Snapshot Spectral Sudoku

Intern proposal
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1 Problem

Many logical puzzles can be represented as constraint satisfaction problems (CSPs), whereby a number of variables must be chosen within defined constraints to find the solution. A good example is a sudoku, where the numbers in a grid must be chosen so that there are no repeating numbers in each row, column and region.

A similar type of CSP is represented in Figure 1a. We have a 2D grid which has been divided into regions, where the boundaries themselves are squares in the grid. The problem is to assign a number from 1 to $N$ to each pixel so that (1) each region contains all numbers and (2) the numbers in each row are entered in ascending order, repeating after reaching $N$. The numbers in the boundary squares can be ignored. In the partial solution of Figure 1a, the second constraint is fulfilled for the first 4 rows, however the first constraint is violated in the top-right green region where both 5 and 6 are absent. Finding a solution for a given grid size, region size and shape, and $N$ is a non-trivial, highly combinatorial, problem.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{a) Illustration of the CSP, where $N = 6$, b) and c): real imagery that define such a CSP (RGB view of a scene and grayscale image, segmented into regions of assumed identical spectra).}
\end{figure}

2 Context

Whilst seemingly an abstract puzzle, this problem is actually highly applicable to the real world problem of snapshot spectral imaging. Spectral imagers measure the spectrum of every pixel in a 2D scene, providing many more spectral bands than a RGB camera, and therefore
proving useful for a wide range of applications. However, the necessity of measuring the 3D information on a 2D sensor requires multiple acquisitions, limiting the frame rate of such systems.

Now let’s assume that a 2D image, for example Figure 1b, can be divided into homogeneous regions of identical spectra using the gray-scale intensity, see Figure 1c. In this case, to obtain the full spectral data for the scene, one only needs to measure the spectrum for each region, instead of for each pixel. This assumption could allow near-snapshot spectral imaging, although the implementation depends on the instrument itself, as the underlying physics of the spectral imager constrains how the spectrum of each pixel can be measured. In fact, finding the best measurement configuration for our prototype system [1] requires solving the CSP defined previously, where each number represents a wavelength band that will be measured in the given pixel. Having all numbers (i.e. bands) represented in a region ensures the full spectrum of the homogeneous region will be measured in a single acquisition.

3 Project

The objective of this internship is to model and solve the problem of configuring the spectral imager so that the information gained at each acquisition is maximized. The candidate will use the physical constraints imposed by the spectral imaging system to define the CSP mathematically, and develop suitable criteria corresponding to real-world requirements. A solver will be developed and tested on model data, and the project can be extended to consider realistic issues such as signal-to-noise ratio, blurring and optical distortion, before being trialled on the prototype system.

4 Required skills and application

The candidate should have a good knowledge on optimization techniques, an interest for algorithms and programming in general. Fluent english is mandatory. Applications (CV, letter of motivation and references if any) should be sent to both Arthur Bit-Monnot and Lizzy Hemsley. The intern will receive a gratification of about 550 Euros per month.