Rigid Networks for Feasible Collaboration and a Taxonomy of Interconnected Systems

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I. Abstract

Collaboration manifests in many ways for multi-agent systems; communication, sensing, indirectly through motion or environmental precepts, physical interaction (end manipulators), higher level notions such as packet routing, and through vehicle dynamics in physically coupled systems (distributed flight arrays, collective transport). Central to the success of distributed systems is the modeling of coordinated objectives and the derivation of control or decision schemes that rely only on limited local information and/or interaction. While the range of research challenges in characterizing collaboration is vast, an underlying thread is the reliance on *topology*, i.e., a necessity for interaction and information exchange to achieve cohesion.

A relatively under-explored topological property in the area of multi-robot systems, *rigidity* has important implications particularly for mission objectives requiring collaboration. For example, its relevance is clear in the context of controlling formations of mobile nodes when only relative sensing information is available [1], [2]. Specifically, the asymptotic stability of a formation is guaranteed when the graph that defines the formation is rigid by construction. Thus, in achieving or maintaining a network's rigidity, it becomes possible to extend the traditional pre-defined formation methodology, to that of *dynamic* formations, precisely as rigidity guarantees correctness over time. The idea of formation *persistence*, i.e., the ability of a formation to remain stable in the face of external perturbation, is also supported by rigidity. As demonstrated for example in [3], minimally persistent co-leader formations are achieved if certain properties on the minors of the rigidity matrix are maintained. Rigidity becomes a necessary (and in certain settings sufficient) condition for localization tasks with distance or bearing-only measurements [4]. The ability of a network to self-localize is of clear importance across various application contexts, and for example in [4] it is shown that if the rigidity conditions for localizability for traditional noiseless systems are satisfied, and measurement errors are small enough, then the network will be approximately localizable, providing a connection between robustness and rigidity. Finally, the idea of *global rigidity* [5] further strengthens the guarantees of formation stability and localizability, as the uniqueness of a given topological embedding is more easily characterized. It is clear then that network rigidity acts as a fundamental precursor to both important spatial behaviors and information-driven objectives, making it a strong motivation in terms of robotic control. We point out that it is typical in the literature to *assume* rigidity properties of a robotic network in order to achieve multi-agent behaviors, however few works provide means of evaluating or achieving network rigidity in a dynamic manner.

In our contribution, we will discuss our current work in decentralized rigidity evaluation and control, with a particular emphasis on achieving feasibility in multi-robot collaboration. To this end, we will demonstrate a decentralization of an algorithm that determines in $O(n^2)$ time the combinatorial rigidity of a network, and a spanning edge set defining the minimally rigid subcomponent of the graph. Specifically, we propose a leader election procedure based on distributed auctions that manages the sequential nature of a pebble game algorithm in a decentralized setting. Further, an asynchronous messaging scheme preserves local-only agent interaction, as well as robustness to delays, failures, etc. Next, by assuming a generalized node/edge metric for measuring network performance, we show how to find the *optimal* rigid subgraph embedded in a multi-robot network. Such an optimization is attractive as the resulting graph not only holds the guarantees associated with rigidity, but also considers network cost, e.g., for localizable and edge optimal sensor embeddings or mobile networks. Finally, we demonstrate the integration of spatial topology control, with our methods for evaluating network rigidity, to achieve rigidity control in a multi-robot team. Requiring rigidity control only during *transitions* in network topology, our solution yields guaranteed generic rigidity (with direct applicability, as in [4]), infinitesimal rigidity in *almost all* team configurations, and by-construction complexity and robustness advantages due to non-continuous operation.

To connect with the focus of the sister workshop *On the centrality of decentralization in multi-robot systems: holy grail or false idol?*, we also would like to suggest a high-level direction for multi-robot research. The study of interconnected systems is remarkably complex and highly susceptible to fragmentation especially due to the diversity of the research communities involved, ranging from computer science to automation. Both a high level view of the fundamental topics that drive interconnected systems, and a fine-grained understanding of each topic is required to truly make progress in the field, and to provide an accessible starting point to new research. An effective approach to attain such goals would be to construct a taxonomy of interconnected systems.

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For example, in studying the topological underpinnings of collaboration, we can take a taxonomic approach by characterizing the need for topology into two arenas: spatial and information-driven collaborations. In terms of spatial collaboration, we can cite aggregative or swarming behaviors, area coverage, flocking, leader-following, formation and shape control, and rendezvous as important objectives where cohesion is achieved in team motion. Informational collaboration which acts on sensed or communicated system state, includes as examples consensus and agreement, joint estimation and filtering, task assignment, localization, and cooperative inference; objectives which form the foundation of *intelligence* in collaboration. These behaviors we believe act as building blocks which may form the basis of increasingly complex multi-agent systems and joint objectives. In studying these building blocks, we hope to identify the theoretical and application-driven novelties for each, and specifically the topological shortcomings or assumptions, which bridge theory and application. Methodically addressing subareas at the forefront of interconnected system research in this way may lead towards an understanding of the current open problems in each subarea, the relationships (bridges) between subareas, ultimately yielding a roadmap for new researchers connecting theory and application. Therefore, we hope that such a taxonomic view will facilitate useful multi-robot behaviors in practice, and importantly, the applications under which such assumptions can be achieved. While we aim to focus on our recent rigidity work, we will engage participants in high-level discussions of the notion of a taxonomic approach to multi-robot research.

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