}^T \in \mathbb{R}^{n^\mu\times n^\mu}$$

Parameters	Notation	Value	Unit
PVTOL mass	m_0	1	kg
PVTOL inertia	J_0	0.015	kg m^2
dis. vec. betw. P_{C_0} & P_G	$\mathbf{d}_G$	$[0\ 0.1]^T$	m
partial distance of one link	$\mathbf{d}_{\nu^\mu}$	$[0\ 0.1]^T$	m
partial distance of one link	$\tilde{\mathbf{d}}_{\nu^\mu}$	$[0\ 0.1]^T$	m
mass of one link	m_{ν^μ}	0.2	kg
inertia of one link	J_{ν^μ}	0.0007	kg m^2
mass of one motor	$m_{m_{\nu^\mu}}$	0.05	kg
inertia of one motor	$J_{m_{\nu^\mu}}$	0.0003	kg m^2
parametric deviations	δ	2	%
3-sigma Gauss. noise in pos.	$3\sigma_p$	0.001	m
3-sigma Gauss. noise in vel.	$3\sigma_v$	0.005	m/s
3-sigma Gauss. noise in θ_0.	$3\sigma_0$	0.01	rad
3-sigma Gauss. noise in $\dot{\theta}_0$.	$3\sigma_{d0}$	0.1	rad/s
3-sigma Gauss. noise in $\mathbf{q}_r$.	$3\sigma_r$	0.001	rad
3-sigma Gauss. noise in $\dot{\mathbf{q}}_r$.	$3\sigma_{dr}$	0.005	rad/s
Sampling of lin. pos. and vel.	-	100	Hz
Sampling of angle and ang. vel.	-	500	Hz

TABLE I: Summary of the parameters used in the simulations. The parameters employed by the controller are all subject to a random parametric deviation within $\pm 2\%$.

will select the 2*-nd and* 4*-th components of the vector* $\mathbf{v}^\mu$, *using* $\bar{\mathbf{v}}^\mu = \mathbf{S}_{N^\mu}\mathbf{v} = [0\ v_{2^\mu}\ 0\ v_{4^\mu}]^T \subset N^\mu$.

Example 5. *Similar to the same example as above, let's say* $N^\mu = \{1,2,3,4\}$ *and* $K^\mu = \{2,4\}$. *Define a vector, say* $\mathbf{v}^\mu = [v_{1^\mu}\ v_{2^\mu}\ v_{3^\mu}\ v_{4^\mu}]^T \subset N^\mu$. *Then the matrix*

$$\mathbf{S}_{K^\mu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{k^\mu\times n^\mu}$$

can be used to select 2*-nd and* 4*-th elements of* $\mathbf{v}^\mu$ *and put them in order, using* $\bar{\mathbf{v}}^\mu = \mathbf{S}_{K^\mu}\mathbf{v}^\mu = [v_{2^\mu}\ v_{4^\mu}]^T \subset K^\mu$.

Example 6. *Say,* $N^\mu = \{1,2,3,4,5,6,7\}$ *and* $K^\mu = \{1,5,7\}$. *It means that the* μ*-th manipulator has 7 DoF and three of the motors are connected to their links via some elastic elements, which are the first, fifth and the seventh ones. Then we say* $\{\kappa^\mu = 3\} = 7$, *is the third elastically connected motor, which corresponds to the seventh joint/motor of the* μ*-th robotic arm. So, we can say e.g. its absolute motor orientation is* $\theta_{m_{0\{\kappa^\mu=3\}}} = \theta_{m_{07^\mu}}$.

III. Details of the Simulations

The system parameters and the deviations used in the simulations are given in Table I. Moreover, additional simulation results are given in in Fig 1, which is also provided in the video attachment of [1]. There, a grasping task is chosen for the robot. This is divided in 5 phases: *i)* PVTOL+2 arm robot follows a desired trajectory on the plane *ii)* The arm configuration is changed *iii)* Another trajectory is followed with the latest arm configuration *iv)* The two arms grasp two individual point mass objects, with no information on their mass for the controller side (each are 0.25kg), *v)* The robot flies away with the unknown object. In this simulation, 2% of parametric uncertainty for mass, inertia and distance parameters is used, while no noise is added to the measurements. A more composite simulation result with parametric uncertainty, noise and the quantizations in the measurements can be found in [1].

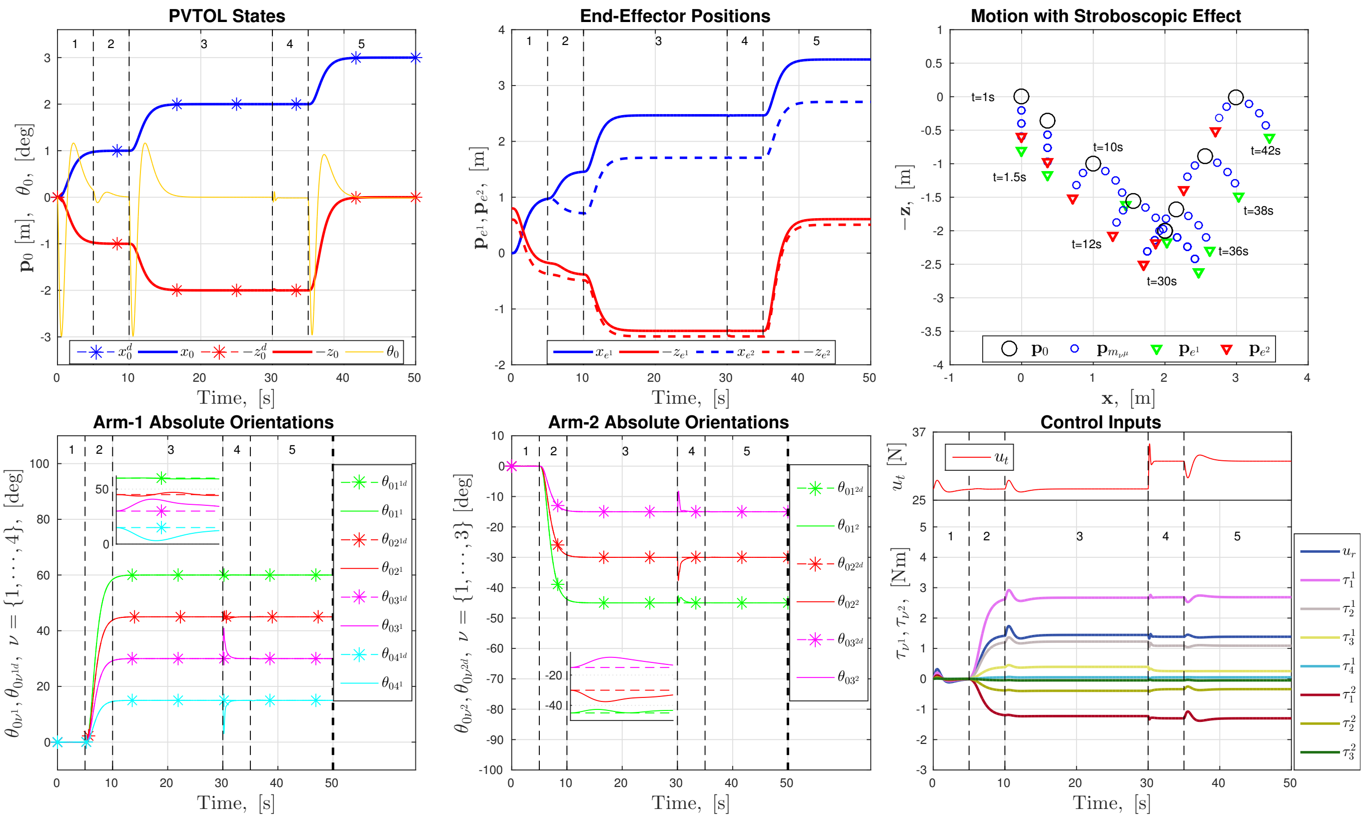

Fig. 1: Simulation results of grasping two individual objects using PVTOL+2 manipulator arms. Notice that each plot (except the third one of the first row) is divided into five different parts. Each part belongs to a different task, as also shown in the video attachment of [1]. The first plot on the first row depicts the PVTOL CoM position and orientation, where the desired trajectories are shown with dashed (starred) curves. The second figure on the first row shows the two end-effector positions of the robot while it is performing the whole task. Notice that the effect of the unknown grasped masses on the end-effector positions is negligible. The third figure is showing the robot configurations at different time instants, where the black circle stands for the PVTOL CoM, the blue circles are the motor/joint positions of the arms, and the triangles are for the end-effectors of the arms, given with different colors. In the second row, the first two plots are showing the absolute link orientations of different robotic arms, where the desired trajectories are given with dashed (starred) curves. We also give here the magnified version of the individual plots, at the time of grasping. Finally, the last plot shows the control inputs of the system, where thrust input is given on the top, separately.

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